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Numerical Approach to A Mathematical Model of Three Species Ecological Ammensalism

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ABSTRACT

The present paper aims at the investigation of a three species ecosystem consisting of a prey (N_1), a predator (N_2) surviving on N_1 and an enemy (N_3) Ammansal with the prey N_1 only. The mathematical model equations are formed by a set of three first order non-linear simultaneous equations in N_1, N_2 and N_3. The equation for N_3 is non-linear but de-coupled with $N_1$and N_2 .The relations among the natural growth rate of Ammensal- prey and the dominance reversal times(t) are identified. More over the interactions among the three species are observed and some conclusions are established by Numerical study.*

AMS Classification: *92 D 25, 92 D 40.*

Keywords: *Ecosystem, prey, predator, Ammensal, Enemy, Normal steady state, stability.*

I. INTRODUCTION

Ecology is a herculean branch of evolutionary biology. This branch of science studies how different types of living beings can live together for generations in the same environment. Sharing the same habitat and interacting with each other in different ways. Without any interaction no organism can live in the natural world. Mathematical modeling of ecosystems was initiated in 1925 by Lotka [13] and in 1931 by Volterra [16]. The general concepts of modeling have been presented in the treatises of Meyer [14], Kapur [9,10] and several authors.. The ecological symbiosis can be broadly classified as Prey-predation, competition, mutualism, commensalism, Ammensalism, and so on. N.C. Srinivas [17] studied the competitive ecosystems of two and three species with limited and unlimited resources. Lakshminarayan and Pattabhiramacharyulu [11, 12] investigated Prey-predator Ecological models with a partial cover for the prey and alternate food for the predator. Recently Acharyulu and Pattabhi Ramacharyulu [1, 2, 3, 4, 5, 6] investigated some remarkable results “on the stability of an enemy-Ammensal species pair with manifold conditions.

The present investigation is a numerical study of three species system: Ammensal–prey, predator and enemy. In all eight equilibrium points are identified based on the model equations and these are spread over three distinct classes: (i) Fully washed out (ii) Semi/partially washed out and (iii) Co-existent states. The relations among the natural growth rate of Ammensal- prey and the dominance reversal times (t*) are identified by employing runge-kutta method of fourth order. More over the interactions among the three species are observed and some conclusions are made in view of dominance reversal time.

The possible states of this model are
i. Fully washed out state.
ii. The predator and enemy are washed out but not the prey.
iii. The prey and the enemy are washed out but not the predator.
iv. The predator is washed out but the prey and the enemy are not.

II. NOTATION ADOPTED

N_1 (t)	:	The population of the Prey-Ammansal Species.
N_2 (t)	:	The population of the predator striving of the prey N_1
N_3 (t)	:	The Population of the enemy to the prey N_1
a_i	:	The natural growth rates of N_i, i = 1,2,3
a_{ii}	:	The rate of decrease of N_i due to insufficient resources of, N_i i = 1,2,3
a_{12}	:	The rate of decrease of the prey (N_1) due to inhibition by the predator (N_2)

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a_{13} : The rate of increase of the Ammansal (N_1) due to its successful promotion by enemy (N_3)
a_{21} : The rate of increase of the predator (N_2) due to its successful attacks on the prey (N_1)
$K_i = a_i / a_{ii}$: Carrying capacities of N_i ,i = 1, 2, 3.
$\alpha = a_{13}/a_{11}$: Co-efficient of Ammensalism.
$P = a_{12}/a_{11}$: Co-efficient of prey inhibition (suffering)
$Q = a_{21}/a_2$: Co-efficient of predator consumption of the prey.

III. BASIC EQUATIONS

The model equations for a three species ecosystem are given by the following system of non-linear ordinary differential equations.

(i) Equation for the growth rate of Prey-Ammensal species (N_1):

$$\frac{dN_1}{dt} = a_{11}N_1 (K_1 - N_1 - PN_2 - \alpha N_3) \qquad (1)$$

(ii) Equation for the growth rate of predator species (N_2):

$$\frac{dN_2}{dt} = a_{22} N_2 (K_2 - N_1 + Q N_1) \qquad (2)$$

(iii) Equation for the growth rate of enemy species (N_3):

$$\frac{dN_3}{dt} = a_{33} N_3 (K_3 - N_3) \qquad (3)$$

Further the variables N_1, N_2 and N_3 are non-negative and the model parameters a_1, a_2, a_3 a_{11}, a_{22}, a_{33}, a_{12}, a_{13}, a_{21}, K_1, K_2, K_3, α , P, and Q are all assumed to be non-negative constants.

