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g^*-closed sets in bigeneralized topological spaces

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ABSTRACT

In this paper we introduce $g^*_{(m,n)}$-closed sets in bigeneralized topological spaces and study some of their properties. We introduce $\mu_{(m,n)}$- $T^*_{1/2}$ and $\mu_{(m,n)}$-$^*T_{1/2}$-spaces as application.

Keywords: *$\mu_{(m,n)}$-closed, $\mu_{(m,n)}$- g^*-closed, $\mu_{(m,n)}$- $T^*_{1/2}$-space, $\mu_{(m,n)}$-$^*T_{1/2}$-space and pairwise-μ- $T^*_{1/2}$-space.*

1. Introduction

The study of g^*- closed sets and g^* - continuity in a topological space was initiated by Veerakumar[6]. g^*- closed sets in bitopological spaces were introduced by M. Sheik John and P. Sundaram [5]. Á. Császár[2] introduced the concepts of generalized neighborhood systems and generalized topological spaces. He also introduced the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In particular, he investigated characterizations for the generalized continuous function by using closure operator defind on generalized topological spaces.

W. Dungthaisong, C. Boonpok and C. Viriyapong [4] introduced the concepts of generalized closed sets in bigeneralized topological spaces. He also introduced the concepts of generalized continuous functions and studied (m,n)- closed sets and (m,n)- open sets in bigeneralized topological spaces.

2. Preliminaries

Let X be a set. A subset μ of exp X is called generalized topology on X and (X, μ) is called a generalized topological space[2], if μ has the following properties
i) $\phi \in \mu$
ii) Any union of elements of μ belongs to μ.

Let μ be a Generalized Topological space (GT) on X, the elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets. For B ⊆ X, let I(B) be the largest μ-open subset of B. I(B) is the union of all μ-open subset of B. Let C(B) be the smallest μ-closed subset which contains B. In other words C(B) is the intersection of all μ-closed subsets which contain B.

Definition 2.1: [3] Let (X, μ) be a generalized topological space. A subset B of X is said to be
i) μ-semi-open iff B $\subseteq c_\mu(i_\mu(B))$
ii) μ-preopen iff B $\subseteq i_\mu(c_\mu(B))$
iii) μ-α-open iff B $\subseteq i_\mu(c_\mu(i_\mu(B)))$
iv) μ-β-open B $\subseteq c_\mu(i_\mu(c_\mu(B)))$

Definition 2.2: [1] Let X be a nonempty set and μ_1,μ_2 be generalized topologies on X. A triple (X, μ_1, μ_2) is said to be a bigeneralized topological space.

Remark 2.3: Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X. The closure of A and the interior of A with respect to μ_m are denoted by c_{μ_m}(A) and i_{μ_m}(A) respectively for m =1, 2. The family of all μ_n-closed sets is denoted by the symbol F_n.

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Definition 2.4: [1] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is called (m, n)-closed if $c_{\mu_m}(c_{\mu_n}$(A)) = A, where $m, n = 1, 2$ and $m \neq n$. The complements of (m, n) closed sets is called (m, n)-open.

Proposition 2.5: [1] Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X. Then A is (m, n)-closed if and only if A is both μ -closed in (X, μ_m) and (X, μ_n).

Definition 2.6: [4] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be (m, n) generalized closed (briefly $\mu_{(m,n)}$-closed) set if c_{μ_n}(A) ⊆ U whenever A ⊆ U and U is μ_m –open set in X, where $m, n = 1, 2$ and $m \neq n$. The complement of $\mu_{(m,n)}$-closed set is said to be (m,n) generalized open (briefly $\mu_{(m,n)}$-open) set.

Definition 2.7: [4] A bigeneralized topological space (X, μ_1, μ_2) is said to be $\mu_{(m,n)}$- $T_{1/2}$- space if, every $\mu_{(m,n)}$-closed set is μ_n-closed, where $m, n = 1, 2$ and $m \neq n$.

