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ON GEODETIC POLYNOMIAL OF GRAPHS WITH EXTREME VERTICES

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ABSTRACT

Let G=(V,E) be a simple graph. A set of vertices S of a graph G is geodetic, if every vertex of G lies on a shortest path between two vertices in S. The geodetic number of G is the minimum cardinality of all geodetic sets of G, and is denoted by g(G). In [10], the concept of geodetic polynomial is defined as $g(G,x)=\sum_{i=g(G)}^{n} g_e(G,i)x^i$ *where* $g_e(G,i)$ *is the number of geodetic sets of G with cardinality i. In this paper, we obtain the geodetic polynomials of the helm graph. Also, we compute the polynomials for some specific graphs.*

***Keywords**: Geodetic polynomial, geodetic set, geodetic number, corona, extreme vertices.*

1. Introduction

Let G= (V, E) be a simple graph of order |V| = n. The distance d (u, v) between two vertices u and v in a connected graph G is the length of a shortest u-v path in G. A u-v path of length d(u, v) is called u-v geodesic. The closed interval I[u, v] consists of all vertices lying on some u-v geodesic of G, while for S $\subseteq$ V, I [S] = $\bigcup_{u,v}$ I [u, v]. A set S of vertices is a geodetic set if I [S] = V, and the minimum cardinality of a geodetic set is the geodetic number g (G). The geodetic number of a graph was introduced in [4, 5]. In [1], the domination polynomial was introduced and some properties have been derived. In [10], the concept of geodetic polynomial was introduced. It is defined as $g(G,x)=\sum_{i=g(G)}^{n} g_e(G,i)x^i$ where G is a graph of order n and $g_e(G,i)$ is the number of geodetic sets of G of cardinality i.

Let h_n^i be the family of geodetic sets of a helm graph H_n with cardinality i and let $g_e(H_n,i)=|h_n^i|$. We call the polynomial $g(H_n,x)=\sum_{i=n}^{2n+1} g_e(H_n,i)x^i$, the geodetic polynomial of the helm graph H_n.

The corona of the two graphs G_1 and G_2, as defined by Frucht and Harary in [10] is the graph

$$G = G_1 o G_2$$

formed from one copy of G_1 and $|V(G_1)|$ copies of G_2, where the i^th^ vertex of G_1 is adjacent to every vertex in the i^th^ copy of G_2.

Let G and H be two graphs, G adding H at u and v denoted by G_u+G_v is defined as $V(G_u+G_v)=V(G)\cup E(H)+uv$. G joining H at u and v denoted by $G_u \odot H_v$ is obtained from G_u+G_v by contracting the edge uv.

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In [9] the concept of extreme geodesic graph was introduced. A vertex is simplicial or extreme if its neighbourhood induces a complete graph. A graph G is an extreme geodesic graph if all extreme vertices form a geodetic set.

In the next section, we constract the geodetic polynomial of helm graph. In section 3, we study the geodetic polynomial of extreme geodesic graph. In section 4, we find the geodetic polynomial of the graph $G \circ K_n$. Where $G \circ K_n$ is the corona of two graph G and K_n. In the last section, we study the polynomial $G_u + H_v$ and $G_u \odot H_v$.

2. Geodetic polynomial of a Helm graph

In this section, we introduce and investigate the geodetic polynomial of helm graph. Let H_n, $n \geq 3$ be the helm graph with 2n+1 vertices.

Definition 2.1: The geodetic polynomial of a Helm graph H_n is defined as $g(H_n, x) = \sum_{i = n}^{2n+1} g_e(H_n, i) x^i$ where $g_e(H_n, i)$ is the number of geodetic sets of H_n with cardinality i.