IV. EQUILIBRIUM STATES

The system under investigation has eight equilibrium states given by $\frac{dNi}{dt} = 0$; i=1,2,3.

A. Fully washed out state:

(i) $$\overline{N_1} = 0, \overline{N_2} = 0; \overline{N_3} = 0 \qquad (4)$$

B. States in which two of the three species are washed out and the third is not.

(ii) $$\overline{N_1} = 0;\ \overline{N_2} = 0;\ \overline{N_3} = K_3 \qquad (5)$$

(iii) $$\overline{N_1} = 0;\ \overline{N_2} = K_2;\ \overline{N_3} = 0 \qquad (6)$$

(iv) $$\overline{N_1} = K_1;\ \overline{N_2} = 0;\ \overline{N_3} = 0 \qquad (7)$$

C. Only one of the three species is washed out while the other two are not

(v) $$\overline{N_1} = 0;\ \overline{N_2} = K_2;\ \overline{N_3} = K_3 \qquad (8)$$

(vi) $$\overline{N_1} = K_1 - \alpha K_3;\ \overline{N_2} = 0;\ \overline{N_3} = K_3 \qquad (9)$$

This state would exist when $K_1 > \alpha K_3$

(vii) $$\overline{N_1} = \frac{K_1 - PK_2}{1+PQ};\ \overline{N_2} = \frac{QK_1 + K_2}{1+PQ}; \overline{N_3} = 0 \qquad (10)$$

This state would exists only when with $K_1 > PK_2$

D. The co-existent state or normal steady state

(viii) $$\overline{N_1} = \frac{K_1 - PK_2 - \alpha K_3}{1+PQ}; \quad \overline{N_2} = \frac{QK_1 + K_2 - Q\alpha K_3}{1+PQ}; \quad \overline{N_3} = K_3 \qquad (11)$$

This would exist only when $K_1 > PK_2 + \alpha K_3$ and $K_3 > K_1/\alpha Q$

V. THE RELATION BETWEEN THE GROWTH RATE OF AMMENSAL-PREY SPECIES AND THE DOMINANCE REVERSAL TIME

The values of the parameters are considered as below:

Fixed parameters: $\mathbf{a_{11}=a_{12}=a_2=N_{10}=N_{20}=0.5;}$

Varying parameters: $\mathbf{a_1= 0.5, 1.0, 1.5, 2.0, a_{22} = 0.5, 1.0, 1.5, 2.0.}$

The values are tabulated as in Table-1.

Table-1

Case	a_1	a_{11}	a_{12}	a_{13}	a_2	a_{22}	a_{21}	a_3	a_{33}	$N_{10}= N_{20}= N_{30}$	t*
1	0.1	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	*
2	0.2	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	*
3	0.3	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	*
4	0.4	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	*
5	0.5	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	*
6	0.6	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{13}*= 34.984
7	0.7	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{12}*= 4.614
8	0.8	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	*
9	0.9	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	*
10	1.0	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	*
11	1.1	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 4.582
12	1.2	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 3.914
13	1.3	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 3.145
14	1.4	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 2.604
15	1.5	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 2.390
16	1.6	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 2.125
17	1.7	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 1.823
18	1.8	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 1.552
19	1.9	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 1.03
20	2.0	0.2	0.3	0.4	0.2	0.4	0.3	0.2	0.3	0.5	t_{23}*= 0.626

The corresponding graphs are illustrated from Figure ((1) to Figure (20) as shown below.

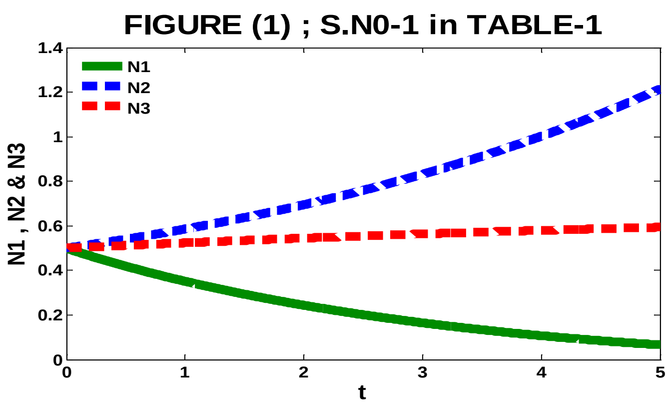

Figure (1): Variation of a_1 vs t*, when a_1=0.1, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5