Definition 2.8: [4] A bigeneralized topological space (X, μ_1, μ_2) is said to be pairwise μ- $T_{1/2}$ - space it is both $\mu_{(1,2)}$- $T_{1/2}$- space and $\mu_{(2,1)}$- $T_{1/2}$- space.

3. (m, n)-g^*-closed sets

Definition 3.1: A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be (m, n)-g^*-closed(briefly $\mu_{(m,n)}$- g^* - closed) set if c_{μ_n}(A) ⊆ U whenever A ⊆ U and U is μ_m- g -open set in X, where $m, n = 1, 2$ and $m \neq n$. The complement of $\mu_{(m,n)}$- g^*-closed set is said to be briefly $\mu_{(m,n)}$- g^*-open set.

Definition 3.2: A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be $\mu_{(m,n)}$- wg – closed set if $c_{\mu_n}(i_{\mu_m}$(A)) ⊆ U whenever A ⊆ U and U is μ_m- open set in X, where $m, n = 1, 2$ and $m \neq n$.

We denote the family of all $\mu_{(m,n)}$- g^*-closed (resp. $\mu_{(m,n)}$- g^*-open) set in (X, μ_1, μ_2) is $\mu_{(m,n)}$- g^*-C(X)) (resp. $\mu_{(m,n)}$- g^*-O(X)), where $m, n = 1, 2$ and $m \neq n$.

Proposition 3.3: If A is (m, n)-closed subset of (X, μ_1, μ_2) then A is $\mu_{(m,n)}$- g^* - closed, where $m, n = 1, 2$ and $m \neq n$.

Proof: Let A ⊆ U and U is μ_m- g -open set. Since A is (m, n)-closed set, c_{μ_n}(A) = A ⊆ U. Hence A is $\mu_{(m,n)}$- g^* - closed.

The converse of the above proposition is not true as seen from the following example.

Example 3.4: Let X = {a, b, c}, μ_1 = {ϕ, {a}, {a, b}, X} and μ_2 = {ϕ, {c}, {b, c}}. Then the subset {c} is $\mu_{(1,\ 2)}$- g^* - closed but not (1, 2)-closed.

Proposition 3.5: If A is μ_n-closed subset of (X, μ_1, μ_2), then A is a $\mu_{(m,n)}$- g^* - closed set, where $m, n = 1, 2$ and $m \neq n$.

The converse of the above proposition is not true as seen from the following example.

Example 3.6: In Example 3.4, {c} is $\mu_{(1,\ 2)}$- g^* -closed but not μ_2-closed

Proposition 3.7: In a bigeneralized topological space (X, μ_1, μ_2), every $\mu_{(m,n)}$- g^*-closed set is (i) $\mu_{(m,n)}$ -closed (ii) $\mu_{(m,n)}$-wg -closed set, where $m, n = 1, 2$ and $m \neq n$.

Proof:
(i) Let A ⊆ U and U is a μ_m- open set. Since A is $\mu_{(m,n)}$- g^*-closed and every μ_m- open set is μ_m- g-open and c_{μ_n}(A) ⊆ U. Hence A is $\mu_{(m,n)}$ –closed.

(ii) Let A ⊆ U and U is μ_m- open set. Since A is $\mu_{(m,n)}$- g^*-closed i_{μ_m}(A) ⊆ U which implies $c_{\mu_n}(i_{\mu_m}$(A)) ⊆ c_{μ_n}(A) ⊆ U. Hence A is $\mu_{(m,n)}$-wg-closed.

The converse of the above proposition is not true as seen from the following example.

Example 3.8: In Example 3.4, the subset {b} is $\mu_{(1,\ 2)}$ - closed and $\mu_{(1,\ 2)}$-wg-closed but not a $\mu_{(1,2)}$- g^*-closed.

Remark 3.9: The union of two $\mu_{(m,n)}$- g^*-closed sets need not be a $\mu_{(m,n)}$- g^*-closed set as seen from the following example.

