

EFFECT OF HALL CURRENT AND THERMO-DIFFUSION ON CONVECTIVE HEAT AND MASS TRANSFER FLOW OF A VISCOUS, ROTATING FLUID THROUGH A POROUS MEDIUM PAST A VERTICAL POROUS PLATE

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ABSTRACT

We analyse the combined effects of Hall currents and radiation on the convective heat and mass transfer of a viscous electrically conducting rotating fluid through a porous medium past a vertical porous plate. in the presence of heat generating sources. The equations governing the flow, heat and mass transfer are solved exactly to obtain the velocity, temperature and concentration and are analysed for different variations of the the governing parameters $G, D^{-1}, M, m, N, N_1, Sc, So$ and α . The shear stress and the rate of heat and mass transfer are numerically evaluated for different sets of the parameters. The influence of the Hall currents, thermo-diffusion and radiation effects on the flow, heat and mass transfer characteristics are discussed in detail.

Key Words: Hall Effects, Soret effect, rotating fluid, heat and Mass transfer, vertical plate.

1. INTRODUCTION

Free convection and mass transfer flow in porous medium has received considerable attention due to its numerous applications in geophysics and energy related problems. Such types of applications include natural circulation in isothermal reservoirs, aquifers, porous insulation in heat storage bed, grain storage, extraction of thermal energy and thermal insulation design. Studies associated with flows through porous medium in rotating environment have some relevance in geophysical and geothermal applications. Many aspects of the motion in a rotating frame of reference of terrestrial and planetary atmosphere are influenced by the effects of rotation of the medium. Also buoyancy and rotational effects are often comparable in geophysical processes. Convective transport in a rotating atmosphere over a locally heated surface gives rise to dust storms (typhoons) and other atmosphere circulations.

The unsteady flow of a rotating viscous fluid in a channel maintained by non-torsional oscillations of one or both the boundaries has been studied by several authors to analyse the growth and development of boundary layers associated with geothermal flows for possible applications in geophysical fluid dynamics [2, 3, 4, 6-8, 9, 11]. Later Singh et al [12] studied free convection in Mhd flow of a rotating viscous liquid in porous media. Singh et al [13] have also studied free convective Mhd flow of a rotating viscous fluid in a porous medium past a infinite vertical porous plate. Recently In all these investigations, the effects of Hall currents are not considered. However, in a partially ionized gas, there occurs Hall currents when the strength of the impressed magnetic field is very strong. These Hall effects play a significant role in determining the flow features. Yamanishi [15] have discussed the Hall effects on the steady hydromagnetic flow between two parallel plates. Debnath [3] has studied the effects of Hall currents on unsteady hydromagnetic flow past a porous plate in a rotating fluid system and the structure of the steady and unsteady flow is investigated. Alam et al [1] have studied unsteady free convective heat and mass transfer flow in a rotating system with Hall currents, viscous dissipation and Joule heating. Taking Hall effects into account Krishna et al [5] have investigated Hall effects on the unsteady hydromagnetic boundary layer flow. Rao et al [9] have analysed Hall effects on unsteady hydromagnetic flow. Sivaprasad et al [14] have studied Hall effects on unsteady Mhd free and forced convection flow in a porous rotating channel.

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In this paper we analyse the combined effects of Hall currents and radiation on the convective heat and mass transfer of a viscous electrically conducting rotating fluid through a porous medium past a vertical porous plate. In the presence of heat generating sources. The equations governing the flow, heat and mass transfer are solved exactly to obtain the velocity, temperature and concentration and are analysed for different variations of the governing parameters $G, D^{-1}, M, m, N, N_1, Sc, So$ and α . The shear stress and the rate of heat and mass transfer are numerically evaluated for different sets of the parameters. The influence of the Hall currents, thermo-diffusion and radiation effects on the flow, heat and mass transfer characteristics are discussed in detail.