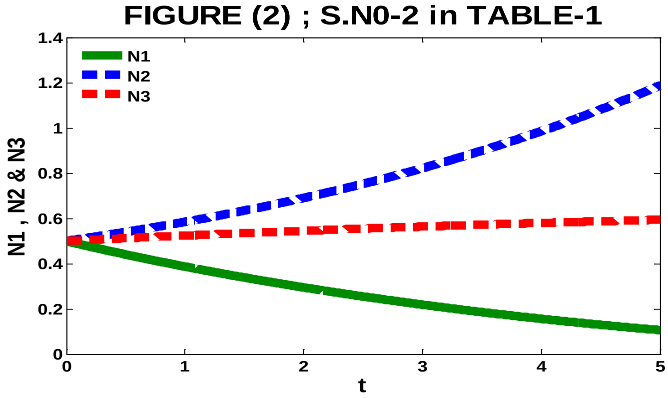

Figure (2): Variation of a_1 vs t*,when a_1=0.2, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4 , a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5

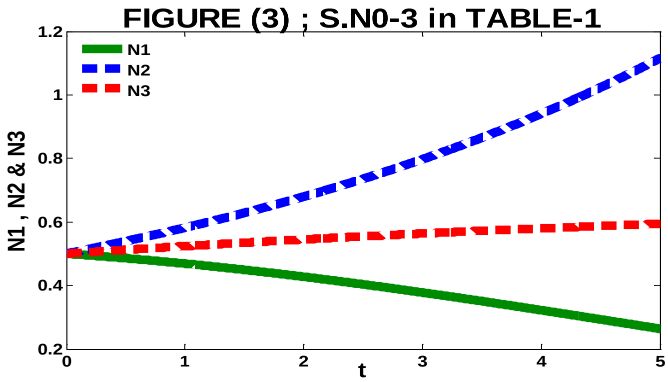

Figure (3): Variation of a_1 vs t*,when a_1=0.3, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5

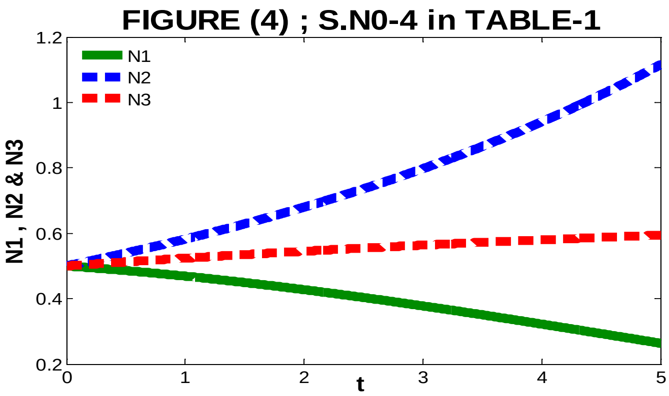

Figure (4): Variation of a_1 vs t*, when a_1=0.4, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5

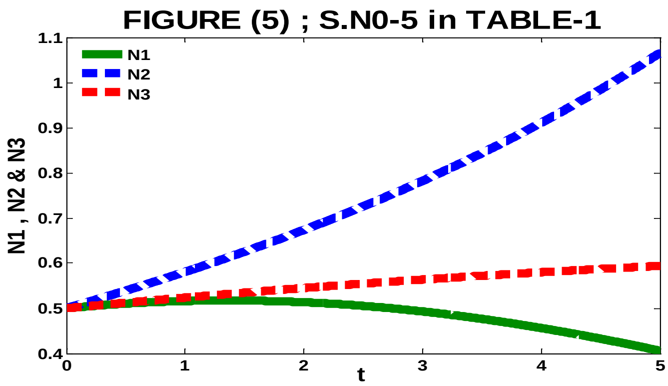

Figure (5): Variation of a_1 vs t*,when a_1=0.5, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5