Example 3.10: If X = {a, b, c, d}, μ_1 = {ϕ, {a, b}, {b, c}, {a, b, c}} and μ_2 = {ϕ, {a, b, d}, {b, c, d}, X}, then {a} and {c} are $\mu_{(1,2)}$- g^*-closed but {a} ∪ {c}= {a, c} is not a $\mu_{(1,2)}$- g^*-closed.

Remark 3.11: The intersection of two $\mu_{(m,n)}$- g^*-closed sets need not be a $\mu_{(m,n)}$- g^*-closed set as seen from the following example.

Example 3.12: In Example 3.4, {a, b} and {b, c} are $\mu_{(1,2)}$- g^*-closed but {a, b} ∩ {b, c} = {b} is not $\mu_{(1,2)}$- g^*-closed.

Proposition 3.13: In a bigeneralized topological space (X, μ_1, μ_2), if $\mu_1 \subseteq \mu_2$ then $\mu_{(2,1)}$- g^*-C(X) ⊆ $\mu_{(1,2)}$- g^*-C(X).

Proof: Let A be a $\mu_{(2,1)}$- g^*-closed set and U be a μ_1- g-open set containing A. Since $\mu_1 \subseteq \mu_2$, we have c_{μ_2}(A) ⊆ c_{μ_1}(A) and μ_1-C(X) ⊆ μ_2-C(X). Since A ∈ $\mu_{(2,1)}$- g^*-C(X), c_{μ_1}(A) ⊆ U. Therefore c_{μ_2}(A) ⊆ U, U is $\mu_1 - g$-open. Thus A ∈ $\mu_{(1,\ 2)}$- g^*-C(X).

The converse of the above proposition is not true as seen from the following example.

Example 3.14: Let X = {a, b, c}, μ_1 = {ϕ, {a}, {a, b}} and μ_2 = {ϕ, {a}}. Then $\mu_{(1,\ 2)}$- g^*-C(X) = {{c}, {b, c}, {a, c}, X} and $\mu_{(2,1)}$- g^*-C(X) = {{c}, {b, c}, X}. Here $\mu_{(2,1)}$- g^*-C(X) ⊆ $\mu_{(1,2)}$- g^*-C(X), but $\mu_1 \not\subset \mu_2$.

Proposition 3.15: For each x of (X, μ_1, μ_2), {x} is μ_m- g -closed or {x}^{c} is $\mu_{(m,n)}$- g^*-closed, where $m, n = 1, 2\ and\ m \neq n$.

Proof: Let x ∈X and {x} be not μ_m- g - closed. Then X – {x} is not μ_m- g - open, if X is μ_m- g -open then X is only μ_m- g - open set which contains X – {x} and so X – {x} is $\mu_{(m,n)}$- g^*-closed and if X is not μ_m- g-open then X – {x} is $\mu_{(m,n)}$- g^*-closed.

Proposition 3.16: Let A be a subset of a bigeneralized topological space (X, μ_1, μ_2). If A is $\mu_{(m,n)}$- g^*-closed then c_{μ_n}(A) –A contains no non empty μ_m- g -closed set, where $m, n = 1, 2\ and\ m \neq n$.

Proof: Let A be an $\mu_{(m,n)}$- g^*-closed set and F≠ ϕ is μ_m- g-closed set such that F ⊆ c_{μ_n}(A)–A. Since A ∈$\mu_{(m,n)}$- g^*-C(X), we have c_{μ_n}(A) ⊆ X – F. Thus F ⊆ (c_{μ_n}(A)) ∩ (X–c_{μ_n}(A)) = ϕ, which is a contradiction to our assumption. Then c_{μ_n}(A) – A contains no nonemptyμ_m- g-closed set.

The converse of the above proposition is not true as seen from the following example.