Let h_n^i be the family of geodetic sets of helm H_n with cardinality i and let $g_e(H_n, i) = |h_n^i|$

Lemma 2.2: The following properties hold for helm graph

i) $g(H_n) = n$

ii) $h_n^i = \phi$ if and only if i<n or i> 2n+1

iii) If a helm graph G with 2k+1 vertices, then every geodetic set of G must contain atleast k vertices

Proof: Proof is obvious

Theorem 2.3: For every $n \geq 5$,

i) If h_n^i is the family of geodetic sets of H_n with cardinality i, then $|h_n^{n+j}| = |h_{n-1}^{n+j-1}| + |h_{n-1}^{n+j-2}|$

ii) $g(H_n, x) = x^2 g(H_{n-1}, x) + g(H_{n-1}, x)$

iii) $g(H_n, x) = x^n (1 + x)^{n+1}$

Proof: i) In H_n, we consider the geodetic sets of H_n with cardinality n+j, j=0,1,2,…. n+1. H_n has 2n+1 vertices. Geodetic sets of H_n must contain n vertices. Now there remain (n+1) vertices and we have to choose j vertices from these (n+1) vertices. Therefore, there are $(n+1)C_j$ geodetic sets in H_n with cardinality n+j. That is $g_e(H_n, n+j) = (n+1)C_j$. Similarly $g_e(H_n, n+j-1) = nC_j$ and $g_e(H_n, n+j-2) = nC_{j-1}$. But $(n+1)C_j = nC_j + nC_{j-1}$.

Therefore $g_e(H_n, n+j) = g_e(H_{n-1}, n+j-1) + g_e(H_{n-1}, n+j-2)$

Hence $|h_n^{n+j}| = |h_{n-1}^{n+j-1}| + |h_{n-1}^{n+j-2}|$

ii) By (i) above, we have $|h_n^{n+j}| = |h_{n-1}^{n+j-1}| + |h_{n-1}^{n+j-2}|$

$$j = 0 \Rightarrow |h_n^n| = |h_{n-1}^{n-1}| + |h_{n-1}^{n-2}| \Rightarrow x^n |h_n^n| = x^n |h_{n-1}^{n-1}| + x^n |h_{n-1}^{n-2}|$$
$$j = 1 \Rightarrow |h_n^{n+1}| = |h_{n-1}^{n-2}| + |h_{n-1}^{n-1}| \Rightarrow x^{n+1} |h_n^{n+1}| = x^{n+1} |h_{n-1}^{n-2}| + x^{n+1} |h_{n-1}^{n-1}|$$

$$j = 2 \Rightarrow \left|h_n^{n+2}\right| = \left|h_{n-1}^{n-1}\right| + \left|h_{n-1}^{n}\right| \Rightarrow x^{n+2}\left|h_n^{n+2}\right| = x^{n+2}\left|h_{n-1}^{n-1}\right| + x^{n+2}\left|h_{n-1}^{n}\right|$$

$$\vdots \qquad\qquad \vdots$$

$$j = n \Rightarrow \left|h_n^{2n}\right| = \left|h_{n-1}^{2n-1}\right| + \left|h_{n-1}^{2n-2}\right| \Rightarrow x^{2n}\left|h_n^{2n}\right| = x^{2n}\left|h_{n-1}^{2n-1}\right| + x^{2n}\left|h_{n-1}^{2n-2}\right|$$

$$j = n+1 \Rightarrow \left|h_n^{2n+1}\right| = \left|h_{n-1}^{2n}\right| + \left|h_{n-1}^{2n-1}\right| \Rightarrow x^{2n+1}\left|h_n^{2n+1}\right| = x^{2n+1}\left|h_{n-1}^{2n}\right| + x^{2n+1}\left|h_{n-1}^{2n-1}\right|$$

Adding all these, we get

$$\begin{aligned} x^n\left|h_n^n\right| + x^{n+1}\left|h_n^{n+1}\right| + x^{n+2}\left|h_n^{n+2}\right| + \ldots + x^{2n}\left|h_n^{2n}\right| + x^{2n+1}\left|h_n^{2n+1}\right| \\ = \left(x^n\left|h_{n-1}^{n-1}\right| + x^{n+1}\left|h_{n-1}^{n}\right| + x^{n+2}\left|h_{n-1}^{n+1}\right| + \ldots + x^{2n}\left|h_{n-1}^{2n-1}\right| + x^{2n+1}\left|h_{n-1}^{2n}\right|\right) \\ + \left(x^n\left|h_{n-1}^{n-2}\right| + x^{n+1}\left|h_{n-1}^{n-1}\right| + x^{n+2}\left|h_{n-1}^{n}\right| + \ldots + x^{2n}\left|h_{n-1}^{2n-2}\right| + x^{2n+1}\left|h_{n-1}^{2n-1}\right|\right) \end{aligned}$$