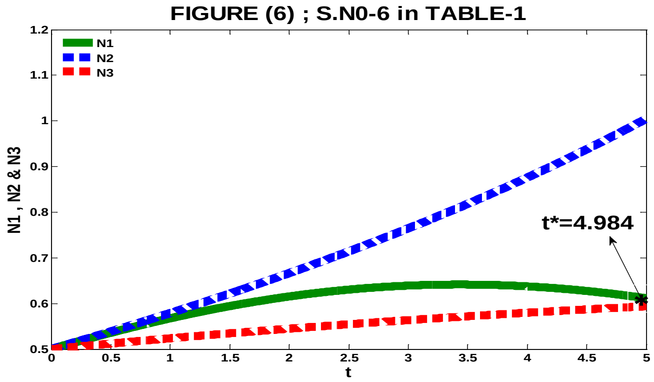

Figure (6): Variation of a_1 vs t*,when a_1=0.6, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5

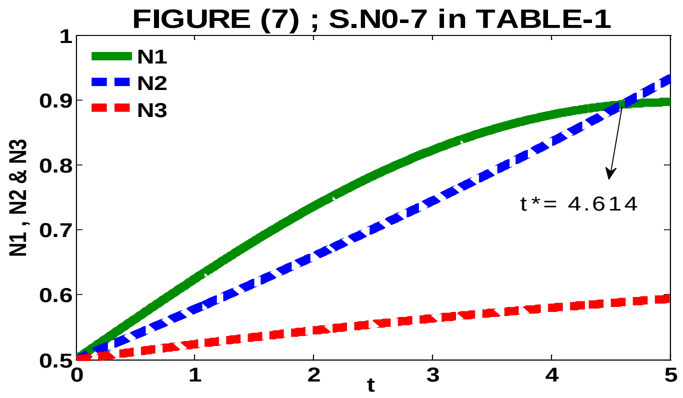

Figure (7): Variation of a_1 vs t*,when a_1=0.7, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5

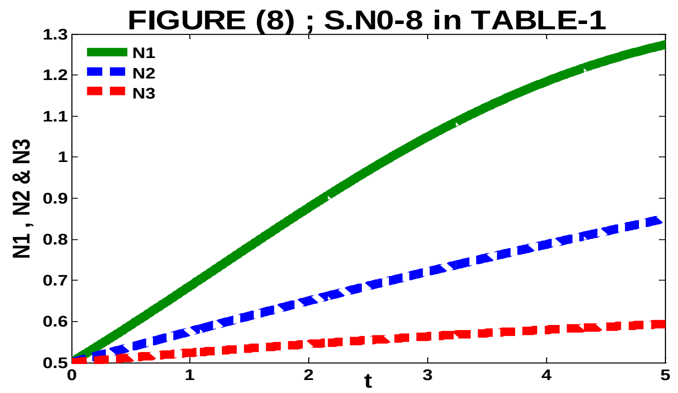

Figure (8): Variation of a_1 vs t*,when a_1=0.8, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5

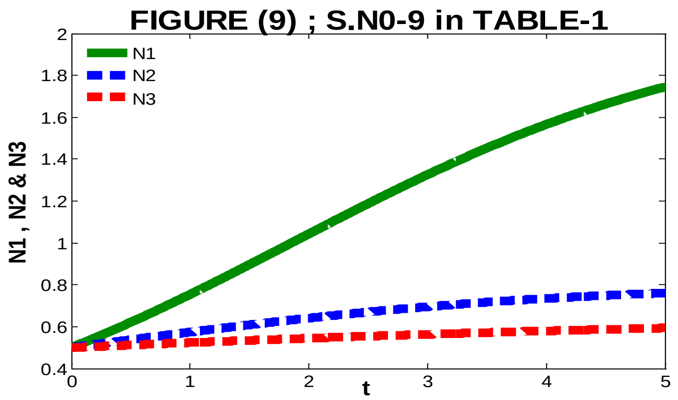

Figure (9): Variation of a_1 vs t*,when a_1=0.9, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5

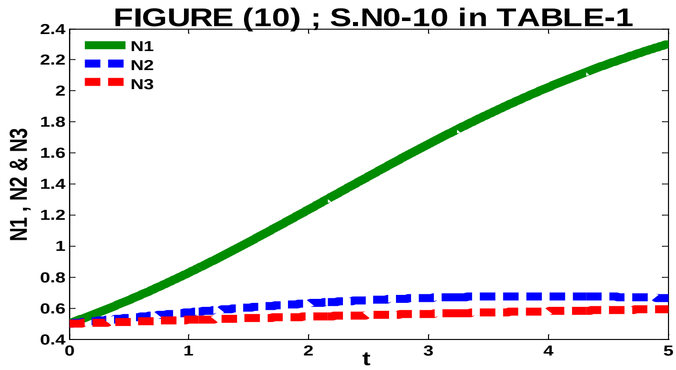

Figure (10): Variation of a_1 vs t*,when a_1=1.0, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

