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SOME COMMON FIXED POINT THEOREMS WITH COMPATIBLE MAPPINGS AND CONVEXITY IN METRIC SPACE

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ABSTRACT

In this note we establish some common fixed point theorems of compatible mappings in the setting of convex metric spaces. Several results are also derived as special cases.

Keywords: *Fixed point, Compatible mappings, Convex metric spaces.*

1. INTRODUCTION

The concept of the commutativity has generalized in several ways.For this Sessa S. [13] has introduced the concept of weakly commuting and Gerald Jungck [5] initiated the concept of compatibility. It can be easily verified that when the two mappings are commuting then they are compatible but not conversely.

The study of common fixed point of mappings satisfying contractive type conditions has been a useful tool for obtaining more comprehensive fixed point theorems[3,6,7,8,9,10] during the last three decades.

Jungck [5] introduced more generalized commuting mappings called compatible mappings. Which are more general then commuting and weakly commuting mappings.

On the other hand, Takahanshi [14] introduced a notion of convex metric spaces, many authers have discussed the existence of fixed point and the convergence of iterative processes for non-expansive mappings in convex metric spaces.

The main purpose of this paper is to present fixed point results for a pair of maps satisfying a new contractive condition by using the concept of compatible maps in a convex metric space. Our main results are generalizations of some known results in [1, 2, 4, 6, 11, 12].

2. PRELIMINARIES

Definition 2.1: The pair (A, B) of self-mappings of a metric space (X, d) is said to be compatible on X if whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to t \in X$ then $d(BAx, ABx) \to 0$.

Definition 2.2: Let (X, d) be a metric space and $J = [0,1]$.A mapping $W : X \times X \times J \to X$ is called a convex structure on X if for each $(x, y, \lambda) \in X \times X \times J$ and $u \in X$,

$$d\left(u, W\left(x, y, \lambda\right)\right) \le \lambda d\left(u, x\right) + \left(1-\lambda\right) d\left(u, y\right)$$

A metric space X together with a convex structure W is called a convex metric space.

Definition 2.3: A nonempty subset K of a convex metric space (X, d) with a convex structure W is said to be convex if for all $(x, y, \lambda) \in K \times K \times J$, $W(x, y, \lambda) \in K$.

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Definition 2.4: Let K be a convex subset of a convex metric space (X,d) with a convex structure W. A mapping $I:K\to K$ is said to be W-affine if for all $(x,y,\lambda)\in K\times K\times J$, $IW(x,y,\lambda)=W(Ix,Iy,\lambda)$

Definition 2.5: A sequence $\{x_n\}$ in a metric space (X,d) is said to be convergent to a point $x\in X$ denoted by $\lim_{n\to\infty} x_n = x$ if $\lim_{n\to\infty} d(x_n,x)=0$.

3. MAIN RESULTS

Theorem 3.1: Let (X,d) be a complete convex metric space with a convex structure W and K is a nonempty closed convex subset of X. Suppose that the mappings T and I are compatible self –maps on K satisfying the following condition

$$d(Tx,Ty)\le ad(Ix,Iy)+b\max\{d(Ix,Tx),d(Iy,Ty)\}+c\max\ \{d(Iy,Tx),d(Ix,Ty)\}$$
$$+d\max\left\{d(Ix,Iy),d(Ix,Tx),d(Iy,Ty),\frac{1}{2}\big(d(Iy,Tx)+d(Ix,Ty)\big)\right\}\quad \forall x,y\in K$$

Where a, b, c, d are non-negative real numbers such that $a+b+2c+d=1$ and $\dfrac{2(a+c)(b+c)}{\left(2-\overline{b+c}\right)}>d$. If $T(K)\subset I(K)$ and I is W-affine and continuous then there exists a unique common fixed point z of T and I, and T is continuous at z.

Proof: Let $x=x_0$ be an arbitrary point in K and choose some points x_1,x_2 and x_3 in K such that $Tx=Ix_1, Tx_1=Ix_2, Tx_2=Ix_3$ It is possible since $T(K)\subset I(K)$.

