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TWO PHASE RETRIAL QUEUEING SYSTEM WITH IMPATIENT CUSTOMERS, ADDITIONAL OPTIONAL SERVICE, SERVER BREAKDOWN, DELAYED REPAIR AND RESERVED TIME

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ABSTRACT

Single server retrial queue with general retrial time is considered. Customers may balk or renege at particular times. The server is subject to breakdowns with repairs. The repair is not immediate and it starts after a random amount of time. While the server is being repaired, the interrupted customer can either remains in the service position or leaves and return by maintaining its rights to the server. The probability generating function is employed to obtain joint distribution of the server state and queue length. The probability of an empty system, availability of the server, failure frequency and the mean number of customers in the retrial queue are derived.

Key words: *Retrial Queue, Additional Optional Service, Breakdown, Delayed repair, Reserved Time.*

INTRODUCTION

Queueing systems with repeated attempts are appropriate for modeling the processes in communication networks, where a customer meeting a busy server tries its luck again after a random time. The increasing interest in this topic is mainly motivated by the development of new facilities in telecommunication technology such as 'repeat last number', 'ring back when free' and so on. Recent work on retrial queues includes Aissani [1], Artalejo [2, 3] and Nathan [9]. Several authors have studied two phase queueing models, interested readers can refer to [7, 8].

Systems with a repairable sever are worth investigating from the queueing theory as well as reliability view pint, since the performance may be heavily affected by the breakdowns. Queueing System with random breakdowns have been studied by numerous researchers including Wang [12], Wang and Li [11], Atecia et. al [4], Geni Gupur [6], Aissani [1], Baskar et. al [5] and Rehab et.al [10].

In this paper, two phase retrial queueing system with general retrial times, allowing balking of new arriving customers and reneging of customers in the retrial queue is discussed. The server is subject to breakdown and repair. We also include the concept of delay time and reserved time.

THE MATHEMATICAL MODEL

Single server retrial queue is considered. Primary customers arrive in a Poisson process with rate λ. There is no waiting room in front of the server. If an arriving primary customer finds the server idle, it begins service immediately and leaves the system after service completion. If the server is found to be blocked, the arriving primary customer either enters a retrial queue with probability p or leaves the system with probability 1– p. The retrial time is generally distributed with distribution function A(x) and Laplace Steltjes transform A*(s). The retrial customer cancels its attempt for service if a primary customer arrives first and either returns to its position in the retrial queue with probability r or quits the system with probability 1– r.

The server provides two phases of service – essential and optional. The first phase service is needed to all customers. As soon as the first phase service of a customer is complete, then with probability τ he may immediately enter into second optional service or else with probability 1– τ he may leave the system. Let $B_i(x)$ and $B_i^*(x)$ be the distribution and Laplace Steltjes transform of the ith phase service time with moments μ_{in}, i = 1, 2, n $\geq$ 1.

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It is assumed that the server is subject to breakdown when it is busy. The life time of the server is exponentially distributed with rate α_i, i = 1, 2. Once the system breaks down the repair does not start immediately. There is some delay to start the repair. The delay time distribution is general with distribution function $D_i(x)$ with moments v_{in}, i = 1, 2, $n \geq 1$. The repair time is also generally distributed with distribution function $H_i(x)$, Laplace Steltjes transform $H_i^*(x)$ with moments h_{in}, i = 1, 2, n ≥ 1.

Upon failure of the server, interrupted customer in the ith phase (i = 1, 2) either remains in the service position with probability q_i until the server is up or enters retrial orbit with probability $1-q_i$ and keeps returning at times exponentially distributed with mean $\dfrac{1}{\theta_i}$ until the server is repaired. If the interrupted customer enters the retrial orbit, the server after repair must wait for the same customer to return. This waiting time of the server is referred as reserved time. The server is not allowed to accept new customers until the customer in service leaves the system. The server is said to be blocked, if the server is busy, under delay, under repair or reserved.