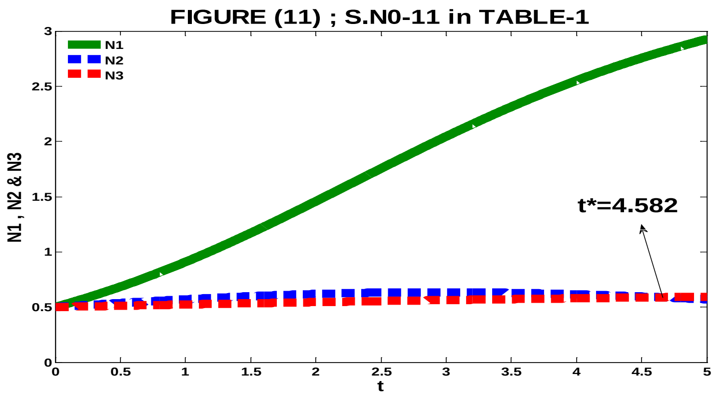

Figure (11): Variation of a_1 vs t*,when a_1=1.1, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, $_{33}$=0.4, N_{10} =N_{20}= N_{10} =0.5.

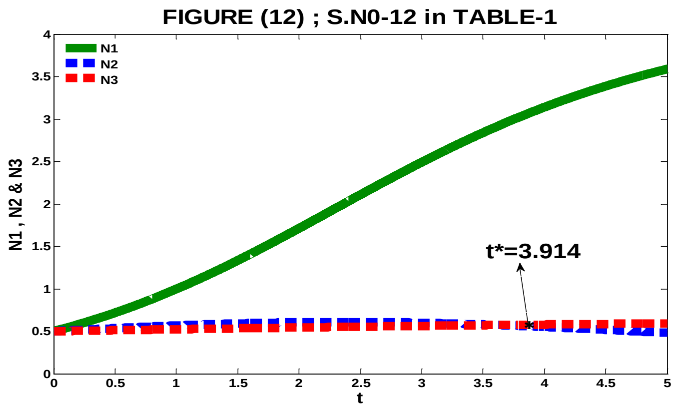

Figure (12): Variation of a_1 vs t*,when a_1=1.2, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

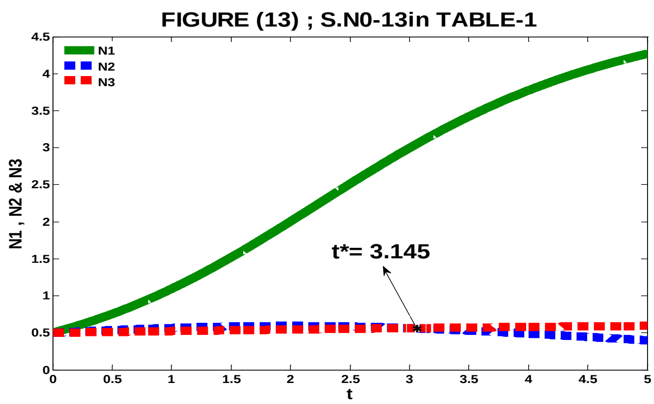

Figure (13): Variation of a_1 vs t*,when a_1=1.3, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

FIGURE (14) ; S.N0-14 in TABLE-1

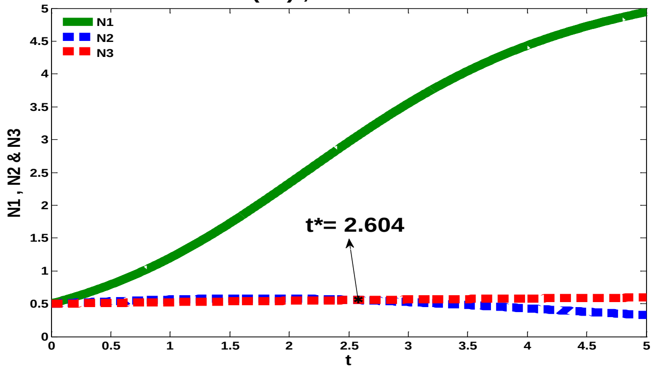

Figure (14): Variation of a_1 vs t*,when a_1=1.4, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