2. FORMULATION and SOLUTION OF THE PROBLEM

We consider the steady flow of an incompressible, viscous, electrically conducting, rotating fluid through a porous medium bounded by a vertical plate. In the undisturbed state, both the plates and fluid rotate with same angular velocity Ω and are maintained at constant temperature and concentration. Further the plates are cooled or heated by constant temperature gradient in some direction parallel to the plane of the plates. We choose a cartesian co-ordinate system $O(x, y, z)$ such that the plate is at $z=0$ and the z -axis coincide with the axis of rotation of the plates. The steady hydro magnetic boundary layer equations of motion with respect to a rotating frame moving with angular velocity Ω under Boussinesq approximations are

$$-W_0 \frac{\partial u}{\partial z} - 2\Omega v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} + (\mu_e H_0 J_y u) - \left(\frac{\nu}{k}\right)u - \rho \bar{g} \quad (2.1)$$

$$-W_0 \frac{\partial v}{\partial z} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} - (\mu_e H_0 J_x v) - \left(\frac{\nu}{k}\right)v \quad (2.2)$$

The energy equation is

$$\rho_0 C_p (-W_0 \frac{\partial T}{\partial z}) = k_1 \frac{\partial^2 T}{\partial z^2} + Q(T_\infty - T) - \frac{\partial(q_r)}{\partial z} + \mu(u_z^2 + v_z^2) + \frac{\sigma \mu_e^2 H_0^2}{\rho_0} (u^2 + v^2) \quad (2.3)$$

The diffusion equation is

$$p (-W_0 \frac{\partial C}{\partial z}) = D_1 \frac{\partial^2 C}{\partial z^2} + k_{11} \frac{\partial^2 T}{\partial z^2} \quad (2.4)$$

Equation of State is

$$\rho - \rho_\infty = -\beta \rho_\infty (T - T_\infty) - \beta^* \rho_\infty (C - C_\infty) \quad (2.5)$$

where u, v are the velocity components along x and y directions respectively, p is the pressure including the centrifugal force, ρ is the density, ν is the permeability constant, ν is the coefficient of kinematic viscosity, k_1 is the thermal diffusivity, D_1 is the chemical molecular diffusivity, β is the coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with mass fraction, Q is the strength of the heat source, k_{11} is the cross diffusivity and q_r is the radiative heat flux.

When the strength of the magnetic field is very large we include the Hall current so that the generalized Ohm's law is modified to

$$\bar{J} + \omega_e \tau_e \bar{J} \times \bar{H} = \sigma (\bar{E} + \mu_e \bar{q} \times \bar{H}) \quad (2.6)$$

where q is the velocity vector. H is the magnetic field intensity vector. E is the electric field, J is the current density vector, ω_e is the cyclotron frequency, τ_e is the electron

collision time, σ is the fluid conductivity and μ_e is the magnetic permeability.

Neglecting the electron pressure gradient, ion-slip and thermo-electric effects and assuming the electric field $E=0$, equation (2.6) reduces

$$j_x + m j_y = \sigma \mu_e H_0 v \quad (2.7)$$

$$j_y - m j_x = -\sigma \mu_e H_0 u \quad (2.8)$$

where $m = \omega_e \tau_e$ is the Hall parameter.

On solving equations (2.7) & (2.8) we obtain

$$j_x = \frac{\sigma\mu_e H_0}{1+m^2} (v + mu) \quad (2.9)$$

$$j_y = \frac{\sigma\mu_e H_0}{1+m^2} (mv - u) \quad (2.10)$$

By using Rosseland approximation the radiative heat flux is

$$q_r = -\frac{4\sigma^*}{3\beta_R} \frac{\partial(T'^4)}{\partial y} \quad (2.11)$$

It should be observed that by using Rosseland approximation the present analysis is limited to optically thick fluids. We assume that the temperature differences within the flow are sufficiently small so that T'^4 may be expressed as a linear function of the temperature. This is accompanied by expanding T'^4 in a Taylor series about T_∞ and neglecting higher order terms as

$$T'^4 \cong 4T_\infty^3 - 3T_\infty^4 \quad (2.12)$$

where σ^* is Stefan-Boltzman constant and β_R is the mean absorption coefficient. Far away from the plate we have

$$0 = -\frac{\partial p_\infty}{\partial x} - \rho_\infty g \quad (2.13)$$

Using the equations(2.5),(2.12)&(2.13)the equations of motions governing the flow through a porous medium with respect to a rotating frame are given by

$$-W_o \frac{\partial u}{\partial z} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} + \frac{(\sigma\mu_e^2 H_0^2)}{\rho_o(1+m^2)} (mv - u) - \left(\frac{\nu}{k}\right)u + \beta g \rho_\infty (T - T_\infty) + \beta^* g \rho_\infty (C - C_\infty) \quad (2.14)$$

$$-W_o \frac{\partial v}{\partial z} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} - \frac{(\sigma\mu_e^2 H_0^2)}{\rho_o(1+m^2)} (v + mu) - \left(\frac{\nu}{k}\right)v \quad (2.15)$$