JOINT DISTRIBUTION OF THE SERVER STATE AND ORBIT SIZE

The state of the system at time t can be described by the Markov Processes $\{N(t): t \geq 0\}$= $\{J(t),\ J^*(t),\ X(t), \xi_0(t),\ \xi_i(t), \xi_j^2(t), \xi_j^3(t), \xi_j^4(t);\ t \geq 0\}$ where J(t) denotes the server state 0, 1, 2, 3 or 4 according as the server is idle, busy, waiting for repair, under repair or in reserved state respectively. $J^*(t)$ denotes the interrupted customers position 0 or 1 according as the customer is in service position or in retrial queue. X(t) denotes the number of customer in the retrial queue at time t. If J(t) = 0 and X(t) > 0 then $\xi_0(t)$ represents the elapsed retrial time. If J(t) = 1, 2, 3 or 4 $\xi_i(t)$ represents the elapsed service time in ith phase (i=1, 2). If J(t) = 2, $\xi_j^2(t)$ represents the elapsed delayed time, if J(t) = 3, $\xi_j^3(t)$ represents the elapsed repair time and if J(t) = 4, $\xi^4(t)$ represents the elapsed reserved time.

Define the probabilities,

$I_0(t)$ = P {J (t) = 0, X (t) = 0}

$I_n(t,x)$ dx = P{ J(t) = 0, X(t) = n, x $\leq \xi_0(t) <$ x + dx } n ≥ 1

For n ≥ 0; i = 1, 2 ; j = 0, 1

$W_{i,n}(t,x)$ dx = P{J(t) = 1, X(t) = n, x $\leq \xi_i(t) <$ x + dx}

$D_{i,j,n}(t,x,y)$ dx dy = P{ J(t) = 2, $J^*(t)$ = j, X(t) = n, x≤ $\xi_i(t)$ < x+dx, y≤ $\xi_j^2(t)$ <y +dy}

$F_{i,j,n}(t,x,y)$ dx dy = P{J(t) = 3, $J^*(t)$ = j, X(t) = n, x ≤ $\xi_i(t)$ < x+dx, y≤ $\xi_j^3(t)$ < y+dy}

$R_{i,n}(t,x,y)$ dx dy = P{J(t) = 4, X(t) = n, x ≤ $\xi_i(t)$ < x + dx, y ≤ $\xi^4(t)$ < y + dy}

Let $\eta(x) = \dfrac{a(x)}{1-A(x)}$, $\mu_i(x) = \dfrac{b_i(x)}{1-B_i(x)}$, $\gamma_i(x) = \dfrac{d_i(x)}{1-D_i(x)}$, $\beta_i(x) = \dfrac{h_i(x)}{1-H_i(x)}$.

Theorem 1: The necessary and sufficient condition for the system is to be stable is $K_1' + \tau\ K_2' < 1 - r + r\,A^*(\lambda)$

where $K_i' \ = \ \lambda p \mu_{i1}\left[1+\alpha_i\left(\dfrac{1-q_i}{\theta_i} + v_{i1} + h_{i1}\right)\right]$ i = 1, 2.

Proof: Let $S_i^{(k)}$ be the generalized service time of the k[th] customer in i[th] phase service. Then $\{S_i^{(k)}\}$ are independently and identically distributed with Laplace transform

$$\wp_i^*(s) = B_i^*\left(s+\alpha_i-\alpha_i\left(\frac{q_i s+\theta_i}{s+\theta_i}H_i^*(s)\,D_i^*(s)\right)\right)$$

and expected value $E\left(S_i^{(k)}\right) = \mu_{i1}\left[1+\alpha_i\left(\frac{1-q_i}{\theta_i}+v_{i1}+h_{i1}\right)\right]$ i = 1, 2

and $E(S^{(k)}) = E(S_1^{(k)}) + \tau\, E(S_2^{(k)})$

Suppose the retrial queue has a large number of customers in the following discussion. Let P(S) and P(I) be denote respectively the probabilities that the system is blocked and idle. Le E $(S^{(k)})$ be the expected blocked time and E(I) be the expected idle time then

$$P(S) = \frac{E(S^{(k)})}{E(S^{(k)})+E(I)} \text{ and } P(I) = \frac{E(I)}{E(S^{(k)})+E(I)}$$

The arrival rate at the retrial queue when the system is blocked is $\lambda p P(S)$.