FIGURE (15) ; S.N0-15 in TABLE-1

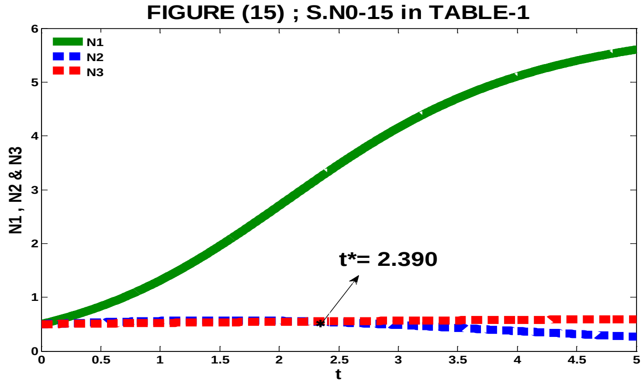

Figure (15): Variation of a_1 vs t*,when a_1=1.5, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

FIGURE (16) ; S.N0-16 in TABLE-1

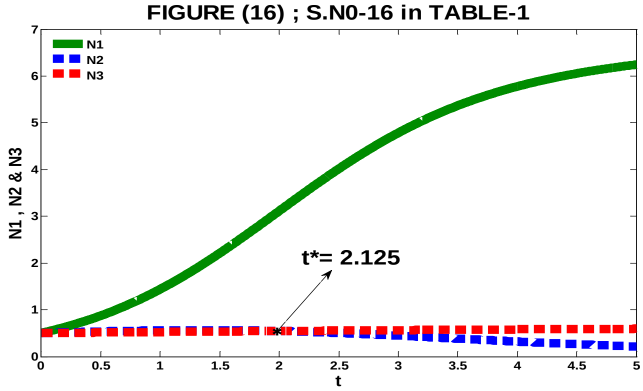

Figure (16): Variation of a_1 vs t*, when a_1=1.6, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

FIGURE (17) ; S.N0-17 in TABLE-1

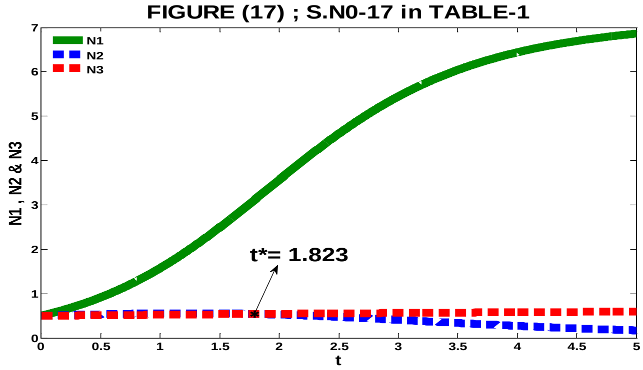

Figure (17): Variation of a_1 vs t*, when a_1=1.7, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

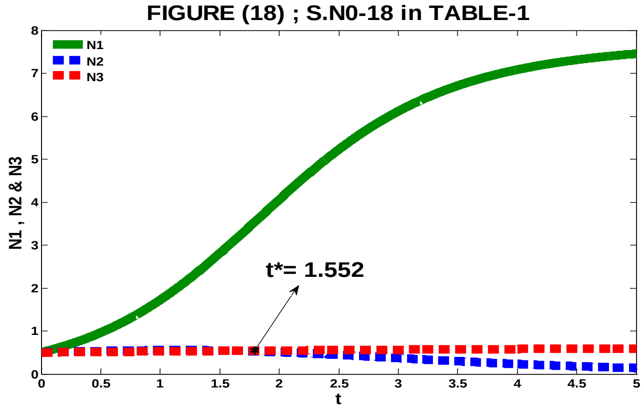

Figure (18): Variation of a_1 vs t*, when a_1=1.8, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

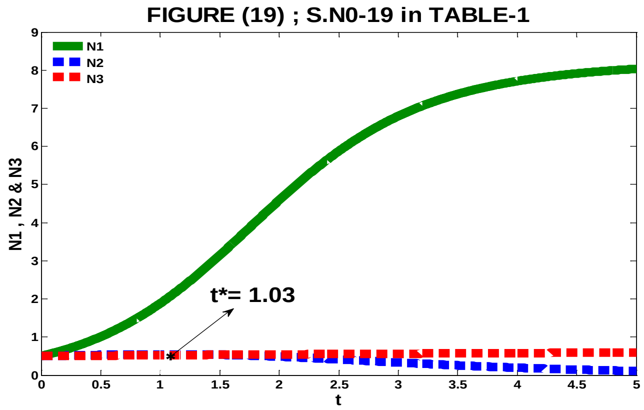

Figure (19): Variation of a_1 vs t*, when a_1=1.6, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