Combining (2.14) & (2.15) we get

$$-W_o \frac{\partial q}{\partial z} - 2i\Omega q = -\frac{(\sigma\mu_e^2 H_0^2)}{\rho_o(1+m^2)} (1 - im)q - \left(\frac{\nu}{k}\right)q + \nu \frac{\partial^2 q}{\partial z^2} \quad (2.16)$$

where

$$q = u + iv$$

The non-dimensional variables are

$$z' = \frac{z}{(\nu/W_o)}, \quad q' = \frac{q}{W_o}$$

$$\theta = \frac{(T - T_{1\infty})}{T_w - T_\infty} \quad C = \frac{(C - C_{1\infty})}{C_w - C_\infty}$$

The equations governing the flow, heat and mass transfer are (dropping the dashes)

$$\frac{\partial^2 q}{\partial z^2} + \frac{\partial q}{\partial z} - \lambda^2 q = -G(\theta + \gamma\theta^2 + NC) \quad (2.17)$$

$$\frac{\partial^2 \theta}{\partial z^2} + P_1 \frac{\partial \theta}{\partial z} - \alpha_1 \theta = -P_1 E_c \left(\frac{\partial q}{\partial z} \cdot \frac{\partial q}{\partial z}\right) - P_1 Ec M_1^2 (q \cdot \bar{q}) \quad (2.18)$$

$$\frac{\partial^2 C}{\partial z^2} - Sc \frac{\partial C}{\partial z} - kC = -\frac{ScSo}{N} \frac{\partial^2 \theta}{\partial z^2} \quad (2.19)$$

where

$$\lambda^2 = (D^{-1} + 2iE^{-1} + \frac{M^2(1-im)}{1+m^2}), E = \frac{\nu}{L^2\Omega} \quad (\text{Ekman number})$$

$$D^{-1} = \frac{L^2}{k} \quad (\text{Darcy parameter}), G = \frac{\beta g L^4 (T_w - T_\infty)}{\nu^2} \quad (\text{Grashof number})$$

$$N = \frac{\beta^* (C_w - C_\infty)}{\beta (T_w - T_\infty)} \quad (\text{Buoyancy ratio}), N_1 = \frac{\beta_R k}{4\sigma^* T_e^3} \quad (\text{Radiation parameter})$$

$$P = \frac{\nu}{k} \quad (\text{Prandtl number}), \alpha = \frac{QL^2}{k_1} \quad (\text{Heat source parameter})$$

$$Sc = \frac{\nu}{D_1} \quad (\text{Schmidt number}), So = \frac{k_{11}\beta^*}{\beta\nu} \quad (\text{Soret Number})$$

$$P_1 = \frac{3N_1P}{3N_1 + 4} \quad \alpha_1 = \frac{3N_1\alpha}{3N_1 + 4}$$

The boundary conditions are

$$\begin{aligned} q &= 0 && \text{on } z=0 \\ \theta &= 1, C = 1 && \text{on } z=0 \\ q &\rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 && \text{as } z \rightarrow \infty \end{aligned} \quad (2.20)$$

Solving equations (2.17)-(2.19) subject to the boundary conditions (2.20) we obtain

$$\begin{aligned} q &= (a_7 - a_4) \exp(-m_1 z) + a_4 \exp(-m_2 z) - a_7 \exp(-scz) \\ \theta &= \exp(-m_2 z) \end{aligned}$$

$$C = (1 + a_3) \exp(-scz) - a_3 \exp(-m_2 z)$$

where

$$\lambda^2 = M^2 + D^{-1} + 2iE^{-1} \quad m_1 = \frac{1 + \sqrt{1 + \lambda^2}}{2}$$

$$m_2 = \frac{P_1 + \sqrt{P_1^2 + 4\alpha_1}}{2} \quad a_2 = \frac{ScSo m_2^2}{N}$$

$$a_3 = a_2 - 1 \quad a_4 = G(Na_3 - 1)$$

$$a_5 = GN(1 + a_3) \quad a_6 = \frac{a_4}{m_2^2 - m_2 - \lambda^2}$$

$$a_7 = \frac{a_5}{Sc^2 - Sc - \lambda^2}$$

3. SHEAR STRESS, NUSSELT NUMBER AND SHERWOOD NUMBER

Shear stress on the wall $z=0$ is given by

$$\begin{aligned} \tau &= \left(\frac{dq}{dz} \right)_{z=0} \\ &= (a_7 - a_4)m_1 - a_4 m_2 + a_7 Sc \end{aligned}$$