The exit rate from the retrial queue by entering service is $A^*(\lambda)\frac{P(I)}{E(I)}$.

The exit rate from the retrial queue by leaving the system is $\{1-A^*(\lambda)\}\{1-r\}\frac{P(I)}{E(I)}$

The Total exit rate from the retrial queue is $\{1-r+rA^*(\lambda)\}\frac{P(I)}{E(I)}$

The system is stable if and only if total arrival rate is less than the exit rate.

Hence, $K_1' + \tau\, K_2' < 1 - r + r\,A^*(\lambda)$ is necessary and sufficient condition for the system to be stable.

STEADY STATE RESULTS

The steady state equations are

$$\lambda I_0 = (1-\tau)\int_0^\infty W_{1,0}(x)\,\mu_1(x)\,dx + \int_0^\infty W_{2,0}(x)\,\mu_2(x)\,dx \tag{1}$$

$$\frac{dI_n(x)}{dx} = -(\lambda+\eta(x))\,I_n(x),\ n \geq 1 \tag{2}$$

$$\frac{dW_{i,n}(x)}{dx} = -(p\lambda+\mu_i(x)+\alpha_i)\,W_{i,n}(x) + \int_0^\infty F_{i,0,n}(x,y)\,\beta_i(y)\,dy + \theta_i$$

$$\int_0^\infty R_{i,n}(x,y)\,dy + p\lambda(1-\delta_{0n})\,W_{i,n-1}(x)\ ,\ n \geq 0;\ i = 1, 2 \tag{3}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right]D_{i,j,n}(x,y) = -(p\lambda+\gamma_i(y))\,D_{i,j,n}(x,y) + p\lambda(1-\delta_{0n})\,D_{i,j,n-1}(x,y),\ n \geq 0;\ i = 1, 2;\ j = 0, 1 \tag{4}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right] F_{i,j,n}(x,y) = -(p\lambda+\beta_i(y))F_{i,j,n}(x,y) + p\lambda(1-\delta_{0n})F_{i,j,n-1}(x,y),\ n\ge 0;\ i=1,2;\ j=0,1 \tag{5}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right] R_{i,n}(x,y) = -(p\lambda+\theta_i)R_{i,n}(x,y) + p\lambda(1-\delta_{0n})R_{i,n-1}(x,y),\ n\ge 0;\ i=1,2 \tag{6}$$

with boundary conditions,

$$\lambda I_n(0) = (1-\tau)\int_0^\infty W_{1,n}(x)\,\mu_1(x)\,dx + \int_0^\infty W_{2,n}(x)\mu_2(x)\,dx,\ n\ge 1 \tag{7}$$

$$W_{1,0}(0) = \lambda I_0 + \int_0^\infty I_1(x)\eta(x)\,dx + (1-r)\lambda\int_0^\infty I_1(x)\,dx \tag{8}$$

$$W_{1,n}(0) = \int_0^\infty I_{n+1}(x)\eta(x)\,dx + (1-r)\lambda\int_0^\infty I_{n+1}(x)\,dx + r\lambda\int_0^\infty I_n(x)\,dx,\ n\ge 1 \tag{9}$$

$$W_{2,n}(0) = \tau\int_0^\infty W_{1,n}(x)\,\mu_1(x)\,dx,\ n\ge 0 \tag{10}$$

$$D_{i,0,n}(x,0) = q_i\,\alpha_i\,W_{i,n}(x),\ i=1,2;\ n\ge 0 \tag{11}$$

$$D_{i,1,n}(x,0) = (1-q_i)\,\alpha_i\,W_{i,n}(x),\ i=1,2;\ n\ge 0 \tag{12}$$

$$F_{i,j,n}(x,0) = \int_0^\infty D_{i,j,n}(x,y)\,\gamma_i(y)\,dy,\ i=1,2;\ j=0,1;\ n\ge 0 \tag{13}$$

$$R_{i,n}(x,0) = \int_0^\infty F_{i,1,n}(x,y)\,\beta_i(y)\,dy,\ i=1,2;\ n\ge 0 \tag{14}$$

