International Journal of Mathematical Archive-3(6), 2012, 2321-2328
Available online through www.ijma.info ISSN 2229 – 5046

SOME COMMON FIXED POINT THEOREMS WITH COMPATIBLE MAPPINGS AND CONVEXITY IN METRIC SPACE

Deepak Singh Kaushal* & S. S. Pagey
Sagar Institute of Science, Technology & Research, Ratibad, Bhopal (M.P.), India
Institue for Excellence in Higher Education, Bhopal (M.P.), India

(Received on: 03-06-12; Accepted on: 20-06-12)

ABSTRACT

In this note we establish some common fixed point theorems of compatible mappings in the setting of convex metric spaces. Several results are also derived as special cases.

Keywords: *Fixed point, Compatible mappings, Convex metric spaces.*

---

1. INTRODUCTION

The concept of the commutativity has generalized in several ways.For this Sessa S. [13] has introduced the concept of weakly commuting and Gerald Jungck [5] initiated the concept of compatibility. It can be easily verified that when the two mappings are commuting then they are compatible but not conversely.

The study of common fixed point of mappings satisfying contractive type conditions has been a useful tool for obtaining more comprehensive fixed point theorems[3,6,7,8,9,10] during the last three decades.

Jungck [5] introduced more generalized commuting mappings called compatible mappings. Which are more general then commuting and weakly commuting mappings.

On the other hand, Takahanshi [14] introduced a notion of convex metric spaces, many authers have discussed the existence of fixed point and the convergence of iterative processes for non-expansive mappings in convex metric spaces.

The main purpose of this paper is to present fixed point results for a pair of maps satisfying a new contractive condition by using the concept of compatible maps in a convex metric space. Our main results are generalizations of some known results in [1, 2, 4, 6, 11, 12].

2. PRELIMINARIES

Definition 2.1: The pair (A, B) of self-mappings of a metric space (X, d) is said to be compatible on X if whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to t \in X$ then $d(BAx, ABx) \to 0$.

Definition 2.2: Let (X, d) be a metric space and $J = [0,1]$.A mapping $W: X \times X \times J \to X$ is called a convex structure on X if for each $(x, y, \lambda) \in X \times X \times J$ and $u \in X$,

$$d\left(u, W\left(x, y, \lambda\right)\right) \le \lambda d\left(u, x\right) + \left(1-\lambda\right) d\left(u, y\right)$$

A metric space X together with a convex structure W is called a convex metric space.

Definition 2.3: A nonempty subset K of a convex metric space (X, d) with a convex structure W is said to be convex if for all $(x, y, \lambda) \in K \times K \times J$, $W(x, y, \lambda) \in K$.

***Corresponding author:** *Deepak Singh Kaushal*, Sagar Institute of Science, Technology & Research, Ratibad, Bhopal (M.P.), India*

Definition 2.4: Let K be a convex subset of a convex metric space (X,d) with a convex structure W. A mapping $I:K\to K$ is said to be W-affine if for all $(x,y,\lambda)\in K\times K\times J$, $IW(x,y,\lambda)=W(Ix,Iy,\lambda)$

Definition 2.5: A sequence $\{x_n\}$ in a metric space (X,d) is said to be convergent to a point $x\in X$ denoted by $\lim_{n\to\infty} x_n = x$ if $\lim_{n\to\infty} d(x_n,x)=0$.

3. MAIN RESULTS

Theorem 3.1: Let (X,d) be a complete convex metric space with a convex structure W and K is a nonempty closed convex subset of X. Suppose that the mappings T and I are compatible self –maps on K satisfying the following condition

$$d(Tx,Ty)\le ad(Ix,Iy)+b\max\{d(Ix,Tx),d(Iy,Ty)\}+c\max\{d(Iy,Tx),d(Ix,Ty)\}$$
$$+d\max\left\{d(Ix,Iy),d(Ix,Tx),d(Iy,Ty),\frac{1}{2}\big(d(Iy,Tx)+d(Ix,Ty)\big)\right\}\quad \forall x,y\in K$$

Where a, b, c, d are non-negative real numbers such that $a+b+2c+d=1$ and $\dfrac{2(a+c)(b+c)}{\left(2-\overline{b+c}\right)}>d$. If $T(K)\subset I(K)$ and I is W-affine and continuous then there exists a unique common fixed point z of T and I, and T is continuous at z.