Local rate of heat transfer across the walls (Nusselt Number) is given by

$$\begin{aligned}(Nu)_{z=0} &= \left(\frac{d\theta}{dz}\right)_{z=0} \\ &= m_2\end{aligned}$$

Rate of mass transfer (Sherwood Number) is given by

$$\begin{aligned}(Sh)_{z=0} &= \left(\frac{dC}{dz}\right)_{z=0} \\ &= a_3 m_2 - (1 + a_3) Sc\end{aligned}$$

4. DISCUSSION OF THE NUMERICAL RESULTS:

The effect of Hall currents and thermo-diffusion on convective Heat and Mass transfer flow of a viscous fluid past a vertical plate is investigated. We take $p = 0.71$ in this analysis.

The velocity component (u) is shown in figs 1-8 for different G , D^{-1} , M , m , E^{-1} , Sc , S_0 , N , N_1 and α . Fig.1 represents the velocity with Grashof number G . It is found that $u > 0$ for $G > 0$ and $u < 0$ for $G < 0$. $|u|$ experiences an enhancement with increase in $|G|$ ($\lesseqgtr 0$). The variation of u with D^{-1} or M shows that lesser the permeability of the porous medium / higher the Lorentz force smaller $|u|$ in the flow region. [An increase in the Hall current parameter (m) enhances $|u|$ in the region]. With respect to variation of u with m shows that the velocity changes from negative to positive as we move away from the plate. The region of transition from negative to positive enhances with increase in m (fig. 2 & 3). From fig.4 we find that u reduces with E^{-1} in the entire flow region. The variation of u with Schmidt number (Sc) shows that lesser the molecular diffusivity smaller $|u|$ in the flow region and for further lowering of the molecular diffusivity the velocity changes from positive to negative as we move away from the wall (fig.5). With respect to solet parameter S_0 we observe that $u > 0$ for $S_0 > 0$ and $u < 0$ for $S_0 < 0$. $|u|$ enhances with increase in $|S_0|$ (<0) and reduces with S_0 (<0) (fig.6). When the molecular buoyancy force dominates over the thermal buoyancy force the velocity depreciates in the flow region irrespective of the directions of the buoyancy forces (fig.7). $|u|$ reduces with radiation parameter N_1 and heat source parameter α (fig.8).

The non-dimensional temperature (θ) is shown in figs 9 & 10 for different α and N_1 . The temperature gradually reduces from its prescribed value 1 on $y = 0$ and attains the value '0' for away from the wall. An increase in the heat source parameter α reduces the temperature. Also the temperature experiences a depreciation with increase in the radiation parameter N_1 (fig.9).

The concentration distribution (C) is exhibited in fig 10 -13 for different Sc , S_0 , N and N_1 . From fig. 10 we find that lesser the molecular diffusivity smaller the concentration in the flow region. An increase in $S_0 > 0$ enhances the actual concentration while an increase in $S_0 < 0$ leads to depreciation in the actual concentration (fig.11). When the molecular buoyancy force dominates over the thermal buoyancy force the actual concentration depreciates when the buoyancy

forces act in the same direction while for the forces acting in opposite directions it experiences an enhancement in the flow region. An increase in the radiation parameter enhances u in the neighborhood of the wall and it depreciates for away from the wall (fig.13).

The shear stress at $y = 0$ is shown in tables 1 & 2 for different G , D^{-1} , M , m , N and N_1 . It is found that the shear stress enhances with increase in G . Lesser the permeability of the porous medium / higher the Lorentz force larger $|Sh|$ at $y = 0$. An increase in the Hall parameter m reduces $|\tau|$ at the wall. Also it experiences an enhancement with increase in D^{-1} (table 1). From table 2, we find that lesser the molecular diffusivity smaller $|\tau|$ at $y = 0$ and for further lowering of the molecular diffusivity larger $|\tau|$. The variation of τ with solet parameter S_0 shows that $|\tau|$ enhances with $|S_0|$ ($\lesseqgtr 0$). Also it enhances with $N > 0$ and reduces with increase in $|N|$ (<0) An increase in N_1 or α leads to a depreciation in $|\tau|$ (table 3).

The Nusselt number (Nu) at the wall $y = 0$ is shown in table 4. It is found that the rate of heat transfer enhances with increase in the radiation parameter N_1 and heat source parameter α .

