International Journal of Mathematical Archive-3(6), 2012, 2369-2374
Available online through www.ijma.info ISSN 2229 – 5046

SOME SPECIAL TYPES OF SQUARE DIFFERENCE GRAPHS

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(Received on: 12-06-12; Revised & Accepted on: 30-06-12)

ABSTRACT

***I** defined a new labeling and a new graph called square difference labeling and the square difference graph. Let G be a (p, q) graph. G is said to be a square difference graph if there exists a bijection f: V(G) $\rightarrow$ { 0,1, …., p-1} such that the induced function f* : E(G) $\rightarrow$ N given by f*(uv) =| $[f(u)]^2$ - $[f(v)]^2$| for every uv $\in$ E(G) are all distinct. A graph which admits square difference labeling is called square difference graph. I discussed the square difference labeling for some graphs like cycles, complete graphs, cycle cactus, ladder, lattice grids and wheels. In this paper my discussion is some graphs like mK_3, mC_n, path union of some K_3, some C_n graphs and duplication of vertices by an edge to some star graphs ans crown graphs are square difference graphs. Also proved a path is an Odd square difference graph and star is a perfect square difference graph.*

Keywords: *Square difference labeling, square difference graph, Odd square difference graph, Perfect square graph, Path union of C_n, the comb graph.*

INTRODUCTION

We begin with simple, finite, connected and undirected graph G = (V (G), E (G)) with p vertices and q edges. For all other terminology and notations I follow Harary [2]. Here brief summary of definitions which is used for the present investigations are given.

If the vertices of the graph are assigned values subject to certain conditions is known as **graph labeling.** A dynamic survey on graph labeling is regularly updated by Gallian [3] and it is published by Electronic Journal of Combinatory. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades.

Previously K.A. Germina, S. Arumugam and V. Ajitha defined Square sum labeling [1] and myself proved square difference labeling for some graphs [9] and also some references [5], [6] and [7] are used. Some concepts are used from B. West [8] and Gary Chartrand, Ping Zhang [4]

Definition 1.1: Let G = (V (G), E (G)) be a graph .G is said to be square difference labeling if there exist a bijection f: V(G)$\rightarrow${0,1,2,….p-1} such that the induced function f^*:E(G) $\rightarrow$ N given by

$$f^*(u\,v) = |[f(u)]^2 - [f(v)]^2| \text{ is injective.}$$

Definition 1.2: A square difference graph with weights as odd numbers are called Odd square difference graph.

Definition 1.3: Let G_1, G_2 … G_n, n $\geq$ 2 be n copies of a fixed graph G. The graph G obtained by adding an edge between G_i and G_{i+1} for i = 1, 2… n-1 is called path union of G.

Definition 1.4: A square difference graph with weights as perfect square numbers are called Perfect square difference graph.

Definition 1.5: Let G be a graph with fixed vertex v. The comb of G is the graph (P_m ; G) obtained from m copies of G and the path P_m : u_1,u_2, …, u_n by joining u_i with the vertex v of the i^{th} copy of G by means of an edge, for 1$\leq$ i$\leq$ m.

Definition 1.6: Duplication of a vertex v_k by an edge e = v_k'v_k" in a graph G produces a new graph G_1 such that N(v_k') $\cap$N(v_k") = v_k

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2. MAIN RESULTS

Theorem 2.1: The path P_n is an Odd square difference graph.

Proof: Let the graph G be a path P_n. Let |V (G)| = n and |E (G)| = n-1.The mapping f: V (G) → {0, 1, …., n-1} is defined by $f(u_i) = i - 1$, $1 \leq i \leq n - 1$ and the induced function f^*: E(G) → N defined by $f^*(xy) = |[f(x)]^2 - [f(y)]^2|$. The edges of the path have distinct values in increasing order hence a one to one mapping. In this labeling one interesting thing is the weights are odd numbers, starting from 1 it goes on increasing as 1, 3, 5, … 2n – 3. Hence this graph can also be called as Odd square difference graph.

Example: 2.2: The path P_4 is an Odd square difference graph.