Proof: Let $x=x_0$ be an arbitrary point in K and choose some points x_1,x_2 and x_3 in K such that $Tx=Ix_1, Tx_1=Ix_2, Tx_2=Ix_3$ It is possible since $T(K)\subset I(K)$.

For $k=1,2,3$ we have

$$\begin{aligned} d(Tx_k,Ix_k) &= d(Tx_k,Tx_{k-1})\\ &\le ad(Ix_k,Ix_{k-1})+b\max\{d(Ix_k,Tx_k),d(Ix_{k-1},Tx_{k-1})\}+c\max\{d(Ix_{k-1},Tx_k),d(Ix_k,Tx_{k-1})\}\\ &\quad +d\max\left\{d(Ix_k,Ix_{k-1}),d(Ix_k,Tx_k),d(Ix_{k-1},Tx_{k-1}),\frac{1}{2}\big(d(Ix_{k-1},Tx_k)+d(Ix_k,Tx_{k-1})\big)\right\}\\ &\le ad(Ix_{k-1},Tx_{k-1})+b\max\{d(Ix_k,Tx_k),d(Ix_{k-1},Tx_{k-1})\}+c\max\{d(Ix_{k-1},Tx_{k-1}),d(Ix_k,Tx_k)\}\\ &\quad +d\max\left\{d(Ix_{k-1},Tx_{k-1}),d(Ix_k,Tx_k),\frac{1}{2}\big(d(Ix_{k-1},Tx_{k-1})+d(Ix_k,Tx_k)\big)\right\} \end{aligned}$$

If

$d(Tx_k,Ix_k)>d(Tx_{k-1},Ix_{k-1})$ then we have

$$\Rightarrow d(Tx_k,Ix_k)<(a+b+2c+d)d(Tx_k,Ix_k)$$
$$\Rightarrow d(Tx_k,Ix_k)<d(Tx_k,Ix_k)\qquad \text{Since}\quad a+b+2c+d=1$$

Which is a contradiction and therefore

$$d(Ix_k,Tx_k)\le d(Ix_{k-1},Tx_{k-1})\qquad \text{Where } k=1,2,3$$

Now

$$\begin{aligned}
d(Ix_1,Tx_2) &= d(Tx,Tx_2)\\
&\le ad(Ix_0,Ix_2)+b\max\{d(Ix,Tx),d(Ix_2,Tx_2)\}+c\max\ \{d(Ix_2,Tx),d(Ix,Tx_2)\}\\
&\quad +d\max\left\{d(Ix,Ix_2),d(Ix,Tx),d(Ix_2,Tx_2),\frac{1}{2}\big(d(Ix,Tx_2)+d(Ix_2,Tx)\big)\right\}\\
&\le a\{d(Ix,Tx)+d(Ix_1,Tx_1)\}+b\,d(Ix,Tx)\\
&\quad +c\max\ \{d(Ix_1,Tx_1),d(Ix,Tx)+d(Ix_1,Tx_1)+d(Ix_2,Tx_2)\}\\
&\quad +d\max\left\{\begin{array}{l}d(Ix,Tx)+d(Ix_1,Tx_1),d(Ix,Tx),d(Ix,Tx),\\ \frac{1}{2}\big(d(Ix,Tx)+d(Ix_1,Tx_1)+d(Ix_2,Tx_2)+d(Ix_1,Tx_1)\big)\end{array}\right\}\\
&=(2a+b+3c+2d)\,d(Ix,Tx)\\
&=(2-\overline{b+c})\,d(Ix,Tx)
\end{aligned}$$

Let $z=W\left(x_{2,}x_3,\frac{1}{2}\right)$ then we know that $z\in K$ and $Iz=W\left(Ix_2,Ix_3,\frac{1}{2}\right)=W\left(Tx_1,Tx_2,\frac{1}{2}\right)$