The Sherwood number (Sh) at $y = 0$ is shown in table 5 for different parametric values. It is found that the rate of mass transfer reduces in magnitude with $\alpha \leq 5$ and enhances with higher $\alpha \leq 10$. Lesser the molecular diffusivity larger $|Sh|$ at the wall. Also it enhances with increase in the solet parameter $|S_0|$ ($\lesseqgtr 0$). When the molecular buoyancy force dominates

over the thermal buoyancy force the rate of mass transfer experiences an enhancement in magnitude irrespective of the directions of the buoyancy forces. Also it depreciates with increase in the radiation parameter N_1 .

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FIGURES:

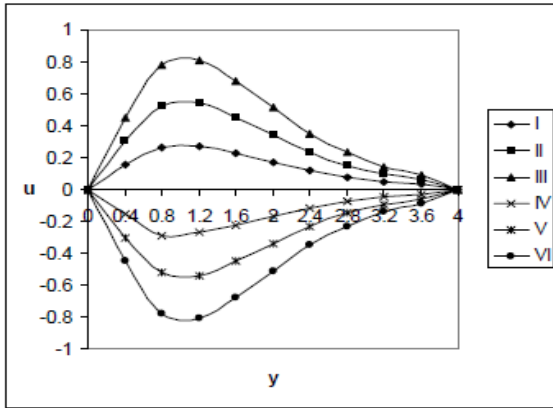