The normalizing condition is

$$I_0 + \sum_{n=1}^{\infty}\int_0^\infty I_n(x)\,dx + \sum_{n=0}^{\infty}\sum_{i=1}^{2}\Big[\int_0^\infty W_{i,n}(x)\,dx + \sum_{j=0}^{1}\Big[\int_0^\infty\int_0^\infty D_{i,j,n}(x,y)\,dx\,dy$$
$$+\int_0^\infty\int_0^\infty F_{i,j,n}(x,y)\,dx\,dy\Big] + \int_0^\infty\int_0^\infty R_{i,n}(x,y)\,dx\,dy\Big] = 1 \tag{15}$$

Define Probability generating function as follows:

$I(z,x) = \sum_{n=1}^{\infty} I_n(x)z^n$, $\qquad W_i(z,x) = \sum_{n=0}^{\infty} W_{i,n}(x)z^n$,

$D_{i,j}(z,x,y) = \sum_{n=0}^{\infty} D_{i,j,n}(x,y)z^n$, $\qquad F_{i,j}(z,x,y) = \sum_{n=0}^{\infty} F_{i,j,n}(x,y)z^n$,

$R_i(z,x,y) = \sum_{n=0}^{\infty} R_{i,n}(x,y)z^n$

Theorem 2: If $K_1' + \tau\, K_2' < 1 - r + r\, A^*(\lambda)$, then the partial generating function of the steady state distributions of $\{ N(t);\, t \ge 0 \}$ under different server states are obtained as,

$$I(z) = \frac{I_0\, z[1 - A^*(\lambda)][1 - B_1^*(G_1(p\lambda(1-z)))][1 - \delta + \delta B_2^*(G_2(p\lambda(1-z)))]}{D(z)} \tag{16}$$

$$W_1(z) = \frac{\lambda I_0 (1-z)[1 - r + rA^*(\lambda)][1 - B_1^*(G_1(p\lambda(1-z)))]}{D(z)\, G_1(p\lambda(1-z))} \tag{17}$$

$$W_2(z) = \frac{\lambda I_0 (1-z)[1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))[1 - B_2^*(G_2(p\lambda(1-z)))]}{D(z)\, G_2(p\lambda(1-z))} \tag{18}$$

$$D_{1,0}(z) = \frac{q_1 \alpha_1 I_0\ [1 - r + rA^*(\lambda)][1 - B_1^*(G_1(p\lambda(1-z)))][1 - D_1^*(p\lambda(1-z))]}{p\, D(z)\, G_1(p\lambda(1-z))} \tag{19}$$

$$D_{1,1}(z) = \frac{(1-q_1)\alpha_1 I_0\ [1 - r + rA^*(\lambda)][1 - B_1^*(G_1(p\lambda(1-z)))][1 - D_1^*(p\lambda(1-z))]}{p\, D(z)\, G_1(p\lambda(1-z))} \tag{20}$$

$$D_{2,0}(z) = \frac{q_2 \alpha_2 I_0 [1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))[1 - B_2^*(G_2(p\lambda(1-z)))][1 - D_2^*(p\lambda(1-z))]}{p\, D(z)\, G_2(p\lambda(1-z))} \tag{21}$$

$$D_{2,1}(z) = \frac{(1-q_2)\alpha_2 I_0 [1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))[1 - B_2^*(G_2(p\lambda(1-z)))][1 - D_2^*(p\lambda(1-z))]}{p\, D(z)\, G_2(p\lambda(1-z))} \tag{22}$$

$$F_{1,0}(z) = \frac{q_1 \alpha_1 I_0\ [1 - r + rA^*(\lambda)][1 - B_1^*(G_1(p\lambda(1-z)))]D_1^*(p\lambda(1-z))[1 - H_1^*(p\lambda(1-z))]}{p\, D(z)\, G_1(p\lambda(1-z))} \tag{23}$$

