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EFFECTS OF A MAGNETIC FIELD ON THE FREE CONVECTIVE FLOW OF JEFFREY FLUID PAST AN INFINITE VERTICAL POROUS PLATE WITH CONSTANT HEAT FLUX

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1. INTRODUCTION

The study of flow and heat transfer for an electrically conducting fluid past a porous plate under has attracted the interest of many investigators in view of its applications in many engineering problems such as magnetohydrodyanmic (MHD) generator, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and the boundary layer control in the field of aerodynamics. The Jeffrey model is relatively simpler linear model using time derivatives instead of convected derivatives, for example the Oldroyd-B model does, it represents a rheology different from the Newtonian. Yamamoto and Yoshida (1974) have studied the flow of a viscous fluid past an infinite vertical porous plate with convective acceleration. An exact solution for the flow of a non-Newtonian fluid past an infinite porous plate was studied by Rajagopal and Gupta (1984). Ogulu and Amos (2005) have investigated the free convective flow of a non-Newtonian fluid past a vertical porous plate.

The MHD boundary layer flow over a semi-infinite plate with an aligned magnetic field in the presence of a pressure gradient was studied by Gribben (1965). Raptis and Kafousias (1982) have discussed the influence of a magnetic field upon the steady free convection flow through a porous medium bounded by an infinite vertical plate with constant suction velocity, and when the plate temperature is also constant. Effects of a magnetic field on the free convective flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux was investigated by Raptis (1983). Raptis (1986) have studied mathematically the case of time-varying two-dimensional natural convective heat transfer of an incompressible, electrically-conducting viscous fluid via a highly porous medium bounded by an infinite vertical porous plate. Takhar and Ram (1991) have investigated the effects of Hall currents on hydromagnetic free convection boundary layer flow through a porous medium past a plate, using harmonic analysis. Takhar and Ram (1994) have studied the MHD free porous convection heat transfer of water at 4°C through a porous medium. Unsteady MHD convection flow of a polar fluid past a vertical moving porous plate in a porous medium was investigated by Kim (2001). Kim and Lee (2003) have discussed the MHD oscillatory flow of a micropolar fluid over a vertical porous plate. Heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium was studied by Kim (2004). Mohamed and Abo-Dahab (2009) have investigated the influence of chemical reaction and thermal radiation on the heat and mass transfer in MHD micropolar fluid flow over a vertical moving porous plate in a porous medium with heat generation.

In view of these, we studied the effects of a magnetic field on the free convective flow of Jeffrey fluid past an infinite vertical porous plate with constant heat flux. A perturbation series solution was used to obtain velocity field and temperature field for small values of Eckert number Ec. The effects of various emerging parameter on the velocity field and temperature field are discussed in detail through graphs.

2. MATHEMATICAL FORMULATION

For a Jeffrey fluid, the constitutive equation for the extra stress tensor S is given by

$$S = \frac{\mu}{1+\lambda_1}\left[\dot{\gamma} + \lambda_2 \ddot{\gamma}\right] \tag{2.1}$$

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where λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, μ is the viscosity function, $\dot{\gamma}$ is the shear rate and dots over the quantities indicate differentiation with respect to time.

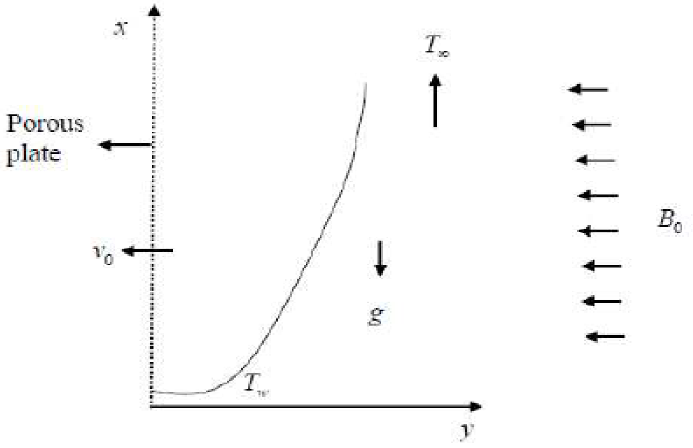

Fig. 1 Physical model

We consider the steady laminar two – dimensional convection flow of an incompressible Jeffrey fluid through a porous medium over a semi-infinite vertical porous moving plate in the presence of a magnetic field. It is assumed that the transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible when comparing with applied magnetic field. The x-axis is taken along the planar surface in the upward direction and the y-axis is taken to be normal to it. Due to the semi-infinite plane surface assumption, the flow variables are functions of y only. Fig. 1 shows the physical model of the problem.