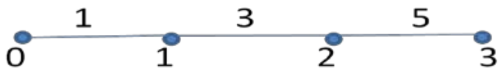

Fig-1: Odd square difference graph of the path P_4

Example: 2.3: The path P_5 is an Odd square difference graph.

Fig-2: Odd square difference graph of the path P_5

Theorem 2.4: Star graphs$K_{1,n}$ are perfect square difference graphs.

Proof: Let $v_1, v_2, \ldots, v_n$ be the pendent vertices of the star graph $K_{1,n}$ and c be the apex vertex Let |V (G)| = n +1 and |E (G)| = n. The mapping f: V (G) → {0, 1, …., n} is defined by $f(u_i) = i$ $1 \leq i \leq n$ and $f(c) = 0$. $1 \leq i \leq n - 1$ and the induced function f^*: E(G) → N defined by $f^*(xy) = |[f(x)]^2 - [f(y)]^2|$ is an increasing function and also one to one. The weights of the edges are distinct also perfect squares as 1,4, 9, 16, … ,n^2. Hence the Star graphs are perfect square difference graphs.

Example: 2.5: The Star graphs$K_{1,5}$ is a perfect square difference graph.

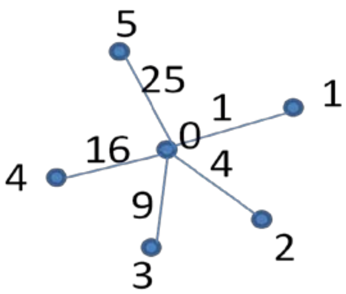

Fig-3: perfect square difference graph of the Star graphs$K_{1,5}$

Example: 2.6: The Star graphs$K_{1,6}$ is a perfect square difference graph.

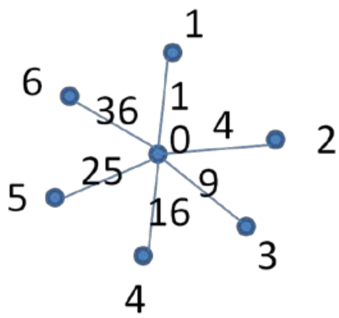

Fig-4: perfect square difference graph of the Star graphs$K_{1,6}$

Theorem 2.6: mK_3 is a square difference graph.

Proof: Let $G_1, G_2, \dots, G_n$ be m copies of K_3. Let v_1, v_2, v_3 be the vertices of G_1, v_4, v_5, v_6 be the vertices of G_2, v_7, v_8, v_9 be the vertices of G_3 … , $v_{3n-2}, v_{3n-1}, v_{3n}$ be the vertices of G_n. Let $|V(G)| = 3n$, $|E(G)| = 3n$ and let us define $f: V(G) \rightarrow \{0, 1, \dots, 3n-1\}$ by $f(u_i) = i - 1$, $1 \leq i \leq 3n$ and the induced function $f^*: E(G) \rightarrow N$ defined by $f^*(xy) = |[f(x)]^2 - [f(y)]^2|$ is an increasing function, hence f^* is an one to one function. Each edges having distinct weights such that $f^*(e_i) \neq f^*(e_j)$ for every ei $\neq e_j$. Here one interesting factor is in each G_i, i = 1,2, … , n. is the weight of one edge of K_3 is equal to sum of the other two.. Also the weights of one edge of K_3 is (m+ 3) multiple of 4, m> 0. This can be verified in the examples 2.7 and 2.8

Example 2.7: The graph $6K_3$ is a square difference graph.

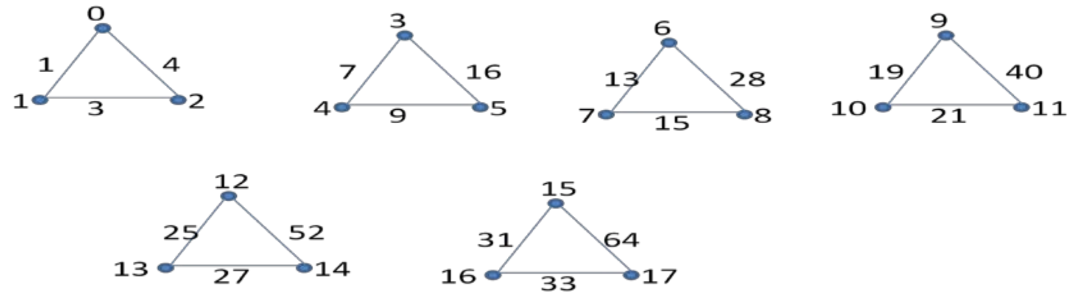

Fig-5: square difference graph of 6K3

Example 2.8: The graph $7K_3$ is a square difference graph.