$$F_{1,1}(z) = \frac{(1-q_1)\alpha_1 I_0\ [1 - r + rA^*(\lambda)][1 - B_1^*(G_1(p\lambda(1-z)))]D_1^*(p\lambda(1-z))[1 - H_1^*(p\lambda(1-z))]}{p\, D(z)\, G_1(p\lambda(1-z))} \tag{24}$$

$$F_{2,0}(z) = \frac{q_2 \alpha_2 I_0 [1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))D_2^*(p\lambda(1-z))[1 - B_2^*(G_2(p\lambda(1-z)))][1 - H_2^*(p\lambda(1-z))]}{p\, D(z)\, G_2(p\lambda(1-z))} \tag{25}$$

$$F_{2,1}(z) = \frac{(1-q_2)\alpha_2 I_0 [1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))D_2^*(p\lambda(1-z))[1 - B_2^*(G_2(p\lambda(1-z)))][1 - H_2^*(p\lambda(1-z))]}{p\, D(z)\, G_2(p\lambda(1-z))} \tag{26}$$

$$R_1(z) = \frac{(1-q_1)\alpha_1 \lambda(1-z) I_0\ [1 - r + rA^*(\lambda)][1 - B_1^*(G_1(p\lambda(1-z)))]D_1^*(p\lambda(1-z))H_1^*(p\lambda(1-z))}{D(z)\, G_1(p\lambda(1-z))[p\lambda(1-z) + \theta_1]} \tag{27}$$

$$R_2(z) = \frac{(1-q_2)\alpha_2 \lambda(1-z) I_0\ [1 - r + rA^*(\lambda)][1 - B_2^*(G_2(p\lambda(1-z)))]\delta B_1^*(G_1(p\lambda(1-z)))D_2^*(p\lambda(1-z))H_2^*(p\lambda(1-z))}{D(z)\, G_2(p\lambda(1-z))[p\lambda(1-z) + \theta_2]} \tag{28}$$

where

$$D(z) = [1 - r\,[1 - A^*(\lambda)](1-z)]\,[1 - \delta + \delta B_2^*(G_2(p\lambda(1-z)))]\, B_1^*(G_1(p\lambda(1-z))) - z \tag{29}$$

$$G_i(x) = x + \alpha_i - \alpha_i \left(\frac{q_{ix} + \theta_i}{x + \theta_i} \right) D_i^*(x) H_i^*(x),\ i = 1, 2 \tag{30}$$

and the probability I_0 is to be determined from the normalizing condition.

Proof: Multiplying equations (1) – (14) by z^n and summing over all possible values of n and solving it, we obtain the following equations,

$$I(z, x) = I(z, 0)\ e^{-\lambda x}\,[1 - A(x)] \tag{31}$$

$$W_i(z, x) = W_i(z,0)\ e^{-G_i(p\lambda(1-z))x}\,[1 - B_i(x)] \tag{32}$$

$$D_{i,j}(z, x, y) = D_{i,j}(z, x, 0)\ e^{-p\lambda(1-z)y}\,[1 - D_i(y)] \tag{33}$$

$$F_{i,j}(z, x, y) = F_{i,j}(z, x, 0)\ e^{-p\lambda(1-z)y}\,[1 - H_i(y)] \tag{34}$$

$$R_i(z, x, y) = R_i(z, x, 0)\ e^{-[p\lambda(1-z)+\theta_i]y} \tag{35}$$

$$I(z, 0) = \frac{\lambda I_0\, z[1 - B_1^*(G_1(p\lambda(1-z)))][1 - \delta + \delta B_2^*(G_2(p\lambda(1-z)))]}{D(z)} \tag{36}$$

$$W_1(z,0) = \frac{\lambda I_0\,(1-z)[1 - r + rA^*(\lambda)]}{D(z)} \tag{37}$$

$$W_2(z,0) = \frac{\lambda I_0\,(1-z)[1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))}{D(z)} \tag{38}$$