Fig.1 Variation of u with G

G	I	II	III	IV	V	VI
	10^3	2×10^3	3×10^3	-10^3	-2×10^3	-3×10^3

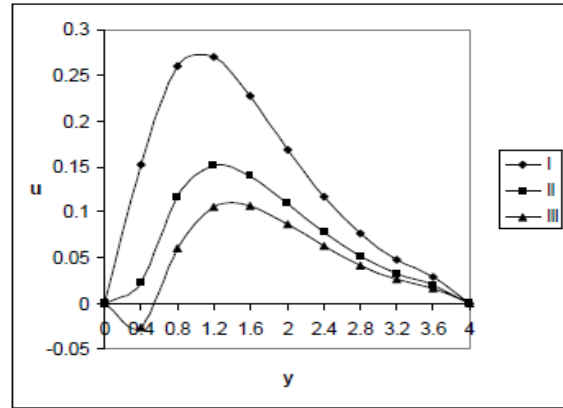


Fig.2 Variation of u with D^{-1}

D^{-1}	I	II	III
	10^2	2×10^2	3×10^2

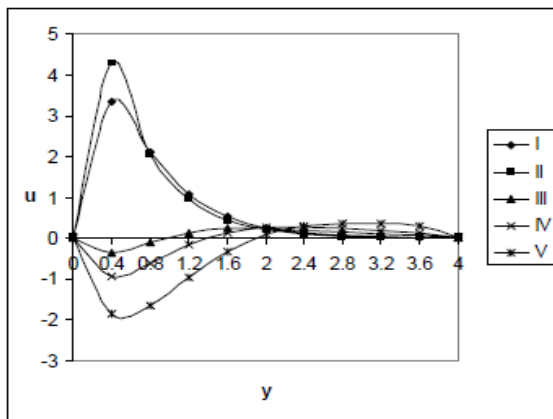


Fig.3 Variation of u with M & m

M	I	II	III	IV	V
m	2	5	10	2	2
	0.5	0.5	1	1.5	2.5

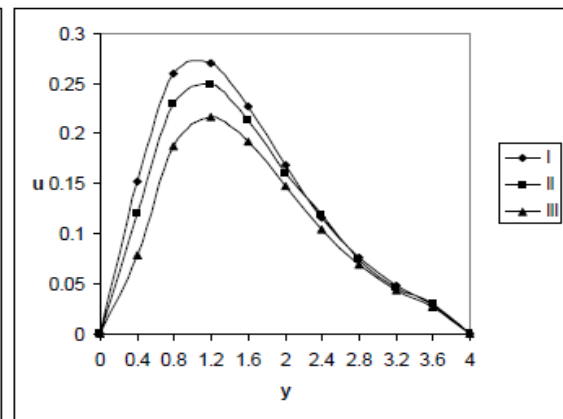


Fig.4 Variation of u with E^{-1}

E^{-1}	I	II	III
	0.5	1.5	2.5

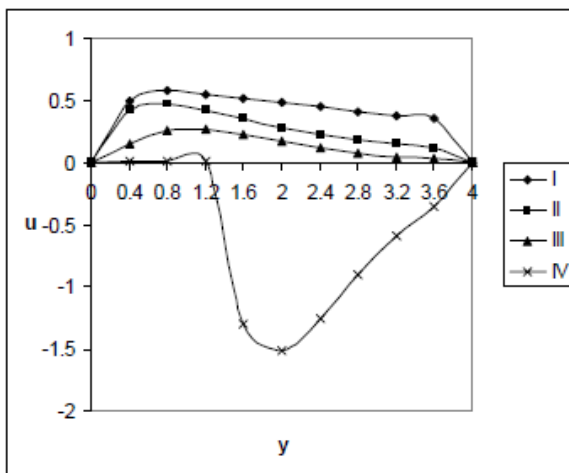


Fig.5 Variation of u with Sc

Sc	I	II	III	IV
	0.24	0.6	1.3	2.01

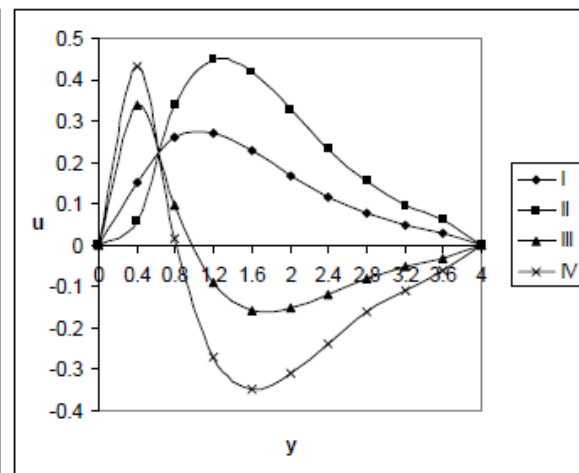


Fig.6 Variation of u with S_0

S_0	I	II	III	IV
	0.5	1.0	-0.5	-1.0

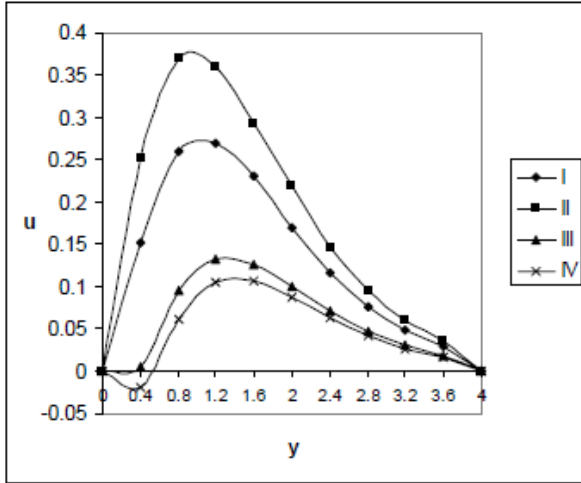