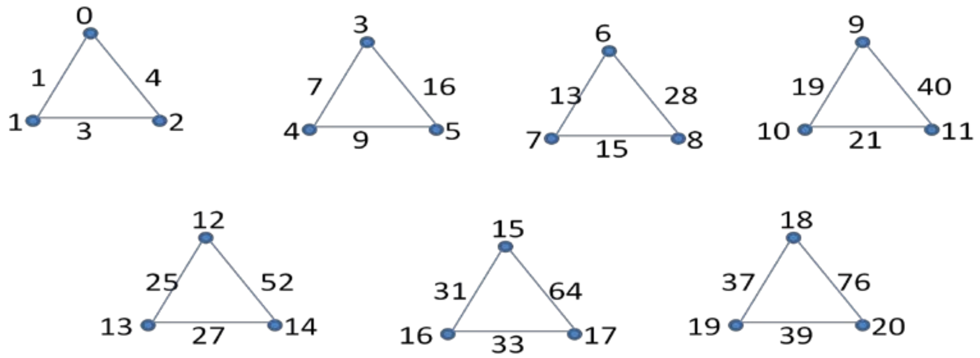

Fig-6: square difference graph of $7K_3$

Theorem 2.9: mC_5 is a square difference graph.

Proof: Let $G_1, G_2, \dots, G_n$ be m copies of C_5. Let v_1, v_2, v_3, v_4, v_5 be the vertices of G_1, $v_6, v_7, v_8, v_9, v_{10}$ be the vertices of G_2, … $v_{mn-4}, v_{mn-3}, v_{mn-2}, v_{mn-1}, v_{mn}$ be the vertices of G_n. Let $|V(G)| = mn$, $|E(G)| = mn$ and let us define $f: V(G) \rightarrow \{0, 1, \dots, mn-1\}$ by $f(u_i) = i - 1$, $1 \leq i \leq mn$ and the induced function $f^*: E(G) \rightarrow N$ defined by $f^*(xy) = |[f(x)]^2 - [f(y)]^2|$ is an increasing function, hence f^* is an one to one function. Each edges having distinct weights such that $f^*(e_i) \neq f^*(e_j)$ for every ei $\neq e_j$. Hence mC_5 is a square difference graph.

Example: 2.10: The graph $4C_5$ is a square difference graph.

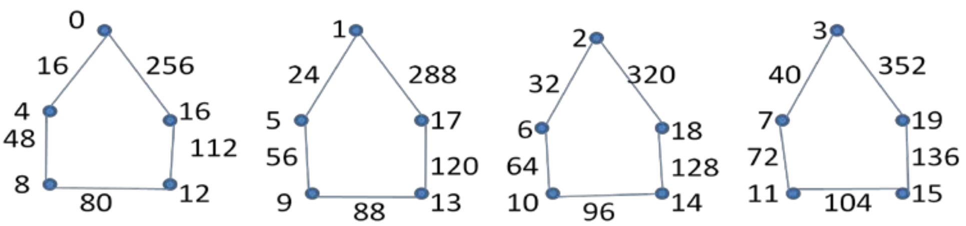

Fig-7: square difference graph of $4C_5$

Theorem 2.11: The comb of the cycle is the graph $(P_m ; C_n)$ is a square difference graph.

Proof: Let $G_1, G_2, \ldots , G_n$ be n copies of C_3 and $P_m : u_1,u_2, \ldots, u_n$ be a path. The comb graph $(P_m; C_n)$ has (m+ mn) vertices and (mn+ 2n+1) edges. Let us define f: V (G) → {0, 1, 2,… (m+ mn) -1} by $f(u_i) = i -1$, $1 \leq i \leq (m+ mn) -1$ such that the induced function $f^*:E(G) \rightarrow N$ given by $f^*(u\ v) = |[f(u)]^2 - [f(v)]^2|$ is injective and the images are distinct. This graph is the combination of the above two graphs one is odd square difference graph and the sum of two edges of C_3 is equal to the third edge. The edges of C_3 and the edge connecting the path and C_3's are the multiples of m+n+1.