Now we obtain

$$\begin{aligned}
d(Iz,Ix_1) &= d\left(Ix_1,W\left(Tx_1,Tx_2,\frac{1}{2}\right)\right)\\
&\le \frac{1}{2}d(Ix_1,Tx_1)+\frac{1}{2}d(Ix_1,Tx_2)\\
&\le \frac{1}{2}d(Ix,Tx)+\frac{1}{2}(2a+b+3c+2d)\,d(Ix,Tx)\\
&= \frac{1}{2}(1+2a+b+3c+2d)\,d(Ix,Tx)\\
&= \frac{3-\overline{b+c}}{2}d(Ix,Tx)
\end{aligned}$$

$$\begin{aligned}
d(Iz,Ix_2) &= d\left(Ix_2,W\left(Tx_1,Tx_2,\frac{1}{2}\right)\right)\\
&\le \frac{1}{2}d(Ix_2,Tx_1)+\frac{1}{2}d(Ix_2,Tx_2)\\
&\le \frac{1}{2}d(Ix_2,Ix_2)+\frac{1}{2}d(Ix,Tx)\\
&= \frac{1}{2}d(Ix,Tx)
\end{aligned}$$

$$\begin{aligned}
d(Iz,Ix_3) &= d\left(Ix_3,W\left(Tx_1,Tx_2,\frac{1}{2}\right)\right)\\
&\le \frac{1}{2}d(Ix_3,Tx_1)+\frac{1}{2}d(Ix_3,Tx_2)\\
&\le \frac{1}{2}d(Ix_2,Tx_2)+\frac{1}{2}d(Ix_3,Ix_3)\\
&= \frac{1}{2}d(Ix,Tx)
\end{aligned}$$

Again we obtain

$$d(Iz,Tz) = d\left(Tz, W\left(Tx_1, Tx_2, \frac{1}{2}\right)\right)$$

$$\leq \frac{1}{2} d(Tz,Tx_1) + \frac{1}{2} d(Tz,Tx_2)$$

$$\leq \frac{a}{2} d(Iz,Ix_1) + \frac{b}{2} \max\{d(Iz,Tz), d(Ix_1,Tx_1)\} + \frac{c}{2} \max\ \{d(Ix_1,Tz), d(Iz,Tx_1)\}$$

$$+ \frac{d}{2} \max\left\{d(Iz,Ix_1), d(Iz,Tz), d(Ix_1,Tx_1), \frac{1}{2}\left(d(Ix_1,Tz) + d(Iz,Tx_1)\right)\right\}$$

$$+ \frac{a}{2} d(Iz,Ix_2) + \frac{b}{2} \max\{d(Iz,Tz), d(Ix_2,Tx_2)\} + \frac{c}{2} \max\ \{d(Ix_2,Tz), d(Iz,Tx_2)\}$$

$$+ \frac{d}{2} \max\left\{d(Iz,Ix_2), d(Iz,Tz), d(Ix_2,Tx_2), \frac{1}{2}\left(d(Ix_2,Tz) + d(Iz,Tx_2)\right)\right\}$$

$$\leq \frac{a}{4}\left(3 - \overline{b+c}\right) d(Ix,Tx) + \frac{b}{2} \max\{d(Iz,Tz), d(Ix,Tx)\}$$

$$+ \frac{c}{2} \max\ \left\{\frac{1}{2}(3 - \overline{b+c}) d(Ix,Tx) + d(Iz,Tz), \frac{1}{2} d(Ix,Tx)\right\}$$

$$+ \frac{d}{2} \max\left\{\frac{1}{2}(3 - \overline{b+c}) d(Ix,Tx), d(Iz,Tz), d(Ix,Tx), \frac{4 - \overline{b+c}}{4} d(Ix,Tx) + \frac{1}{2} d(Iz,Tz)\right\}$$

$$+ \frac{a}{4} d(Ix,Tx) + \frac{b}{2} \max\{d(Iz,Tz), d(Ix,Tx)\} + \frac{c}{2} \max\ \left\{\frac{1}{2} d(Ix,Tx) + d(Iz,Tz), \frac{1}{2} d(Ix,Tx)\right\}$$

$$+ \frac{d}{2} \max\left\{\frac{1}{2} d(Ix,Tx), d(Iz,Tz), d(Ix,Tx), \frac{1}{2}\left(d(Ix,Tx) + d(Iz,Tz)\right)\right\}$$

If $d(Iz,Tz) > d(Ix,Tx)$ then we have

$$d(Iz,Tz) \leq \left[1 - \frac{1}{8}\left(2c^2 - 2d + 2ab + 2ac + 2bc + bd + cd\right)\right] d(Iz,Tz)$$

This is a contradiction since $\dfrac{2(a+c)(b+c)}{\left(2 - \overline{b+c}\right)} > d$ and $a + b + 2c + d = 1$ implies that