$$\begin{aligned} \sum_{1=n}^{2n}\left|h_n^i\right|x^i = x\left[x^{n-1}\left|h_{n-1}^{n-1}\right| + x^n\left|h_{n-1}^{n}\right| + x^{n+1}\left|h_{n-1}^{n+1}\right| + \ldots + x^{2n-1}\left|h_{n-1}^{2n-1}\right|\right] \\ + x^2\left[x^{n-1}\left|h_{n-1}^{n-1}\right| + x^n\left|h_{n-1}^{n}\right| + \ldots + x^{2n-2}\left|h_{n-1}^{2n-2}\right| + x^{2n-1}\left|h_{n-1}^{2n-1}\right|\right] \end{aligned}$$

Since $h_{n-1}^{2n} = 0$, $h_{n-1}^{n-2} = 0$

$$\sum_{i=n}^{2n}\left|h_n^i\right|x^i = x\sum_{i=n-1}^{2n-1}\left|h_{n-1}^i\right|x^i + x^2\sum_{i=n-1}^{2n-1}\left|h_{n-1}^i\right|x^i$$

ie $\sum_{i=n}^{2n} g_e(H_n, i)\, x^i = x\sum_{i=n-1}^{2n-1} g_e(H_{n-1}, i)x^i + x^2\sum_{i=n-1}^{2n-1} g_e(H_{n-1}, i)x^i$

Therefore, $g(H_n, x) = xg(H_{n-1}, x) + x^2 g(H_{n-1}, x)$

iii) By induction on n. The result is true for $n=5$, because

$g(H_5, x) = x^5 + 6x^6 + 15x^7 + 20x^8 + 15x^9 + 6x^{10} + x^{11}$. Assume that the result is true for all natural numbers less than n. We prove the result for n. We have $g(H_n, x) = x^{n-1}(1+x)^n$.

Now $g(H_n, x) = x^2 g(H_{n-1}, x) + xg(H_{n-1}, x)$

$$\begin{aligned} &= x^2\left[x^{n-1}(1+x)^n\right] + x\left[x^{n-1}(1+x)^n\right] \\ &= x^{n-1}(1+x)^n\left[x^2 + x\right] \\ &= x^{n-1}(1+x)^n x[1+x] \\ &= x^n(1+x)^{n+1} \end{aligned}$$

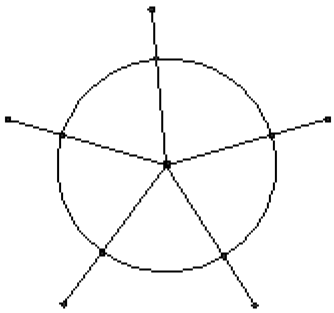
Therefore the result is true for all n.

Using theorem 2.3 we obtain $g_e(H_n, i)$ for $4 \le n \le 9$ as shown in the table1. There are interesting relationships between numbers in this table.

Table1:
$g_e(H_n, j)$, the number geodetic set of H_n with cardinality j.

n \ j	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
H_4	1	5	10	10	5	1										
H_5	0	1	6	15	20	15	6	1								
H_6	0	0	1	7	21	35	35	21	7	1						
H_7	0	0	0	1	8	28	42	70	42	28	8	1				
H_8	0	0	0	0	1	9	36	84	126	126	84	36	9	1		
H_9	0	0	0	0	0	1	10	45	120	210	252	210	120	45	10	1

Example 1.4: A helm graph H_5 with 11 vertices.



Theorem 1.5: The following properties hold for the coefficients of $g(H_n, x)$, for all $n \ge 4$

i) $g_e(H_n, n) = 1$
ii) $g_e(H_n, 2n+1) = 1$
iii) $g_e(H_n, n+1) = n+1$
iv) $g_e(H_n, 2n) = n+1$
v) $g_e(H_n, n+2) = \dfrac{n(n+1)}{2}$
vi) $g_e(H_n, 2n-1) = \dfrac{n(n+1)}{2}$
vii) $g_e(H_n, n+3) = \dfrac{n(n-1)(n+1)}{6}$
viii) $g_e(H_n, 2n-2) = \dfrac{n(n-1)(n+1)}{6}$
ix) If $S_n = \sum_{j=n}^{2n+1} g_e(H_n, j)$, then for every $n \ge 4$, $S_n = 2(S_{n-1})$ with initial value $S_4 = 32$
x) S_n = Total number of geodetic sets in $H_n = 2^{n+1}$