Under these assumptions the equations which govern the flow are given by

$$\frac{\partial v}{\partial y} = 0 \tag{2.2}$$

$$v\frac{\partial u}{\partial y} = g\beta\left(T - T_\infty\right) + \frac{\nu}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{2.3}$$

$$v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.4}$$

where u, v are the dimensional velocities along the x and y, respectively. ρ is the density, ν is the kinematic viscosity, g is the acceleration due to gravity, σ is the electrical conductivity of the fluid, B_0 is the magnetic field strength, T is the temperature, c_p is the specific heat constant and k is the thermal conductivity.

The boundary conditions are

$$u = 0, \qquad \frac{\partial T}{\partial y} = -\frac{q}{k} \qquad \text{at} \qquad y = 0 \tag{2.5}$$

$$u \to 0, \qquad T \to T_\infty \qquad \text{as} \qquad y \to \infty \tag{2.6}$$

where q is the constant heat flux per unit area at the plate and T_∞ is the free stream temperature.

Integrating Eq. (2.2) for constant suction, we get

$$v = -v_0 \tag{2.7}$$

where $v_0 > 0$ is the constant suction velocity at the plate and the negative sign indicates that the suction velocity is directed towards the plate.

Introducing the following non-dimensional quantities

$$\bar{y}=\frac{v_0 y}{\nu},\ \bar{u}=\frac{u}{v_0},\ \theta=\left(\frac{T-T_\infty}{q\nu}\right)kv_0,\ \Pr=\frac{\rho\nu c_p}{k},\ Ec=\frac{kv_0^3}{q\nu c_p},$$

$$Gr=\frac{g\beta\nu^2 q}{kv_0^4},\ M^2=\frac{\sigma\nu B_0^2}{\rho v_0^2}$$

into the Equations (2.3) and (2.4) and using Eq. (2.7), we obtain (dropping bars)

$$\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}-M^2 u=-Gr\theta \tag{2.8}$$

$$\frac{\partial^2\theta}{\partial y^2}+\Pr\frac{\partial\theta}{\partial y}=-\Pr Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.9}$$

The corresponding non-dimensional boundary conditions are

$$u=0,\qquad \frac{\partial\theta}{\partial y}=-1\qquad \text{at}\qquad y=0 \tag{2.10}$$

$$u\to 0,\qquad \theta\to 0\qquad \text{as}\qquad y\to\infty \tag{2.11}$$

3. SOLUTION

Since the Eq. (2.9) is a non-linear differential equation, it is not possible to get closed form solution. But, Ec is small for incompressible fluids, so we expand the velocity field and temperature field as

$$u=u_0+Ecu_1+\mathrm{O}\left(Ec^2\right) \tag{3.1}$$

$$\theta=\theta_0+Ec\theta_1+\mathrm{O}\left(Ec^2\right) \tag{3.2}$$

Substituting Equations (3.1) and (3.2) into the Equations (2.8) – (2.11), we get

3.1 Zeroth order system $\left(Ec^0\right)$

$$\frac{1}{1+\lambda_1}\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial u_0}{\partial y}-M^2 u_0=-Gr\theta_0 \tag{3.3}$$

$$\frac{\partial^2\theta_0}{\partial y^2}+\Pr\frac{\partial\theta_0}{\partial y}=0 \tag{3.4}$$

The corresponding non-dimensional boundary conditions are

$$u_0=0,\qquad \frac{\partial\theta_0}{\partial y}=-1\qquad \text{at}\qquad y=0 \tag{3.5}$$

$$u_0\to 0,\qquad \theta_0\to 0\qquad \text{as}\qquad y\to\infty \tag{3.6}$$

3.2 First order system $\left(Ec\right)$

$$\frac{1}{1+\lambda_1}\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial u_1}{\partial y}-M^2 u_1=-Gr\theta_1 \tag{3.7}$$

$$\frac{\partial^2\theta_1}{\partial y^2}+\Pr\frac{\partial\theta_1}{\partial y}=-\Pr\left(\frac{\partial u_0}{\partial y}\right)^2 \tag{3.8}$$

The corresponding non-dimensional boundary conditions are

$$u_1 = 0, \frac{\partial \theta_1}{\partial y} = 0 \qquad \text{at} \qquad y = 0 \tag{3.9}$$

$$u_1 \to 0, \qquad \theta_1 \to 0 \qquad \text{as} \qquad y \to \infty \tag{3.10}$$

3.3 Solution of order zero

Solving the Equations (3.3) and (3.4) using the boundary conditions (3.5) and (3.6), we get

$$u_0 = \frac{Gr}{\Pr}\frac{(1+\lambda_1)}{A}\left[e^{-m_1 y} - e^{-\Pr y}\right] \tag{3.11}$$

$$\theta_0 = \frac{1}{\Pr}e^{-\Pr y} \tag{3.12}$$

where $m_1 = \frac{1+\lambda_1+\sqrt{(1+\lambda_1)^2+4N^2}}{2}$, $N = (1+\lambda_1)M^2$ and $A = \Pr^2 - (1+\lambda_1)\Pr - N$.