$$D_{1,0}(z, x, 0) = \frac{q_1 \alpha_1 \lambda I_0\,[1-z][1 - r + rA^*(\lambda)]e^{-G_1(p\lambda(1-z))x}\,[1 - B_1(x)]}{D(z)} \tag{39}$$

$$D_{1,1}(z, x, 0) = \frac{(1-q_1)\alpha_1 \lambda I_0\,[1-z][1 - r + rA^*(\lambda)]e^{-G_1(p\lambda(1-z))x}\,[1 - B_1(x)]}{D(z)} \tag{40}$$

$$D_{2,0}(z, x, 0) = \frac{q_2 \alpha_2 \lambda I_0\,[1-z][1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))e^{-G_2(p\lambda(1-z))x}\,[1 - B_2(x)]}{D(z)} \tag{41}$$

$$D_{2,1}(z, x, 0) = \frac{(1-q_2)\alpha_2 \lambda I_0\,[1-z][1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))e^{-G_2(p\lambda(1-z))x}\,[1 - B_2(x)]}{D(z)} \tag{42}$$

$$F_{1,0}(z, x, 0) = \frac{q_1 \alpha_1 \lambda I_0\,[1-z][1 - r + rA^*(\lambda)]D_1^*(p\lambda(1-z))e^{-G_1(p\lambda(1-z))x}\,[1 - B_1(x)]}{D(z)} \tag{43}$$

$$F_{1,1}(z, x, 0) = \frac{(1-q_1)\alpha_1 \lambda I_0\,[1-z][1 - r + rA^*(\lambda)]D_1^*(p\lambda(1-z))e^{-G_1(p\lambda(1-z))x}\,[1 - B_1(x)]}{D(z)} \tag{44}$$

$$F_{2,0}(z, x, 0) = \frac{q_2 \alpha_2 \lambda I_0\,[1-z][1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))D_2^*(p\lambda(1-z))e^{-G_2(p\lambda(1-z))x}\,[1 - B_2(x)]}{D(z)} \tag{45}$$

$$F_{2,1}(z, x, 0) = \frac{(1-q_2)\alpha_2 \lambda I_0\,[1-z][1 - r + rA^*(\lambda)]\delta B_1^*(G_1(p\lambda(1-z)))D_2^*(p\lambda(1-z))e^{-G_2(p\lambda(1-z))x}\,[1 - B_2(x)]}{D(z)} \tag{46}$$

$$R_1(z,x,0)=\frac{(1-q_1)\alpha_1\lambda(1-z)I_0\,[1-r+rA^*(\lambda)]e^{-G_1(p\lambda(1-z))x}\,[1-B_1(x)]D_1^*(p\lambda(1-z))H_1^*(p\lambda(1-z))}{D(z)} \tag{47}$$

$$R_2(z,x,0)=\frac{(1-q_2)\alpha_2\lambda(1-z)I_0\,[1-r+rA^*(\lambda)]e^{-G_2p\lambda(1-z))x}\,[1-B_2(x)]\delta B_1^*(G_1(p\lambda(1-z)))D_2^*(p\lambda(1-z))H_2^*(p\lambda(1-z))}{D(z)} \tag{48}$$

Using the equations (36) to (48) and integrating with respect to x and y from 0 to ∞ we get the results in equations (16) to (28).

Using the normalizing condition we obtain I_0 as $[1-r+rA^*(\lambda)-K_1'-\tau K_2']/T_1$

where $T_1 = (1-r+rA^*(\lambda)+\lambda(1-r+(r-p)A^*(\lambda))[(K_1'+\tau K_2')/p]$

Corollary 1: Let P(z) be the PGF of the orbit size distribution of this model then under the steady state condition we have

$$P(z)=I_0+I(z)+\sum_{i=1}^{2}[W_i(z)+\sum_{j=0}^{1}[D_{i,j}(z)+F_{i,j}(z)]+R_i(z)]$$

$$=I_0\{(1-r+rA^*(\lambda))[1-(1-p+pz)[1-\tau+\tau B_2^*(G_2(p\lambda(1-z)))]B_1^*(G_1(p\lambda(1-z)))$$
$$-pzA^*(\lambda)\{1-B_1^*(G_1(p\lambda(1-z)))[1-\tau+\tau B_2^*(G_2(p\lambda(1-z)))]\}\}/[pD(z)]$$