For $k=1,2,3$ we have

$$d(Tx_k,Ix_k)=d(Tx_k,Tx_{k-1})$$
$$\le ad(Ix_k,Ix_{k-1})+b\max\{d(Ix_k,Tx_k),d(Ix_{k-1},Tx_{k-1})\}+c\max\ \{d(Ix_{k-1},Tx_k),d(Ix_k,Tx_{k-1})\}$$
$$+d\max\left\{d(Ix_k,Ix_{k-1}),d(Ix_k,Tx_k),d(Ix_{k-1},Tx_{k-1}),\frac{1}{2}\big(d(Ix_{k-1},Tx_k)+d(Ix_k,Tx_{k-1})\big)\right\}$$
$$\le ad(Ix_{k-1},Tx_{k-1})+b\max\{d(Ix_k,Tx_k),d(Ix_{k-1},Tx_{k-1})\}+c\max\ \{d(Ix_{k-1},Tx_{k-1}),d(Ix_k,Tx_k)\}$$
$$+d\max\left\{d(Ix_{k-1},Tx_{k-1}),d(Ix_k,Tx_k),\frac{1}{2}\big(d(Ix_{k-1},Tx_{k-1})+d(Ix_k,Tx_k)\big)\right\}$$

If

$d(Tx_k,Ix_k)>d(Tx_{k-1},Ix_{k-1})$ then we have

$$\Rightarrow d(Tx_k,Ix_k)<(a+b+2c+d)d(Tx_k,Ix_k)$$
$$\Rightarrow d(Tx_k,Ix_k)<d(Tx_k,Ix_k)\qquad \text{Since}\quad a+b+2c+d=1$$

Which is a contradiction and therefore

$$d(Ix_k,Tx_k)\le d(Ix_{k-1},Tx_{k-1})\qquad \text{Where } k=1,2,3$$

Now

$$\begin{aligned}
d\left(Ix_1,Tx_2\right) &= d\left(Tx,Tx_2\right)\\
&\le ad\left(Ix_0,Ix_2\right)+b\max\left\{d\left(Ix,Tx\right),d\left(Ix_2,Tx_2\right)\right\}+c\max\ \left\{d\left(Ix_2,Tx\right),d\left(Ix,Tx_2\right)\right\}\\
&\quad +d\max\left\{d\left(Ix,Ix_2\right),d\left(Ix,Tx\right),d\left(Ix_2,Tx_2\right),\frac{1}{2}\left(d\left(Ix,Tx_2\right)+d\left(Ix_2,Tx\right)\right)\right\}\\
&\le a\left\{d\left(Ix,Tx\right)+d\left(Ix_1,Tx_1\right)\right\}+b\,d\left(Ix,Tx\right)\\
&\quad +c\max\ \left\{d\left(Ix_1,Tx_1\right),d\left(Ix,Tx\right)+d\left(Ix_1,Tx_1\right)+d\left(Ix_2,Tx_2\right)\right\}\\
&\quad +d\max\left\{\begin{array}{l} d\left(Ix,Tx\right)+d\left(Ix_1,Tx_1\right),d\left(Ix,Tx\right),d\left(Ix,Tx\right),\\ \frac{1}{2}\left(d\left(Ix,Tx\right)+d\left(Ix_1,Tx_1\right)+d\left(Ix_2,Tx_2\right)+d\left(Ix_1,Tx_1\right)\right)\end{array}\right\}\\
&= \left(2a+b+3c+2d\right)d\left(Ix,Tx\right)\\
&= \left(2-\overline{b+c}\right)d\left(Ix,Tx\right)
\end{aligned}$$

Let $z=W\left(x_2,x_3,\frac{1}{2}\right)$ then we know that $z\in K$ and $Iz=W\left(Ix_2,Ix_3,\frac{1}{2}\right)=W\left(Tx_1,Tx_2,\frac{1}{2}\right)$