Example 3.17: Let X = {a, b, c}, μ_1 = {ϕ, {a}, {a, b}} and μ_2 = {ϕ, {b, c}}. If A = ϕ then c_{μ_2}(A) – A= {a} does not contain any non empty μ_1- g - closed set. But A is not $\mu_{(1,\ 2)}$- g^*-closed set.

Proposition 3.18: If A is a $\mu_{(m,n)}$- g^*-closed set of (X, μ_1, μ_2) such that A ⊆ B ⊆ c_{μ_n}(A), then B is also an $\mu_{(m,n)}$- g^*-closed set, where $m, n = 1, 2\ and\ m \neq n$.

Proof: Let A be a $\mu_{(m,n)}$- g^*-closed set and A⊆ B ⊆ c_{μ_n}(A). Let B ⊆ U and U is μ_m- g-open. Then A ⊆U. Since A is $\mu_{(m,n)}$- g^*-closed, we have c_{μ_n}(A) ⊆ U. Since B ⊆ c_{μ_n}(A), then c_{μ_n}(B) ⊆ c_{μ_n}(A) ⊆ U. Hence B is $\mu_{(m,n)}$- g^*-closed.

Proposition 3.19: In a bigeneralized topological space (X, μ_1, μ_2), GO(X, μ_m) ⊆ F_n if and only if every subset of X is a $\mu_{(m,n)}$- g^*-closed set, where $m,\ n = 1, 2\ and\ m \neq n$.

Proof: Suppose that GO(X, μ_m) $\subseteq F_n$. Let A be a subset of X such that A $\subseteq$ U, where U $\in$ GO(X, μ_m). Then c_{μ_n}(A) $\subseteq$ c_{μ_n}(U) $\subseteq$ U. Hence A is $\mu_{(m,n)}$- g^*-closed set.

Conversely, Suppose that every subset of X is $\mu_{(m,n)}$- g^*-closed. Let U $\in$ GO(X, μ_m). Since U is $\mu_{(m,n)}$- g^*-closed, we have c_{μ_n}(U) $\subseteq$ U. Therefore U is a μ_n-closed and hence GO(X, μ_m) $\subseteq F_n$.

Proposition 3.20: Let (X, μ_1, μ_2) be a bigeneralized topological space. If A is a $\mu_{(m,n)}$- g^*-closed set then $c_{\mu_m}(\{x\}) \cap$ A $\neq \phi$ holds for each x $\in c_{\mu_n}$(A), where $m,\ n\ =\ 1, 2\ and\ m\ \neq\ n.$

Proof: Let x $\in c_{\mu_n}$(A). Suppose that $c_{\mu_m}(\{x\}) \cap$ A = ϕ. Then A $\subseteq$ X – $c_{\mu_m}(\{x\})$. Since A is $\mu_{(m,n)}$- g^*-closed and X – $c_{\mu_m}(\{x\})$ is $\mu_m - g$-open . Thus c_{μ_n}(A) $\subseteq$ X – $c_{\mu_m}(\{x\})$. Hence c_{μ_n}(A) $\cap c_{\mu_m}(\{x\})$ = ϕ, which is a contradiction to our assumption. Therefore $c_{\mu_m}(\{x\}) \cap$ A $\neq \phi$.

Proposition 3.21: If A is μ_n-closed subset of (X, μ_1, μ_2) then A is a $\mu_{(m,\ n)}$- wg – closed set, where $m,\ n\ =\ 1, 2\ and\ m\ \neq\ n.$

The converse of the above proposition is not true as seen from the following example.

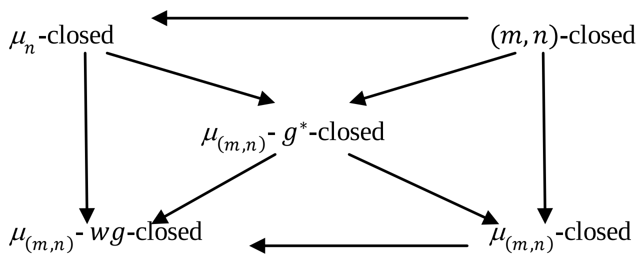
Example 3.22: In Example 3.8, {c} is $\mu_{(1\ ,\ 2)}$- wg-closed but not μ_2-closed.