3.4 Solution of order One

Solving the Equations (3.7) and (3.8) using the Eq. (3.11) and boundary conditions (3.9) and (3.10), we obtain

$$u_1 = c_1 e^{-m_1 y} + \frac{Gr}{\Pr}\frac{A_5}{A}e^{-\Pr y} + GrA_4\left[B_1 e^{-2\Pr y} + B_2 e^{-2m_1 y} - B_3 e^{-(m_1+\Pr)y}\right] \tag{3.13}$$

$$\theta_1 = -\frac{A_5}{\Pr}e^{-\Pr y} - A_4\left[A_1 e^{-2\Pr y} + A_2 e^{-2m_1 y} - A_3 e^{-(m_1+\Pr)y}\right] \tag{3.14}$$

where $A_1 = \frac{1}{2}$, $A_2 = \frac{m_1}{4m_1 - 2\Pr}$, $A_3 = \frac{2\Pr}{m_1+\Pr}$, $A_4 = \frac{1}{\Pr}\left[\frac{Gr(1+\lambda_1)}{A}\right]^2$,

$A_5 = A_4\left[-2\Pr A_1 - 2m_1 A_2 + (m_1+\Pr)A_3\right]$, $B_1 = \frac{A_1}{4\Pr^2 - 2(1+\lambda_1)\Pr - N^2}$,

$B_2 = \frac{A_2}{4m_1^2 - 2(1+\lambda_1)m_1 - N^2}$, $B_3 = \frac{A_3}{(m_1+\Pr)^2 - (1+\lambda_1)(m_1+\Pr) - N^2}$

and $c_1 = -\frac{Gr}{\Pr}\frac{A_5}{A} - GrA_4\left[B_1 + B_2 - B_3\right]$.

The final expressions of the velocity field and temperature field are given by

$$u = \frac{Gr}{\Pr}\frac{(1+\lambda_1)}{A}\left[e^{-m_1 y} - e^{-\Pr y}\right] + Ec\begin{pmatrix} c_1 e^{-m_1 y} + \frac{Gr}{\Pr}\frac{A_5}{A}e^{-\Pr y} \\ +GrA_4\left[B_1 e^{-2\Pr y} + B_2 e^{-2m_1 y} - B_3 e^{-(m_1+\Pr)y}\right]\end{pmatrix} \tag{3.15}$$

$$\theta = \frac{1}{\Pr}e^{-\Pr y} + Ec\left(-\frac{A_5}{\Pr}e^{-\Pr y} - A_4\left[A_1 e^{-2\Pr y} + A_2 e^{-2m_1 y} - A_3 e^{-(m_1+\Pr)y}\right]\right) \tag{3.16}$$

4. DISCUSSION OF THE RESULTS

Fig. 2 shows the effect of material parameter λ_1 on the velocity u for $M = 1$, $Gr = 1$, $\Pr = 0.71$ and $Ec = 0.01$. It is observed that, the velocity u increases with increasing λ_1.

The effect of Hartmann number M on the velocity u for $\lambda_1 = 0.3, Gr = 1, \Pr = 0.71$ and $Ec = 0.01$ is shown in Fig. 3. It is found that, the velocity u decreases with an increase in M.

Fig. 4 depicts the effect of Prandtl number $\Pr$ on the velocity u for $M = 1, Gr = 1, \lambda_1 = 0.3$ and $Ec = 0.01$. It is noted that, the velocity u decreases on increasing $\Pr$.

The effect of Grashof number Gr on the velocity u for $M = 1, \lambda_1 = 0.3, \Pr = 0.71$ and $Ec = 0.01$ is depicted in Fig. 5. It is observed that, the velocity u increases with an increase in Gr.

Fig. 6 illustrates the effect of Eckert number Ec on the velocity u for $M = 1, Gr = 1, \Pr = 0.71$ and $\lambda_1 = 0.3$. It is found that, the velocity u increases on increasing Ec.