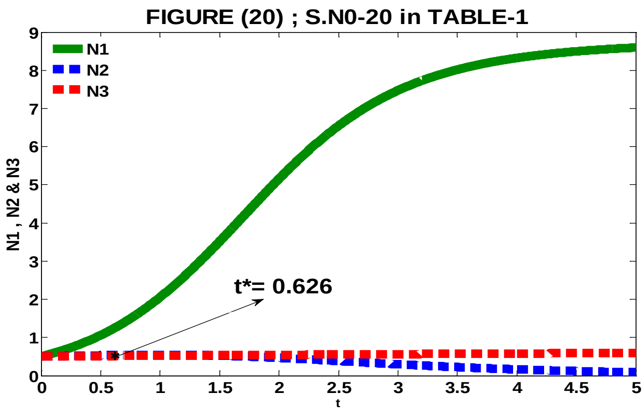

Figure (20): Variation of a_1 vs t*,when a_1=1.6, a_{11}=0.2, a_{12}=0.5, a_2=0.2, a_{22}= 0.4, a_{21}=0.3, a_3 =0.1, a_{13}=0.4, a_{33}=0.4, N_{10} =N_{20}= N_{10} =0.5.

The carrying capacity of Ammensal-prey is obtained by the ratio of the natural growth rate of Ammensal-prey species and the rate of decrease of Ammensal-prey species (due to its own insufficient resources). The values of Carrying capacity of Ammensal-prey species in respect with the derived numerical solutions are tabulated in Table-3 along with the corresponding values of dominance reversal time(t*).

Table-2

S.NO	Carrying Capacity of Ammensal-Prey(K_1)	Dominance reversal time(t_{23}*) (between predator & enemy)
1	5.5	4.582
2	6	3.914

3	**6.5**	**3.145**
4	**7**	**2.604**
5	**7.5**	**2.390**
6	**8**	**2.125**
7	**8.5**	**1.823**
8	**9**	**1.552**
9	**9.5**	**1.03**
10	**10**	**0.626**

VI. Conclusions:

The following facts and results are observed with the help of above numerical study.

Case (i):
(i) Initially Ammensal-prey has declining growth rate, predator has exponential growth rate and enemy has constant growth rate in which the three species are having same initial growth rates.
(ii) Predator prevails over the enemy and Ammensal-prey throughout the duration with prospering growth rate. In addition to this, enemy predominates over the Ammensal-Prey.
(iii)The dominance reversal time (t*) does not occur among the three species.

These are illustrated from Figure (1) to Figure (5).

Case (ii):
(iv) Ammensal –Prey has potential growth rate to dominate over the enemy up to the time instinct (t_{13}*) and then declines in the remaining span of time.
(v) The reversal dominance time occurs only between Ammensal-Prey and enemy,
(vi)The predator is not affected in any manner by enemy or by the growth rate of Ammensal-Prey as shown in Figure (6).

Case (iii):
(vii)The Ammensal –Prey continues its potential growth rate and prevails the predator at reversal dominance time (t_{13}*).Moreover, The Ammensal-Prey dominates over the remaining both species in all through the interval. It is depicted in Figure (7).

Case (iv):
(viii)The predator reins over the enemy in short span of duration and goes down with low growth rate. Even though Predator and enemy have the same initial growth rates,the predator is influenced by the enemy. However, there is a down fall of growth rate in predator. The various dominance reversal times (t*) can be seen from Figure (8) to Figure (20).

By enhancing the growth rate of Ammensal –prey,it is noticed that the predator will lose its entire strength and moves towards the equilibrium point.

VII. Over all Conclusions:

Criterion	Conclusion
The natural growth of Ammensal-prey increases in a three species ecosystem	The predator and enemy gradually decrease and predator will extinct at the end.
	The carrying capacity of Ammensal –Prey increases
	The dominance reversal time t^*_{12} decreases step by step
	The dominance reversal time t^*_{23} decreases gradually

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