Proof: Properties (i) to (viii) hold, by theorem 2.2.

ix) If $S_n = \sum_{j=n}^{2n+1} g_e(H_n, j)$

$$= \sum_{j=n}^{2n+1} \{g_e(H_{n-1,}, j-1) + g_e(H_{n-1}, j-2)\}$$

$$= \sum_{j=n}^{2n+1} g_e(H_{n-1,}, j-1) + \sum_{j=n}^{2n+1} g_e(H_{n-1,}, j-2)$$

$$= \sum_{j=n-1}^{2n-1} g_e(H_{n-1,}, j) + \sum_{j=n-1}^{2n-1} g_e(H_{n-1,}, j)$$

$$= S_{n-1} + S_{n-1}$$

ie, $S_n = 2(S_{n-1})$

x) By induction on n. The result is true for n = 4, since $S_4 = 2^5 = 32$. Assume that the result is true for all natural numbers less than n. Therefore, $S_{n-1} = 2^n$. Now $S_n = 2S_{n-1} = 2.2^n = 2^{n+1}$. Therefore, the result is true for n. Hence, by induction principle, the result is true for all n.

3. Extreme vertices

A vertex is simplicial or extreme it is neighbourhood induces a complete graph. A graph G is an extreme geodesic graph if all extreme vertices form a geodetic set.

Theorem 3.1: If G is an extreme geodesic graph than $g(G,x) = x^r\,(1+x)^{n-r}$ where r is the number of extreme vertices

Proof: Let G be an extreme geodesic graph with n vertices. Let the extreme vertices be $v_1, v_2, \ldots, v_r$. Every vertex lies on the path joining of two extreme vertices. Therefore g(G)=r. The geodetic set of G with cardinality r is $\{v_1, v_2, \ldots., v_r\}$. Therefore $g_e(G,r) = 1$. The geodetic set of G with cardinality r+1 is $(n\text{-}r)C_1$. Therefore $g_e(G, r+1) = (n-r)C_1$. The geodetic set of G with cardinality r+2 is $(n\text{-}r)C_2$. Therefore $g_e(G, r+2) = (n-r)C_2$. Continuing in this way, the geodetic set of G with cardinality n is $(n-r)C_{n-r}$. That is $g_e(G,n) = (n-r)C_{n-r}$. Therefore geodetic polynomial of G is

$$g(G,x) = (n-r)\,C_o x^r + (n-r)\,C_1 x^{r+1} + \ldots + (n-r)\,C_{n-r} x^n$$

$$= x^r\,[\,(n-r)C_o + (n-r)C_1 x + (n-r)C_2 x^2 + \ldots. + (n-r)C_{n-r} x^{n-r}\,]$$

$$= x^r\,[1 + (n-r)C_1 x + (n-r)C_2 x^2 + \ldots.. + x^{n-r}\,]$$

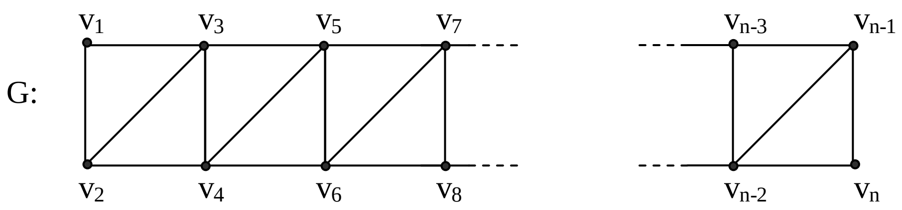
$$= x^r\,[(1+x)^{n-r}\,]$$

Corollary 3.2: In any graph G with n vertices $v_1, v_2 \ldots. v_r$ are pendent vertices and if $g(G) = r$ then $g(G,x) = x^r\,(1+x)^{n-r}$

Proof: As every pendent vertex is extreme, by previous theorem, $g(G,x) = x^r\,(1+x)^{n-r}$.