$2c^2 - 2d + 2ab + 2ac + 2bc + bd + cd > 0.$ Hence we have

$$d(Iz,Tz) \leq \left[1 - \frac{1}{8}\left(2c^2 - 2d + 2ab + 2ac + 2bc + bd + cd\right)\right] d(Ix,Tx)$$

Letting $\lambda = 1 - \frac{1}{8}\left(2c^2 - 2d + 2ab + 2ac + 2bc + bd + cd\right)$ We know that $0 < \lambda < 1.$

Then we have

$$d(Iz,Tz) \leq \lambda d(Ix,Tx)$$

Since x_0 is an arbitrary point in K then there exists a sequence $\{z_n\}$ in K such that

$$d(Iz_0,Tz_0) \leq \lambda d(Ix_0,Tx_0)$$

$$d(Iz_1,Tz_1) \leq \lambda d(Iz_0,Tz_0)$$

....

....

$$d(Iz_n,Tz_n) \leq \lambda d(Iz_{n-1},Tz_{n-1})$$

Which yield that $d\left(Iz_n,Tz_n\right)\le \lambda^{n+1}d\left(Iz_0,Tz_0\right)$ and so we have

$$d\left(Iz_n,Tz_n\right)\to 0$$

Setting

$$K_n=\left\{x\in K:d\left(Ix,Tx\right)\le\frac{1}{n}\right\}\quad,\quad n=1,2,3..................$$

Then $K_n\ne\phi$, $n=1,2,3,$...........and $K_1\supset K_2\supset$................

Obviously

$$\overline{TK_n}\ne\phi \, and \, \overline{TK_n}\supset\overline{TK_{n+1}}, n=1,2,3.........$$

For any $x,y\in K_n$ we have

$$\begin{aligned}d\left(Tx,Ty\right)&\le ad\left(Ix,Iy\right)+b\max\left\{d\left(Ix,Tx\right),d\left(Iy,Ty\right)\right\}+c\max\left\{d\left(Iy,Tx\right),d\left(Ix,Ty\right)\right\}\\&\quad+d\max\left\{d\left(Ix,Iy\right),d\left(Ix,Tx\right),d\left(Iy,Ty\right),\frac{1}{2}\left(d\left(Iy,Tx\right)+d\left(Ix,Ty\right)\right)\right\}\\&\le a\left(\frac{2}{n}+d\left(Tx,Ty\right)\right)+\frac{b}{n}+c\left(\frac{1}{n}+d\left(Tx,Ty\right)\right)+d\left(\frac{2}{n}+d\left(Tx,Ty\right)\right)\\&=\frac{1}{n}\left(2a+b+c+2d\right)+(a+c+d)d\left(Tx,Ty\right)\end{aligned}$$

Which yields that

$$d\left(Tx,Ty\right)\le\frac{1}{n\left(b+c\right)}\left(2a+b+c+2d\right)$$

Therefore we have

$$\lim_{n\to\infty}diam\left(\overline{TK_n}\right)=\lim_{n\to\infty}diam\left(TK_n\right)=0$$

Where $diam\left(\overline{TK_n}\right)$ denotes the diameter of $\left(\overline{TK_n}\right)$.By the Cantor's theorem there exists a point $u\in K$ such that $\{u\}=\bigcap_{n=1}^{n=\infty}\overline{TK_n}$. Since $u\in K$ for each $n=1,2,3,.....,$ there exists a point $x_n\in K_n$ such that $d\left(u,Tx_n\right)<\frac{1}{n}$, and so $Tx_n\to u.$ Further, since $x_n\in K_n$, we have $d\left(Tx_n,Ix_n\right)<\frac{1}{n}$ and $Ix_n\to u.$ Since I is continuous and the pair $\left(T,I\right)$ is compatible, we can induce easily that

$$ITx_n\,,IIx_n,TIx_n\to Iu.$$

Now

$$\begin{aligned}d\left(Tu,Iu\right)&\le d\left(Tu,TIx_n\right)+d\left(TIx_n,Iu\right)\\&\le ad\left(Iu,IIx_n\right)+b\max\left\{d\left(Iu,Tu\right),d\left(IIx_n,TIx_n\right)\right\}+c\max\left\{d\left(IIx_n,Tu\right),d\left(Iu,TIx_n\right)\right\}\\&\quad+d\max\left\{d\left(Iu,IIx_n\right),d\left(Iu,Tu\right),d\left(IIx_n,TIx_n\right),\frac{1}{2}\left(d\left(IIx_n,Tu\right)+d\left(Iu,TIx_n\right)\right)\right\}\end{aligned}$$