Proposition 3.23: In a bigeneralized topological space (X, μ_1, μ_2), every $\mu_{(m,n)}$-closed set is a $\mu_{(m,\ n)} - wg$-closed set, where $m,\ n\ =\ 1, 2\ and\ m\ \neq\ n.$

Proof: Let A be a $\mu_{(m,n)}$-closed set. Let A $\subseteq$ U and U is an μ_m-open. Since A is $\mu_{(m,n)}$-closed, $c_{\mu_n}(i_{\mu_m}$(A)) $\subseteq c_{\mu_n}$(A) $\subseteq$ U. Therefore A is a $\mu_{(m,n)} - wg$-closed set.

The converse of the above proposition is not true as seen from the following example.

Example 3.24: In Example 3.10, {b} is $\mu_{(1,2)} - wg$-closed set. But not $\mu_{(1,2)}$-closed.



4. $\mu_{(m,n)}$- $T^*_{1/2}$- spaces and $\mu_{(m,n)}$-* $T_{1/2}$- spaces

In this section we introduce $\mu_{(m,n)}$- $T^*_{1/2}$ and $\mu_{(m,n)}$-* $T_{1/2}$ and pairwise μ- $T^*_{1/2}$- space in bigeneralized topological spaces and some of their properties.

Definition 4.1: A bigeneralized topological space (X, μ_1, μ_2) is said to be an $\mu_{(m,n)}$- $T^*_{1/2}$-space if every $\mu_{(m,n)}$- g^*-closed set is μ_n-closed, where $m,\ n\ =\ 1, 2\ and\ m\ \neq\ n.$

Definition 4.2: A bigeneralized topological space (X, μ_1, μ_2) is said to be pairwise μ- $T^*_{1/2}$- space if it is both $\mu_{(m,n)}$- $T^*_{1/2}$-space and $\mu_{(n,m)}$- $T^*_{1/2}$, where $m,\ n\ =\ 1, 2\ and\ m\ \neq\ n.$

Definition 4.3: A bigeneralized topological space (X, μ_1, μ_2) is said to be an $\mu_{(m,n)}$- $^*T_{1/2}$space if every $\mu_{(m,n)}$-closed set is $\mu_{(m,n)}$- g^*-closed, where $m,\ n\ =\ 1, 2\ and\ m\ \neq\ n.$

Proposition 4.4: If (X, μ_1, μ_2) is $\mu_{(m,n)}$- $T_{1/2}$- space, then it is an $\mu_{(m,n)}$-$T^*_{1/2}$-space, where $m,\ n\ =\ 1, 2\ and\ m\ \neq\ n.$

The converse of the above proposition is not true as seen from the following example.

Example 4.5: Let X = {a, b, c} μ_1 = {ϕ, {a}} and μ_2 = {ϕ, {a}, {b, c}, X}. Then (X, μ_1, μ_2) is a $\mu_{(1,2)}$-$T^*_{1/2}$-space but not a $\mu_{(1,2)}$- $T_{1/2}$- space.

Proposition 4.6: A bigeneralized topological space (X, μ_1, μ_2) is an $\mu_{(m,n)}$-$T^*_{1/2}$-space if and only if {x} is μ_n-open or μ_m-g-closed for each x ∈ X, where m, n = 1, 2 and m ≠ n

Proof: Suppose that {x} is not μ_m-g-closed. By Proposition 3.14, $\{x\}^c$ is $\mu_{(m,n)}$- g^*-closed. Since (X, μ_1, μ_2) is an $\mu_{(m,n)}$-$T^*_{1/2}$-space, $\{x\}^c$ is μ_n-closed. Therefore {x} is μ_n-open.