The effect of material parameter λ_1 on the temperature θ for $M = 1, Gr = 1, \Pr = 0.71$ and $Ec = 0.01$ is shown in Fig. 7. It is noted that, the temperature θ increases with increasing λ_1.

Fig. 8 shows the effect of Hartmann number M on the temperature θ for $\lambda_1 = 0.3, Gr = 1, \Pr = 0.71$ and $Ec = 0.01$. It is observed that, the temperature θ decreases with increasing M.

The effect of Prandtl number $\Pr$ on the temperature θ for $M = 1, Gr = 1, \lambda_1 = 0.3$ and $Ec = 0.01$ is presented in Fig. 9. It is noted that, the temperature θ decreases on increasing $\Pr$.

Fig. 10 depicts the effect of Grashof number Gr on the temperature θ for $M = 1, \lambda_1 = 0.3, \Pr = 0.71$ and $Ec = 0.01$. It is found that, the temperature θ increases with increasing Gr.

The effect of Eckert number Ec on the temperature θ for $M = 1, Gr = 1, \Pr = 0.71$ and $Ec = 0.01$ is depicted in Fig. 11. It is observed that, the temperature θ increases with an increase in Ec.

5. CONCLUSIONS

In this chapter, we studied the effects of a magnetic field on the free convective flow of Jeffrey fluid past an infinite vertical porous plate with constant heat flux. The governing non-linear equations are solved for the velocity field and temperature field using the perturbation technique. It is observed that, the velocity field u and temperature field θ are increases with increasing λ_1, Gr and Ec, while they decreases with increasing M and $\Pr$.

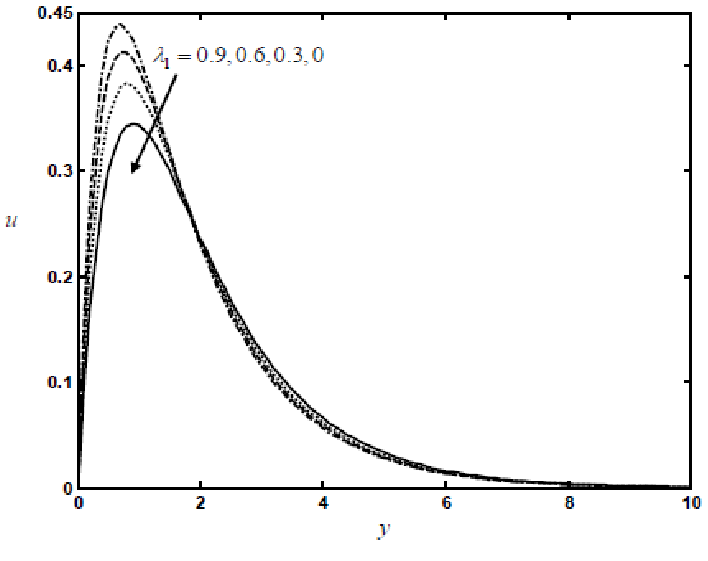

Fig. 2 Effect of material parameter λ_1 on the velocity u for $M = 1, Gr = 1$, $\Pr = 0.71$ and $Ec = 0.01$.

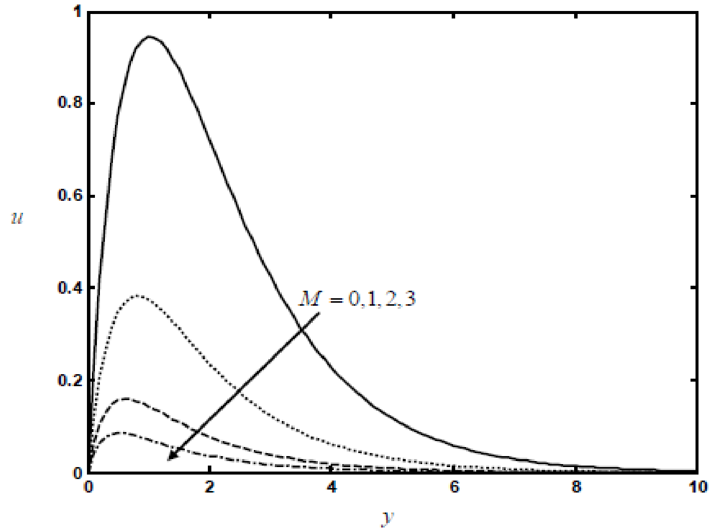

Fig. 3 Effect of Hartmann number M on the velocity u for $\lambda_1 = 0.3, Gr = 1$, $\Pr = 0.71$ and $Ec = 0.01$.