Now letting $n \to \infty$ we have

$$\leq ad(Iu, Iu) + b\max\{d(Iu, Tu), d(Iu, Iu)\} + c\max\{d(Iu, Tu), d(Iu, Iu)\}$$
$$+ d\max\left\{d(Iu, Iu), d(Iu, Tu), d(Iu, Iu), \frac{1}{2}(d(Iu, Tu) + d(Iu, Iu))\right\}$$
$$\Rightarrow d(Tu, Iu) \leq (b + c + d)\, d(Tu, Iu)$$

Which implies that

$$d(Tu, Iu) = 0$$

$$\Rightarrow Tu = Iu$$

Hence

$IIu = ITu = TIu = TTu$ since the pair (T, I) is compatible.

Now

$$d(TTu, Tu) \leq ad(ITu, Iu) + b\max\{d(ITu, TTu), d(Iu, Tu)\} + c\max\{d(Iu, TTu), d(ITu, Tu)\}$$
$$+ d\max\left\{d(ITu, Iu), d(Iu, Tu), d(ITu, TTu), \frac{1}{2}(d(Iu, TTu) + d(ITu, Tu))\right\}$$
$$\Rightarrow d(TTu, Tu) \leq (a + c + d)\, d(TTu, Tu) \qquad \text{Since } a + c + d < 1$$

Which implies that

$$d(TTu, Tu) = 0$$

$$\Rightarrow TTu = Tu$$

Now let $z = Tu$, then z is a common fixed point of $T \,\&\, I$. If z^* is another common fixed point of $T \,\&\, I$. then $d(z, z^*) > 0$.

Now

$$d(z, z^*) = d(Tz, Tz^*)$$
$$\leq ad(Iz, Iz^*) + b\max\{d(Iz, Tz), d(Iz^*, Tz^*)\} + c\max\{d(Iz^*, Tz), d(Iz, Tz^*)\}$$
$$+ d\max\left\{d(Iz, Iz^*), d(Iz, Tz), d(Iz^*, Tz^*), \frac{1}{2}(d(Iz^*, Tz) + d(Iz, Tz^*))\right\}$$
$$\Rightarrow d(z, z^*) \leq (a + c + d)\, d(z, z^*)$$

Which is a contradiction. Hence z is the unique common fixed point of $T \,\&\, I$.

To show that T is continuous at z. Let $\{z_n\} \subset K$ be a sequence converges to z. Since I is continuous then $Iz_n \to Iz$. Now we can deduce that

$$d\left(Tz_n,Tz\right)\le ad\left(Iz_n,Iz\right)+b\max\left\{d\left(Iz_n,Tz_n\right),d\left(Iz,Tz\right)\right\}+c\max\left\{d\left(Iz,Tz_n\right),d\left(Iz_n,Tz\right)\right\}$$
$$+d\max\left\{d\left(Iz_n,Iz\right),d\left(Iz_n,Tz_n\right),d\left(Iz,Tz\right),\frac{1}{2}\left(d\left(Iz,Tz_n\right)+d\left(Iz_n,Tz\right)\right)\right\}$$
$$\le ad\left(Iz,Iz\right)+b\max\left\{d\left(Iz,Tz_n\right),d\left(z,z\right)\right\}+c\max\left\{d\left(Iz,Tz\right)+d\left(Tz,Tz_n\right),d\left(Iz,Tz\right)\right\}$$
$$+d\max\left\{d\left(Iz,Iz\right),d\left(Tz,Tz_n\right),d\left(z,z\right),\frac{1}{2}\left(d\left(Iz,Tz\right)+d\left(Tz,Tz_n\right)+d\left(Iz,Tz\right)\right)\right\}$$
$$\Rightarrow d\left(Tz_n,Tz\right)\le\left(b+c+d\right)d\left(Tz_n,Tz\right)$$

Taking the limit as $n\to\infty$ from which we see that

$$\Rightarrow \limsup_{n\to\infty} d\left(Tz_n,Tz\right)\le\left(b+c+d\right)\limsup_{n\to\infty} d\left(Tz_n,Tz\right)$$
$$\lim_{n\to\infty} d\left(Tz_n,Tz\right)=0 \qquad \text{Since } \; b+c+d<1$$

Which implies that T is continuous at z.