Fig.7 Variation of u with N

	I	II	III	IV
N	1	2	-0.5	-0.8

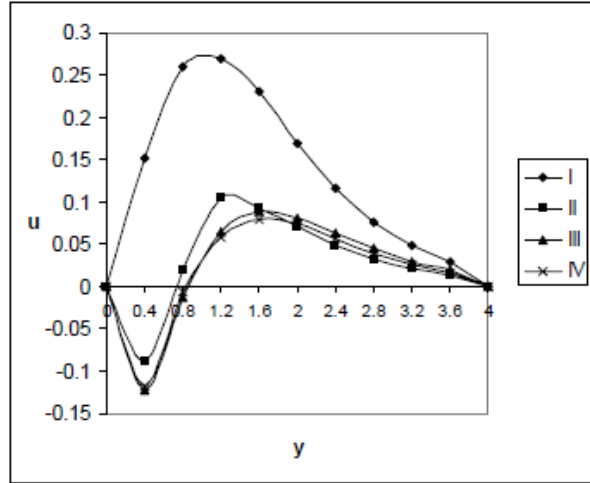


Fig.8 Variation of u with α

	I	II	III	IV
α	2	4	6	8

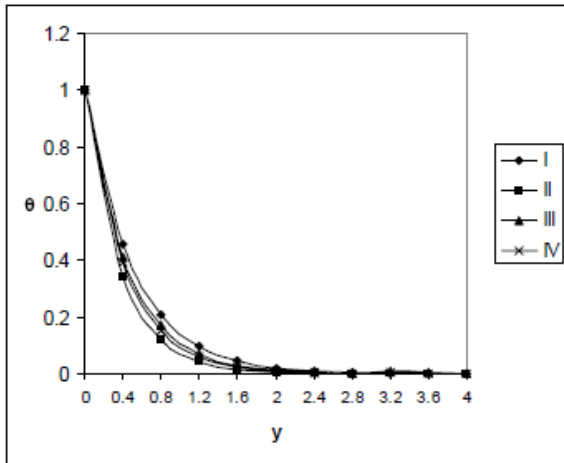


Fig.9 Variation of θ with α & N

	I	II	III	IV
α	2	4	2	2
N1	2	2	4	6

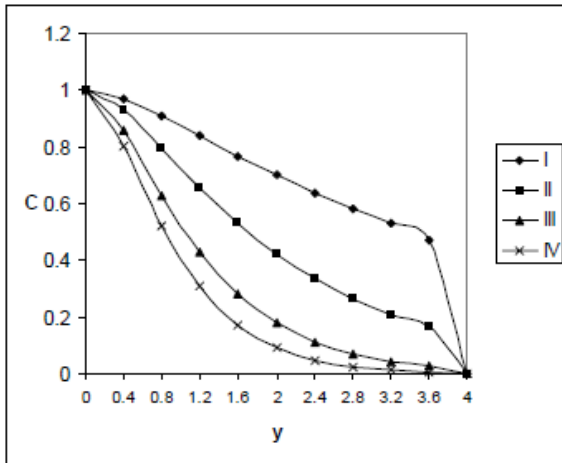


Fig.10 Variation of C with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

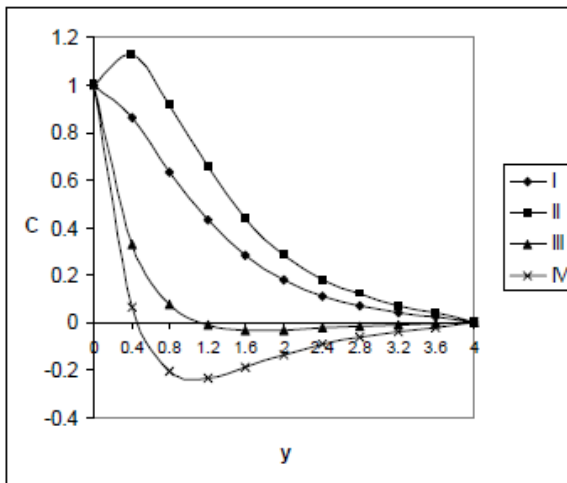


Fig.11 Variation of C with S_0

	I	II	III	IV
S_0	0.5	1.0	-0.5	-1.0

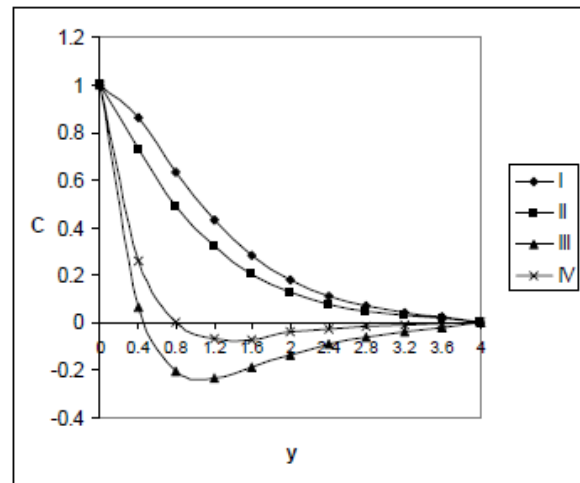


Fig.12 Variation of C with N

	I	II	III	IV
N	1	2	-0.5	-0.8

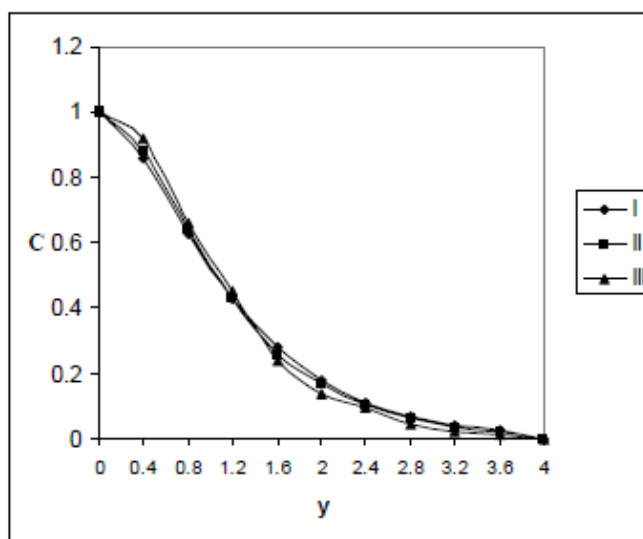


Fig.13 Variation of C with N_1

	I	II	III	IV
N_1	0.5	2	4	6

TABLES:

Table - 1
Sheer Stress (τ) at $z = 0$

G	I	II	III	IV	V	VI	VII	VIII
10^3	27.2077	44.8585	58.2840	36.6831	49.5986	25.0449	24.2602	27.2297
3×10^3	81.6230	134.5755	174.8520	110.0494	148.7960	75.1349	72.7806	81.6892
-10^3	-27.2077	-44.8585	-58.2840	-36.6831	-49.5986	-25.0449	-24.2602	-27.2297
-3×10^3	-81.6230	-134.5755	-174.8520	-110.0494	-148.7960	-75.1349	-72.7806	-81.6892
M	2	2	2	4	6	2	2	2
m	0.5	0.5	0.5	0.5	0.5	1.5	2.5	0.5
E^{-1}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.5

Table - 2
Sheer Stress (τ) at $z = 0$

G	I	II	III	IV	V	VI	VII	VIII	IX	X
10^3	27.2077	27.2298	-15.7463	-8.7425	121.3533	-66.9379	-114.0108	29.4449	23.8518	21.0525
3×10^3	81.6230	86.1296	-47.2388	-26.2276	364.0599	-200.8139	-342.0324	88.3346	71.5555	65.9846
-10^3	-27.2077	-27.2298	15.7463	8.7425	-121.3533	66.9379	114.0108	-29.4449	-23.8518	-21.0525
-3×10^3	-81.6230	-86.1296	47.2388	26.2276	-364.0599	200.8139	342.0324	-88.3346	-71.5555	-65.9846
Sc	1.3	2.01	0.24	0.6	1.3	1.3	1.3	1.3	1.3	1.3
S_0	0.5	0.5	0.5	0.5	1.5	0.5	-1.0	0.5	0.5	0.5
N	1	1	1	1	1	1	1	2	-0.5	-0.8

Table - 3
Sheer Stress (τ) at $z = 0$

G	I	II	III	IV	V
10^3	27.2077	16.7958	12.1637	9.0507	4.5063
3×10^3	81.6230	50.3877	36.4901	27.1519	13.5189
-10^3	-27.2077	-16.7958	-12.1637	-9.0507	-4.5063
-3×10^3	-81.6230	-50.3877	-36.4901	-27.1519	-13.5189
N_1	2	4	10	2	2
α	5	5	5	10	20

Table - 4
Nusselt Number (Nu) at $z = 0$

G	I	II	III	IV	V
2	1.32896	1.51960	1.67809	1.74198	1.9812
5	1.95810	2.22096	2.43688	2.52331	2.59909
10	2.67173	3.01777	3.30015	3.41271	3.51121
N_1	2	4	10	20	100
α	2	2	2	4	6

Table – 5
Sherwood Number (Sh) at $z = 0$

α	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
2	-0.43618	-0.31226	-0.20924	-0.16771	2.16428	-0.67439	-0.08052	-0.20131	1.29147	2.16382	-3.89147
5	-0.02724	0.14362	0.28397	0.34015	0.38941	-0.04211	-0.00503	-0.01257	2.51829	-2.57276	-5.11829
10	0.43663	0.66155	0.84510	0.91826	0.98228	0.67509	0.08061	0.20152	3.90988	-3.03663	-6.50988
N_1	2	4	10	20	100	2	2	2	2	2	2
Sc	1.3	1.3	1.3	1.3	1.3	2.01	0.24	0.6	1.3	1.3	1.3
S_0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.5	-0.5	-1.5

Table – 6
Sherwood Number (Sh) at $z = 0$

α	I	II	III	IV
2	-0.43618	-0.8681	-3.02765	-3.37978
5	-0.02724	-0.6636	-3.84553	-4.46986
10	0.43663	-0.4417	-4.77325	-5.36636
N	1	2	-0.5	-0.8

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