Corollary 3.3: If G is a Triangular ladder graph with n vertices. Then the geodetic polynomial of G is $g(G,x) = x^2\,(1+x)^{n-2}$

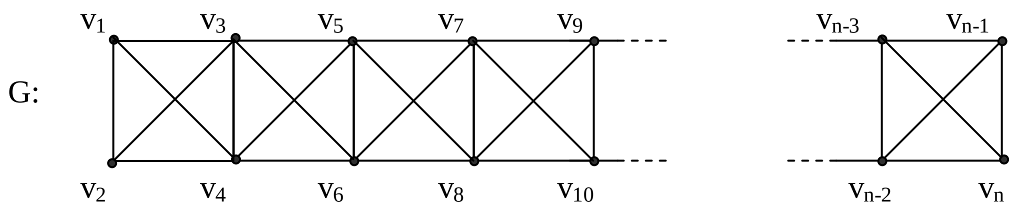
Proof: Triangular ladder graph with n vertices is



In Triangular ladder graph G, v_0, v_n are extreme vertices, and $g(G)=2$. Therefore by previous theorem3.1, $g(G,x)=x^2(1+x)^{n-2}$

Corollary 3.4: If G is a Extended grid graph with n vertices then the geodetic polynomial of G is $g(G,x)=x^r(1+x)^{n-r}$.

Proof: Extended grid graph with n vertices is



In graph G, v_1, v_2, v_n, v_{n-1} are extreme vertices and g(G)=4. Therefore by theorem 3.1 $g(G,x)=x^4(1+x)^{n-4}$.

4. Geodetic polynomial of $G \circ K_m$

Let G be any graph with vertex set $\{v_1, v_2, ...v_n\}$. Join each vertex of G to K_m. By the definition of corona of two graphs, we shall denote this by $G \circ K_m$.

Lemma 4.1: For any graph G of order n, $g(G \circ K_m) = mn$

Proof: If G_1 is a geodetic set of $G \circ K_m$. Then $G_1 = \{u_1, u_2, .., u_m, u_{m+1}, u_{m+2}, ..., u_{2m}, u_{2m+1}, u_{2m+2}, ..., u_{nm}\}$ or $G_1 = \{u_1, u_2, ..., u_m, ..., u_{2m}, ..., u_{nm}\} \cup A$ where $A \subseteq \{v_1, v_2 ... v_n\}$. Therefore $|G_1| \geq mn$. Since $\{u_1, u_2, ..., u_m, ..., u_{2m}, ..., u_{nm}\}$ is a geodetic set of $G \circ K_m$, we have $g(G \circ K_m) = mn$.
By lemma4.1, $g_e(G \circ K_m, p) = 0$ for $p < mn,$ so we shall compute $g_e(G \circ K_m, p)$ for $mn \leq p \leq n(1+m).$

Theorem 4.2: For any graph G of order n and $mn \leq p \leq n(1+m)$, we have $g_e(G \circ K_m, p) = \binom{n}{p-mn}$. Hence, $g(G \circ K_m, x) = x^{mn}(1+x)^n$.

Proof: Suppose that G_1 is a geodetic set of G of size p. when p=mn, the geodetic set with cardinality mn is $G_1 = \{u_1, u_2, ..., u_m, ..., u_{2m}, ..., u_{mn}\}$. Therefore, $g_e(G \circ K_m, mn) = \binom{n}{0}$

When p= mn+1, $g_e(G \circ K_m, mn+1) = \binom{n}{1}$

When p=mn+2, $g_e(G \circ K_m, mn+2) = \binom{n}{2}$. By continuing in this way

When *p=mn+n=n(m+1)* the geodetic set with cordiality *n(m+1)* is
$G_1 = \{u_1, u_2, \dots u_m, ..u_{m+1} ..u_{2m} \dots u_{mn.}\} \cup \{v_1, v_2, \dots v_n\}$.

Therefore $g_e(G \circ K_m, mn+n) = \binom{n}{n}$. In general we conclude that $g_e(G \circ K_m, p) = \binom{n}{p-mn}$.