Example: 2.10: The graph $(P_4; C_3)$ is a square difference graph.

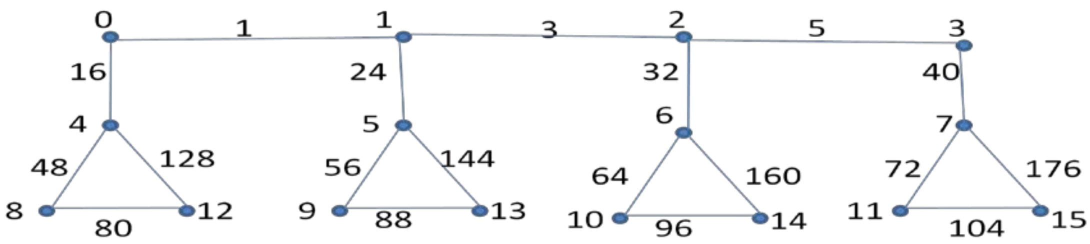

Fig-8: square difference graph $(P_4; C_3)$

Example: 2.11: The graph $(P_4; C_5)$ is a square difference graph.

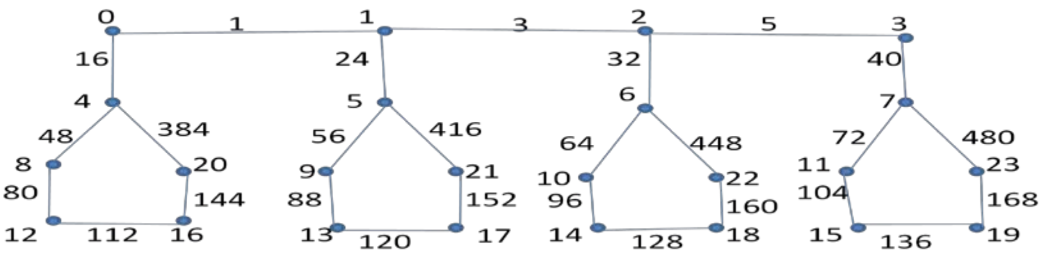

Fig-9: square difference graph $(P_4; C_5)$

Theorem 2.12: The graph obtained by duplication of an arbitrary vertex by a new edge in star $K_{1,n}$ admits a square difference labeling.

Proof: Let Let $v_1, v_2, \ldots , v_n$ be the pendent vertices of the star graph $K_{1,n}$ and c be the apex vertex . Let G be the graph obtained by duplicating an arbitrary vertex v_1 by a new edge $e = v_1' v_1''$. Let $|V(G)| = 3n+1$, $|E(G)| = 4n$ and define f: V(G) → {0, 1, …., 3n+1} by $f(u_i) = i$, $1 \leq i \leq 3n$ and f(c) = 0 and the induced function $f^*: E(G) \rightarrow N$ defined by $f^*(xy) = |[f(x)]^2 - [f(y)]^2|$ is an increasing function. i.e $f^*(e_1) < f^*(e_2) < \ldots < f^*(e_{4n})$. The weights of all the edges of the graph G are distinct. Hence the graph G admits square difference labeling. Therefore G is a square difference graph. This graph is the combination of perfect square difference graph and sum of two sides of K_3 is equal to the third side.

Example: 2.13: The graph obtained by duplication of an arbitrary vertex by a new edge in star $K_{1,5}$ is a square difference graph.