Now we obtain

$$\begin{aligned}
d\left(Iz,Ix_1\right) &= d\left(Ix_1,W\left(Tx_1,Tx_2,\frac{1}{2}\right)\right)\\
&\le \frac{1}{2}d\left(Ix_1,Tx_1\right)+\frac{1}{2}d\left(Ix_1,Tx_2\right)\\
&\le \frac{1}{2}d\left(Ix,Tx\right)+\frac{1}{2}\left(2a+b+3c+2d\right)d\left(Ix,Tx\right)\\
&= \frac{1}{2}\left(1+2a+b+3c+2d\right)d\left(Ix,Tx\right)\\
&= \frac{3-\overline{b+c}}{2}d\left(Ix,Tx\right)
\end{aligned}$$

$$\begin{aligned}
d\left(Iz,Ix_2\right) &= d\left(Ix_2,W\left(Tx_1,Tx_2,\frac{1}{2}\right)\right)\\
&\le \frac{1}{2}d\left(Ix_2,Tx_1\right)+\frac{1}{2}d\left(Ix_2,Tx_2\right)\\
&\le \frac{1}{2}d\left(Ix_2,Ix_2\right)+\frac{1}{2}d\left(Ix,Tx\right)\\
&= \frac{1}{2}d\left(Ix,Tx\right)
\end{aligned}$$

$$\begin{aligned}
d\left(Iz,Ix_3\right) &= d\left(Ix_3,W\left(Tx_1,Tx_2,\frac{1}{2}\right)\right)\\
&\le \frac{1}{2}d\left(Ix_3,Tx_1\right)+\frac{1}{2}d\left(Ix_3,Tx_2\right)\\
&\le \frac{1}{2}d\left(Ix_2,Tx_2\right)+\frac{1}{2}d\left(Ix_3,Ix_3\right)\\
&= \frac{1}{2}d\left(Ix,Tx\right)
\end{aligned}$$

Again we obtain

$$d(Iz,Tz) = d\left(Tz, W\left(Tx_1, Tx_2, \frac{1}{2}\right)\right)$$

$$\leq \frac{1}{2}d(Tz,Tx_1) + \frac{1}{2}d(Tz,Tx_2)$$

$$\leq \frac{a}{2}d(Iz,Ix_1) + \frac{b}{2}\max\{d(Iz,Tz), d(Ix_1,Tx_1)\} + \frac{c}{2}\max\ \{d(Ix_1,Tz), d(Iz,Tx_1)\}$$

$$+\frac{d}{2}\max\left\{d(Iz,Ix_1), d(Iz,Tz), d(Ix_1,Tx_1), \frac{1}{2}\left(d(Ix_1,Tz) + d(Iz,Tx_1)\right)\right\}$$

$$+\frac{a}{2}d(Iz,Ix_2) + \frac{b}{2}\max\{d(Iz,Tz), d(Ix_2,Tx_2)\} + \frac{c}{2}\max\ \{d(Ix_2,Tz), d(Iz,Tx_2)\}$$

$$+\frac{d}{2}\max\left\{d(Iz,Ix_2), d(Iz,Tz), d(Ix_2,Tx_2), \frac{1}{2}\left(d(Ix_2,Tz) + d(Iz,Tx_2)\right)\right\}$$

$$\leq \frac{a}{4}\left(3 - \overline{b+c}\right)d(Ix,Tx) + \frac{b}{2}\max\{d(Iz,Tz), d(Ix,Tx)\}$$

$$+\frac{c}{2}\max\ \left\{\frac{1}{2}(3-\overline{b+c})d(Ix,Tx) + d(Iz,Tz), \frac{1}{2}d(Ix,Tx)\right\}$$

$$+\frac{d}{2}\max\left\{\frac{1}{2}(3-\overline{b+c})d(Ix,Tx), d(Iz,Tz), d(Ix,Tx), \frac{4-\overline{b+c}}{4}d(Ix,Tx) + \frac{1}{2}d(Iz,Tz)\right\}$$

$$+\frac{a}{4}d(Ix,Tx) + \frac{b}{2}\max\{d(Iz,Tz), d(Ix,Tx)\} + \frac{c}{2}\max\ \left\{\frac{1}{2}d(Ix,Tx) + d(Iz,Tz), \frac{1}{2}d(Ix,Tx)\right\}$$

$$+\frac{d}{2}\max\left\{\frac{1}{2}d(Ix,Tx), d(Iz,Tz), d(Ix,Tx), \frac{1}{2}\left(d(Ix,Tx) + d(Iz,Tz)\right)\right\}$$