Therefore Geodetic polynomial of $G \circ K_m$ is $G(G \circ K_m, x) = nC_0 x^{mn} + nC_1 x^{mn+1} + \dots nC_n x^{mn+n}$

$$= x^{mn}\left(nC_o + nC_1 + \dots + nC_n x^n\right)$$

$$g(G \circ K_m, x) = x^{mn}(1+x)^n$$

5. Geodetic polynomial of $G_u + H_v$ and $G_u \odot H_v$

Let G and H be two graphs. G adding H at u and v denoted by $G_u + H_v$ is defined as $V(G_u + H_v) = V(G) \cup V(H)$ and $E(G_u + H_v) = E(G) \cup E(H) + uv$

G joining H at u and v denoted by $G_u \odot H_v$ is obtained from $G_u + H_v$ by contracting the edge uv.

Theorem 5.1: Suppose G and H are two non-trivial graphs and S_1 (S_2 respectively) is a minimum geodetic set of G (H respectively). Let $u \in S_1$ and $v \in S_2$. Then $g(G_u + H_v) = g(G) + g(H) - 2$ and $g(G_u \odot H_v) = g(G) + g(H) - 2$

Theorem 5.2:

(i) The geodetic polynomial of $K_{m_u} + K_{n_v}$ is $g(K_{m_u} + K_{n_v}, x) = x^{m+n}\left(\frac{1}{x^2} + \frac{2}{x} + 1\right)$

(ii) The geodetic polynomial of $K_{m_u} \odot K_{n_v}$ is $g(K_{m_u} \odot K_{n_v}, x) = x^{m+n}\left(\frac{1}{x^2} + \frac{1}{x}\right)$

Proof: By theorem 5.1

$$g(K_{m_u} + K_{n_v}) = m+n-2$$

Let $\{u_1, u_2, \dots, u_{m-1}, u\}$ be the vertex set of K_m.

Let $\{v_1, v_2, \dots, v_{m-1}, v\}$ be the vertex set of K_n.

Therefore $\{u_1, u_2, \dots, u_{m-1}, u, v_1, v_2, \dots v_{n-1}, v\}$ is the vertex set of $K_{m_u} + K_{n_v}$ .The only geodetic set of cardinality m+n-2 is $\{u_1, u_2, \dots, u_{m-1}, v_1, v_2, \dots, v_{n-1}\}$

ie, $g_e(K_{m_u} + K_{n_v}, m+n-2) = 1$. The Geodetic set with cardinality m+n-1 is, $\{u_1, u_2, \dots, u_{m-1}, u, v_1, v_2, \dots, v_{n-1}\}$ and $\{u_1, u_2, \dots, u_{m-1}, v_1, v_2, \dots v_{n-1}\}$

ie, $g_e(K_{m_u} + K_{n_v}, m+n-1) = 2$

The geodetic set with cardinality m+n is $\{u_1, u_2, \dots, u_{m-1}, u, v, v_1, v_2, \dots, v_{n-1}\}$

ie, $g_e(K_{m_u} + K_{n_v}, m+n) = 1$

Therefore $g\left(K_{m_u}+K_{n_v},x\right)=x^{m+n-2}+2x^{m+n-1}+x^{m+n}=x^{m+n}\left(\frac{1}{x^2}+\frac{2}{x}+1\right)$

i) By theorem 5.1

$$g\left(K_{m_u}\odot K_{n_v}\right)=m+n-2$$

Let $\{u_1,u_2,...,u_{m-1},u,v_1,v_2,...,v_{n-1}\}$ be the vertex set $K_{m_u}\odot K_{n_v}$

The only geodetic set of cardinality m+n-2 is $\{u_1,u_2,...,u_{m-1},v_1,v_2,...,v_{n-1}\}$

ie, $g_e\left(K_{m_u}\odot K_{n_v},m+n-2\right)=1$

The geodetic set with cardinality m+n-1 is $\{u_1,u_2,...,u_{m-1},v_1,v_2,...,v_{n-1}\}$

ie, $g\left(K_{m_u}\odot K_{n_V},m+n-1\right)=1$

Since $K_{m_u}\odot K_{n_v}$ contains m+n-1 vertices

$$g\left(K_{m_u}\odot K_{n_v},x\right)=x^{m+n-2}+x^{m+n-1}=x^{m+n}\left(\frac{1}{x^2}+\frac{1}{x}\right).$$

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