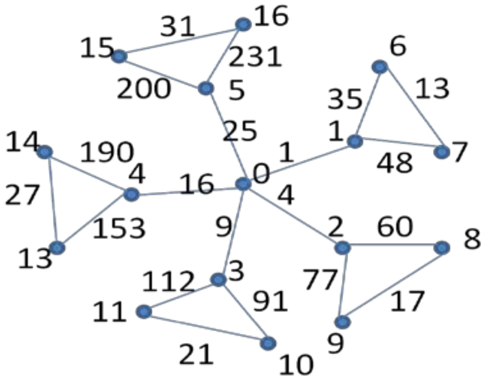

Fig-10: Duplication of a vertex by an edge in a graph $K_{1,5}$ and its square difference labeling

Example: 2.14: The graph obtained by duplication of an arbitrary vertex by a new edge in star $K_{1,4}$ is a square difference graph.

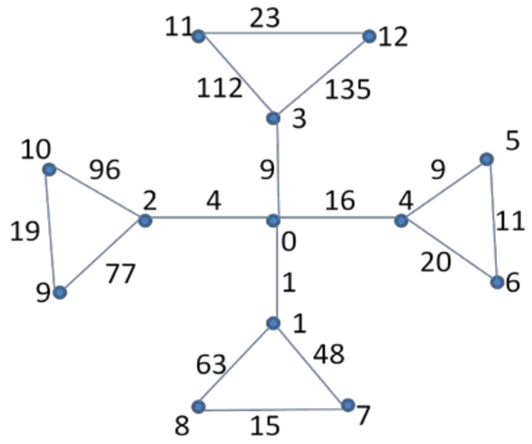

Fig-11: Duplication of a vertex by an edge in a graph $K_{1,4}$ and its square difference labeling

Theorem 2.15: The graph obtained by duplication of an arbitrary vertex by a new edge in a crown graph C_n © K_1 admits a square difference labeling.

Proof: Let Let $v_1, v_2, \dots, v_n$ be the vertices of the crown graph C_n © K_1. Let G be the graph obtained by duplicating an arbitrary vertex v_1 by a new edge $e = v_1' v_1''$. Let $|V(G)| = 4n$, $|E(G)| = 5n$ and define $f: V(G) \rightarrow \{0, 1, \dots, 4n-1\}$ by $f(u_i) = i$, $1 \leq i \leq 4n - 1$ and the induced function $f^*: E(G) \rightarrow N$ defined by $f^*(xy) = |[f(x)]^2 - [f(y)]^2|$ is an increasing function. i.e $f^*(e_1) < f^*(e_2) < \dots < f^*(e_{5n})$. The weights of all the edges of the graph G are distinct. Hence the crown graph C_n © K_1 admits a square difference labeling.

Example: 2.16: The graph obtained by duplication of an arbitrary vertex by a new edge in a crown graph admits a square difference graph.

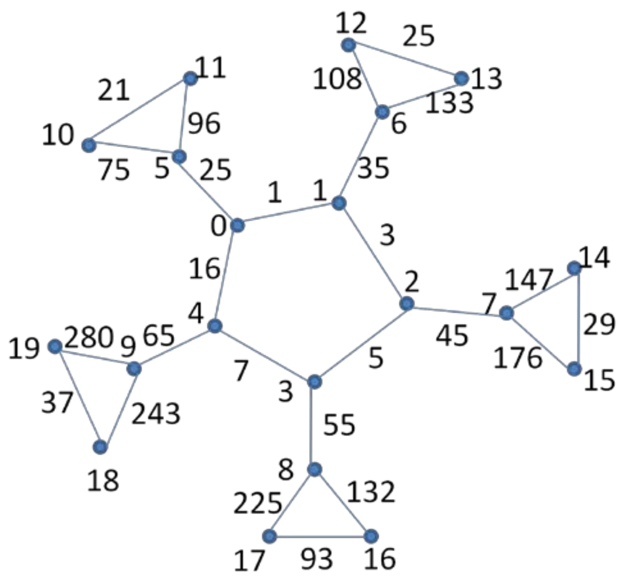

Fig-12: Duplication of a vertex by an edge in a graph C_5© K_1 and its square difference labeling

In this paper some special types of Square difference graphs are discussed. Duplication of a vertex by an edge generates some new graphs. And I proved some graphs generated using duplication concepts are Square difference graphs. Similarly one can verify this Square difference labeling for many more graphs. Some other labeling is also verified for these graphs which I used in this paper.

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Source of support: Nil, Conflict of interest: None Declared