Corollary2: Let $\pi(z)$ be the PGF of the system size distribution of this model then under the steady state condition we have

$$\pi(z)=I_0+I(z)+z\sum_{i=1}^{2}[W_i(z)+\sum_{j=0}^{1}[D_{i,j}(z)+F_{i,j}(z)]+R_i(z)]$$

$$=I_0\{(1-r+rA^*(\lambda))[z-(z-p+pz)[1-\tau+\tau B_2^*(G_2(p\lambda(1-z)))]B_1^*(G_1(p\lambda(1-z)))]$$
$$-pzA^*(\lambda)\{1-B_1^*(G_1(p\lambda(1-z)))[1-\tau+\tau B_2^*(G_2(p\lambda(1-z)))]\}\}/[pD(z)]$$

SOME PERFORMANCE MEASURES

The stationary probabilities of the server state is given by

- Prob{the server is on idle in empty system} = $I_0 = [1-r+rA^*(\lambda)-K_1'-\tau K_2']/T_1$
- Prob{the server is on idle in non empty system} = $I(1) = [1-A^*(\lambda)][K_1'+\tau K_2']/T_1$
- Prob{the server is busy} = $W = [1-r+rA^*(\lambda)]\lambda(\mu_{11}+\tau\mu_{21})/T_1$
- Prob{the server is in delayed mode} = $D = [1-r+rA^*(\lambda)]\lambda(\alpha_1\mu_{11}v_{11}+\tau\alpha_2\mu_{21}v_{21})/T_1$
- Prob{the server is in repair mode} = $F = [1-r+rA^*(\lambda)]\lambda(\alpha_1\mu_{11}h_{11}+\tau\alpha_2\mu_{21}h_{21})/T_1$
- Prob{the server is reserved}=
  $$R=[1-r+rA^*(\lambda)]\lambda[(1-q_1)\alpha_1\mu_{11}/\theta_1+\tau(1-q_2)\alpha_2\mu_{21}/\theta_2]/T_1$$
- Availability of the server = $A = I_0 + I(1) + W + R$
  $$= [(1-r+rA^*(\lambda)+K_1'A^*(\lambda)-A^*(\lambda)\,\tau K_2')$$
  $$+\lambda(1-r+rA^*(\lambda))[\mu_{11}(1+(1-q_1)\alpha_1/\theta_1)+\tau\mu_{21}(1+(1-q_2)\alpha_2/\theta_2)]/T_1$$
- The steady state failure frequency of the server is
  $$F=\lambda(1-r+rA^*(\lambda))(\alpha_1\mu_{11}+\tau\alpha_2\mu_{21})/T_1$$

- The mean number of customer in the queue is

$$L_q = N_2 / [T_1 p] + T_2 N_1 / [pT_1]$$

- The mean number of customer in the System is

$$L_s = [N_2 + (1 - r + r\, A^*(\lambda))\,(\tau\, K_2' + K_1')] / [T_1 p] + T_2 N_1 / [pT_1]$$

where $N_1 = [1 - r + r\, A^*(\lambda)]\,[p + \tau\, K_2' + K_1'] + p\, A^*(\lambda)[\tau\, K_2' + K_1']$

$$N_2 = [1 - r + r\, A^*(\lambda)]\,[p\,\tau\, K_2' + p K_1' + \tau K_2' K_1' + (K_1'' + K_2'')/2]$$
$$+ pA^*(\lambda)\,[\tau\, K_2' + K_1' + \tau K_2' K_1' + (K_1'' + K_2'')/2]$$
$$T_2 = r\,(1 - A^*(\lambda))\,[\tau\, K_2' + K_1' + \tau K_2' K_1' + (K_1'' + K_2'')/2]$$

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