This completes the proof of theorem.

Remark: If we put $c=0$ in main inequality then we will obtain the result of Huang and Li [4].

Corollary 3.2: Let (X,d) be a complete convex metric space with a convex structure W and K is a nonempty closed convex subset of X. Suppose that the mappings T and I are compatible self –maps on K satisfying the following condition

$$d\left(Tx,Ty\right)\le ad\left(Ix,Iy\right)+b\max\left\{d\left(Ix,Tx\right),d\left(Iy,Ty\right),\frac{1}{2}\left(d\left(Ix,Tx\right)+d\left(Iy,Ty\right)\right)\right\}$$
$$+c\max\left\{d\left(Iy,Tx\right),d\left(Ix,Ty\right),\frac{1}{2}\left(d\left(Iy,Tx\right)+d\left(Ix,Ty\right)\right)\right\}$$

Where a,b,c are non-negative real numbers such that $a+b+2c=1$ and $(a+c)(b+c)>0$ .If $T(K)\subset I(K)$ and I is W -affine and continuous then there exists a unique common fixed point z of T and I, and T is continuous at z.

Corollary 3.3: Let (X,d) be a complete convex metric space with a convex structure W and K is a nonempty closed convex subset of X. Suppose that the mappings T and I are compatible self –maps on K satisfying the following condition

$$d\left(Tx,Ty\right)\le ad\left(Ix,Iy\right)+b\max\left\{d\left(Ix,Tx\right),d\left(Iy,Ty\right)\right\}+c\max\left\{d\left(Iy,Tx\right),d\left(Ix,Ty\right)\right\}$$
$$+d\max\left\{d\left(Ix,Iy\right),d\left(Ix,Tx\right),d\left(Iy,Ty\right),d\left(Iy,Tx\right),d\left(Ix,Ty\right)\right\}$$

Where a, b, c, d are non-negative real numbers such that $a+b+2c+2d=1$ and $(a+c+d)(b+c+d)>0$. If $T(K)\subset I(K)$ and I is W -affine and continuous then there exists a unique common fixed point z of T and I, and T is continuous at z.

REFERENCES

[1] Fisher,B. and Sessa S., "On a fixed point theorem of Gregustype", Publ. Math. Debrecen, 34 (1987), 83-89.

[2] Gregus Jr.M., "A fixed point theorem in Banach space", Boll. Un. Mat. Ital., 517(1980), 193-198.

[3] Huang,N.J., "Stability for fixed point iterative procedures of nonlinear mappings in convex metric spaces", J. Gannan Teacher's College, 3(1991), 22-28.

[4] Huang,N.J. and Li,H.X., "Fixed point theorems of compatible mappings in convex metric spaces", Soochow Journal of Mathematics, 22(1996),439-447.

[5] Jungck,G., "Compatible mapping and common fixed points" International J. Math. Math. Sci., 9 (1986), 771-779.

[6] Jungck,G., "On a fixed point theorem of Fisher and Sessa" International J. Math. Math. Sci., 13 (1990), 497-500.

[7] Jungck,G., "Compatible mappings and common fixed points " International J. Math. Math. Sci., 17(1994), 37-40.

[8] Jungck,G., "Coincidence and fixed points for compatible and relatively nonexpansive maps" International J. Math. Math. Sci., 16 (1993),95-100.

[9] Jungck,G., "Compatible mapping and common fixed points(2)" International J. Math. Math. Sci., 11(1988), 265-288.

[10] Jungck,G. and Rhodes B.E., "Some fixed point theorems for compatible maps" International J. Math. Math. Sci., 16 (1993), 417-428.

[11] Li,B.Y. ,"Fixed point theorems for non expansive mappings in convex metric spaces", Appl . Math. Mech., 10 (1989), 173-178.

[12] Mukherjee, R. N. and Verma , V., "A note on fixed point theorem of Gregus", Math. Japonica, 33 (1988), 745-749.

[13] Sessa,S., "On a weak commutativity condition of mapping in a fixed point considerations", Publ. Inst. Math. Debre. , 32(1982), 149-153.

[14] Takahashi,W. "A convexity in metric spaces and nonexpansive mappings" Kodai Math. Sem. Rep. 229(1970) 142-149.

Source of support: Nil, Conflict of interest: None Declared