If $d(Iz,Tz) > d(Ix,Tx)$ then we have

$$d(Iz,Tz) \leq \left[1 - \frac{1}{8}\left(2c^2 - 2d + 2ab + 2ac + 2bc + bd + cd\right)\right]d(Iz,Tz)$$

This is a contradiction since $\dfrac{2(a+c)(b+c)}{\left(2-\overline{b+c}\right)} > d$ and $a+b+2c+d=1$ implies that

$2c^2 - 2d + 2ab + 2ac + 2bc + bd + cd > 0.$ Hence we have

$$d(Iz,Tz) \leq \left[1 - \frac{1}{8}\left(2c^2 - 2d + 2ab + 2ac + 2bc + bd + cd\right)\right]d(Ix,Tx)$$

Letting $\lambda = 1 - \frac{1}{8}\left(2c^2 - 2d + 2ab + 2ac + 2bc + bd + cd\right)$ We know that $0 < \lambda < 1.$

Then we have

$$d(Iz,Tz) \leq \lambda d(Ix,Tx)$$

Since x_0 is an arbitrary point in K then there exists a sequence $\{z_n\}$ in K such that

$$d(Iz_0,Tz_0) \leq \lambda d(Ix_0,Tx_0)$$

$$d(Iz_1,Tz_1) \leq \lambda d(Iz_0,Tz_0)$$

....

....

$$d(Iz_n,Tz_n) \leq \lambda d(Iz_{n-1},Tz_{n-1})$$

Which yield that $d\left(Iz_n, Tz_n\right) \le \lambda^{n+1} d\left(Iz_0, Tz_0\right)$ and so we have

$$d\left(Iz_n, Tz_n\right) \to 0$$

Setting

$$K_n = \left\{ x \in K : d\left(Ix, Tx\right) \le \frac{1}{n} \right\} \quad , \quad n = 1, 2, 3 \ldots\ldots\ldots\ldots\ldots\ldots$$

Then $K_n \ne \phi$, $n = 1, 2, 3, \ldots\ldots\ldots\ldots$ and $K_1 \supset K_2 \supset \ldots\ldots\ldots\ldots\ldots\ldots$

Obviously

$$\overline{TK_n} \ne \phi \, and \, \overline{TK_n} \supset \overline{TK_{n+1}} , n = 1, 2, 3 \ldots\ldots\ldots$$

For any $x, y \in K_n$ we have

$$\begin{aligned} d\left(Tx, Ty\right) &\le ad\left(Ix, Iy\right) + b \max\left\{ d\left(Ix, Tx\right), d\left(Iy, Ty\right) \right\} + c \max \left\{ d\left(Iy, Tx\right), d\left(Ix, Ty\right) \right\} \\ &\quad + d \max\left\{ d\left(Ix, Iy\right), d\left(Ix, Tx\right), d\left(Iy, Ty\right), \frac{1}{2}\left(d\left(Iy, Tx\right) + d\left(Ix, Ty\right) \right) \right\} \\ &\le a\left(\frac{2}{n} + d\left(Tx, Ty\right) \right) + \frac{b}{n} + c\left(\frac{1}{n} + d\left(Tx, Ty\right) \right) + d\left(\frac{2}{n} + d\left(Tx, Ty\right) \right) \\ &= \frac{1}{n}\left(2a + b + c + 2d \right) + (a + c + d) d\left(Tx, Ty\right) \end{aligned}$$

Which yields that

$$d\left(Tx, Ty\right) \le \frac{1}{n\left(b + c\right)}\left(2a + b + c + 2d \right)$$