Conversely, Let A be a $\mu_{(m,n)}$- g^*-closed set. By assumption {x} is μ_n-open (or) μ_m-g-closed for any x ∈ c_{μ_n}(A).

Case (i): Suppose {x} is μ_n-open. Since {x} ∩ A ≠ ϕ, we have x ∈ A.

Case (ii): Suppose {x} is μ_m-g-closed. If x ∉ A, then {x} ⊆ c_{μ_n}(A) – A. This is a contradiction to the Proposition 3.15.

Therefore, X ∈ A.

Proposition 4.7: A bigeneralized topological space (X, μ_1, μ_2) is pairwise μ- $T_{1/2}$ - space then it is pairwise μ-$T^*_{1/2}$-space.

The converse of the above proposition is not true as seen from the following example.

Example 4.8: In Example 4.5, (X, μ_1, μ_2) is pairwise $\mu_{(1,2)}$-$T^*_{1/2}$-space and pairwise $\mu_{(2,1)}$-$T^*_{1/2}$-space and Therefore it is pairwise μ- $T^*_{1/2}$- space. But (X, μ_1, μ_2) is not pairwise μ- $T_{1/2}$-space, since it is not $\mu_{(1,2)}$-$T_{1/2}$-space.

Proposition 4.9: In a bigeneralized topological space (X, μ_1, μ_2), every $\mu_{(m,n)}$- $T_{1/2}$-space is an $\mu_{(m,n)}$- ${}^*T_{1/2}$- space, where $m,\ n = 1,2\ and\ m \neq n$.

The converse of the above proposition is not true as seen from the following example.

Example 4.10: In Example 3.9, then (X, μ_1, μ_2) is a $\mu_{(1,2)}$- ${}^*T_{1/2}$-space but not a $\mu_{(1,2)}$- $T_{1/2}$-space.

Remark 4.11: $\mu_{(m,n)}$-$T^*_{1/2}$-space and $\mu_{(m,n)}$- ${}^*T_{1/2}$- spaces are independent as seen from the example.

Example 4.12: In Example 4.5, (X, μ_1, μ_2) is a $\mu_{(1,2)}$-$T^*_{1/2}$-space but not a $\mu_{(1,2)}$- ${}^*T_{1/2}$-space. In Example 4.10, (X,μ_1, μ_2) is a $\mu_{(1,2)}$- ${}^*T_{1/2}$-space but not a $\mu_{(1,2)}$-$T^*_{1/2}$-space.

Theorem 4.13: A bigeneralized topological space (X, μ_1, μ_2) is a $\mu_{(m,n)}$- $T_{1/2}$-space if and only if it is both $\mu_{(m,n)}$- ${}^*T_{1/2}$ and $\mu_{(m,n)}$-$T^*_{1/2}$, where $m,n = 1,2\ and\ m \neq n$.

Proof: Suppose that (X, μ_1, μ_2) is a $\mu_{(m,n)}$- $T_{1/2}$-space. Then by Proposition 4.9, and Proposition 4.4, (X, μ_1, μ_2) is both $\mu_{(m,n)}$- ${}^*T_{1/2}$ and $\mu_{(m,n)}$-$T^*_{1/2}$.

Conversely, Suppose that (X, μ_1, μ_2) is both $\mu_{(m,n)}$- ${}^*T_{1/2}$ and $\mu_{(m,n)}$-$T^*_{1/2}$. Let A be a $\mu_{(m,n)}$- closed set. Since (X,μ_1, μ_2) is $\mu_{(m,n)}$- ${}^*T_{1/2}$-space, A is a $\mu_{(m,n)}$- g^*-closed set. Since (X, μ_1, μ_2) is $\mu_{(m,n)}$-$T^*_{1/2}$, A is a μ_n-closed set. Therefore (X, μ_1, μ_2) is $\mu_{(m,n)}$- $T_{1/2}$-space.

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