Therefore we have

$$\lim_{n \to \infty} diam\left(\overline{TK_n} \right) = \lim_{n \to \infty} diam\left(TK_n \right) = 0$$

Where $diam\left(\overline{TK_n} \right)$ denotes the diameter of $\left(\overline{TK_n} \right)$. By the Cantor's theorem there exists a point $u \in K$ such that $\{u\} = \bigcap_{n=1}^{n=\infty} TK_n$. Since $u \in K$ for each $n = 1, 2, 3, \ldots..$, there exists a point $x_n \in K_n$ such that $d\left(u, Tx_n\right) < \frac{1}{n}$, and so $Tx_n \to u$. Further, since $x_n \in K_n$, we have $d\left(Tx_n, Ix_n\right) < \frac{1}{n}$ and $Ix_n \to u$. Since I is continuous and the pair $\left(T, I\right)$ is compatible, we can induce easily that

$$ITx_n , IIx_n , TIx_n \to Iu.$$

Now

$$\begin{aligned} d\left(Tu, Iu\right) &\le d\left(Tu, TIx_n\right) + d\left(TIx_n, Iu\right) \\ &\le ad\left(Iu, IIx_n\right) + b \max\left\{ d\left(Iu, Tu\right), d\left(IIx_n, TIx_n\right) \right\} + c \max\left\{ d\left(IIx_n, Tu\right), d\left(Iu, TIx_n\right) \right\} \\ &\quad + d \max\left\{ d\left(Iu, IIx_n\right), d\left(Iu, Tu\right), d\left(IIx_n, TIx_n\right), \frac{1}{2}\left(d\left(IIx_n, Tu\right) + d\left(Iu, TIx_n\right) \right) \right\} \end{aligned}$$

Now letting $n \to \infty$ we have

$$\leq ad(Iu,Iu)+b\max\{d(Iu,Tu),d(Iu,Iu)\}+c\max\{d(Iu,Tu),d(Iu,Iu)\}$$
$$+d\max\left\{d(Iu,Iu),d(Iu,Tu),d(Iu,Iu),\frac{1}{2}\big(d(Iu,Tu)+d(Iu,Iu)\big)\right\}$$
$$\Rightarrow d(Tu,Iu)\leq(b+c+d)\,d(Tu,Iu)$$

Which implies that

$$d(Tu,Iu)=0$$

$$\Rightarrow Tu=Iu$$

Hence

$IIu = ITu = TIu = TTu$ since the pair (T,I) is compatible.

Now

$$d(TTu,Tu)\leq ad(ITu,Iu)+b\max\{d(ITu,TTu),d(Iu,Tu)\}+c\max\{d(Iu,TTu),d(ITu,Tu)\}$$
$$+d\max\left\{d(ITu,Iu),d(Iu,Tu),d(ITu,TTu),\frac{1}{2}\big(d(Iu,TTu)+d(ITu,Tu)\big)\right\}$$
$$\Rightarrow d(TTu,Tu)\leq(a+c+d)\,d(TTu,Tu)\qquad \text{Since } a+c+d<1$$

Which implies that

$$d(TTu,Tu)=0$$

$$\Rightarrow TTu=Tu$$

Now let $z=Tu,$ then z is a common fixed point of $T\,\&\,I.$ If z^* is another common fixed point of $T\,\&\,I.$ then $d(z,z^*)>0.$

Now

$$d(z,z^*)=d(Tz,Tz^*)$$
$$\leq ad(Iz,Iz^*)+b\max\{d(Iz,Tz),d(Iz^*,Tz^*)\}+c\max\{d(Iz^*,Tz),d(Iz,Tz^*)\}$$
$$+d\max\left\{d(Iz,Iz^*),d(Iz,Tz),d(Iz^*,Tz^*),\frac{1}{2}\big(d(Iz^*,Tz)+d(Iz,Tz^*)\big)\right\}$$
$$\Rightarrow d(z,z^*)\leq(a+c+d)\,d(z,z^*)$$

Which is a contradiction. Hence z is the unique common fixed point of $T\,\&\,I.$

To show that T is continuous at $z.$ Let $\{z_n\}\subset K$ be a sequence converges to z. Since I is continuous then $Iz_n \to Iz.$ Now we can deduce that

$$d\left(Tz_n,Tz\right)\le ad\left(Iz_n,Iz\right)+b\max\left\{d\left(Iz_n,Tz_n\right),d\left(Iz,Tz\right)\right\}+c\max\left\{d\left(Iz,Tz_n\right),d\left(Iz_n,Tz\right)\right\}$$
$$+d\max\left\{d\left(Iz_n,Iz\right),d\left(Iz_n,Tz_n\right),d\left(Iz,Tz\right),\frac{1}{2}\left(d\left(Iz,Tz_n\right)+d\left(Iz_n,Tz\right)\right)\right\}$$
$$\le ad\left(Iz,Iz\right)+b\max\left\{d\left(Iz,Tz_n\right),d\left(z,z\right)\right\}+c\max\left\{d\left(Iz,Tz\right)+d\left(Tz,Tz_n\right),d\left(Iz,Tz\right)\right\}$$
$$+d\max\left\{d\left(Iz,Iz\right),d\left(Tz,Tz_n\right),d\left(z,z\right),\frac{1}{2}\left(d\left(Iz,Tz\right)+d\left(Tz,Tz_n\right)+d\left(Iz,Tz\right)\right)\right\}$$
$$\Rightarrow d\left(Tz_n,Tz\right)\le\left(b+c+d\right)d\left(Tz_n,Tz\right)$$

Taking the limit as $n\to\infty$ from which we see that

$$\Rightarrow \lim_{n\to\infty}\sup d\left(Tz_n,Tz\right)\le\left(b+c+d\right)\lim_{n\to\infty}\sup d\left(Tz_n,Tz\right)$$
$$\lim_{n\to\infty} d\left(Tz_n,Tz\right)=0 \qquad \text{Since } \ b+c+d<1$$

Which implies that T is continuous at z.

This completes the proof of theorem.

Remark: If we put $c=0$ in main inequality then we will obtain the result of Huang and Li [4].

Corollary 3.2: Let (X,d) be a complete convex metric space with a convex structure W and K is a nonempty closed convex subset of X. Suppose that the mappings T and I are compatible self –maps on K satisfying the following condition

$$d\left(Tx,Ty\right)\le ad\left(Ix,Iy\right)+b\max\left\{d\left(Ix,Tx\right),d\left(Iy,Ty\right),\frac{1}{2}\left(d\left(Ix,Tx\right)+d\left(Iy,Ty\right)\right)\right\}$$
$$+c\max\left\{d\left(Iy,Tx\right),d\left(Ix,Ty\right),\frac{1}{2}\left(d\left(Iy,Tx\right)+d\left(Ix,Ty\right)\right)\right\}$$

Where a,b,c are non-negative real numbers such that $a+b+2c=1$ and $(a+c)(b+c)>0$ .If $T(K)\subset I(K)$ and I is W -affine and continuous then there exists a unique common fixed point z of T and I, and T is continuous at z.

Corollary 3.3: Let (X,d) be a complete convex metric space with a convex structure W and K is a nonempty closed convex subset of X. Suppose that the mappings T and I are compatible self –maps on K satisfying the following condition

$$d\left(Tx,Ty\right)\le ad\left(Ix,Iy\right)+b\max\left\{d\left(Ix,Tx\right),d\left(Iy,Ty\right)\right\}+c\max\left\{d\left(Iy,Tx\right),d\left(Ix,Ty\right)\right\}$$
$$+d\max\left\{d\left(Ix,Iy\right),d\left(Ix,Tx\right),d\left(Iy,Ty\right),d\left(Iy,Tx\right),d\left(Ix,Ty\right)\right\}$$

Where a, b, c, d are non-negative real numbers such that $a+b+2c+2d=1$ and $(a+c+d)(b+c+d)>0$. If $T(K)\subset I(K)$ and I is W -affine and continuous then there exists a unique common fixed point z of T and I, and T is continuous at z.

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