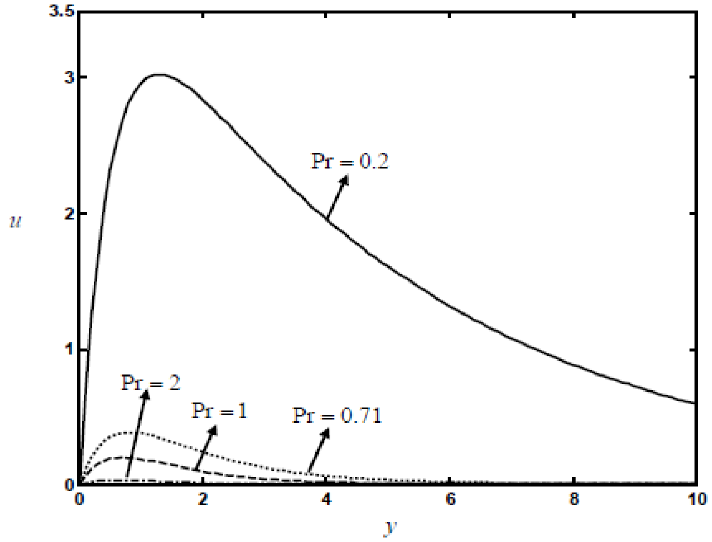

Fig. 4 Effect of Prandtl number Pr on the velocity u for $M = 1, Gr = 1$, $\lambda_1 = 0.3$ and $Ec = 0.01$.

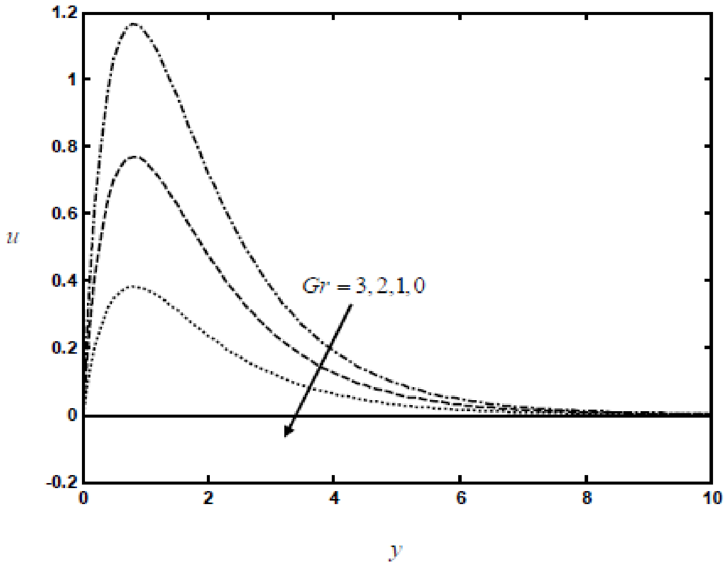

Fig. 5 Effect of Grashof number λ_1 on the velocity u for $M = 1, \lambda_1 = 0.3$, $\Pr = 0.71$ and $Ec = 0.01$.

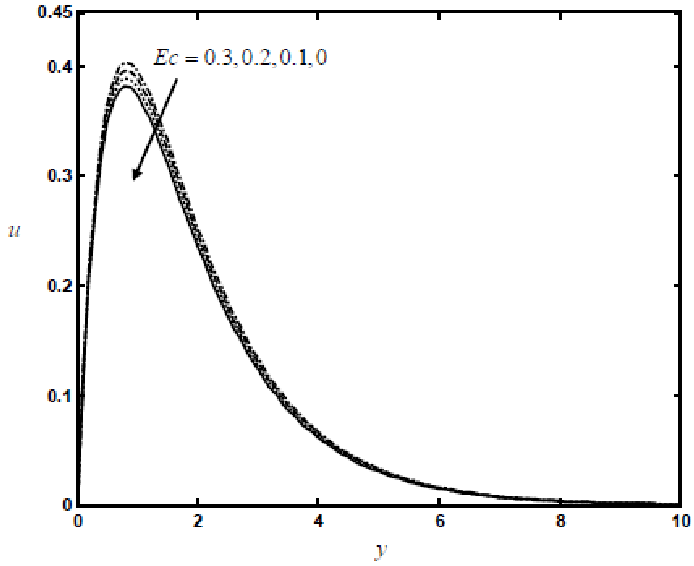

Fig. 6 Effect of Eckert number Ec on the velocity u for $M = 1, Gr = 1$, $\Pr = 0.71$ and $\lambda_1 = 0.3$.

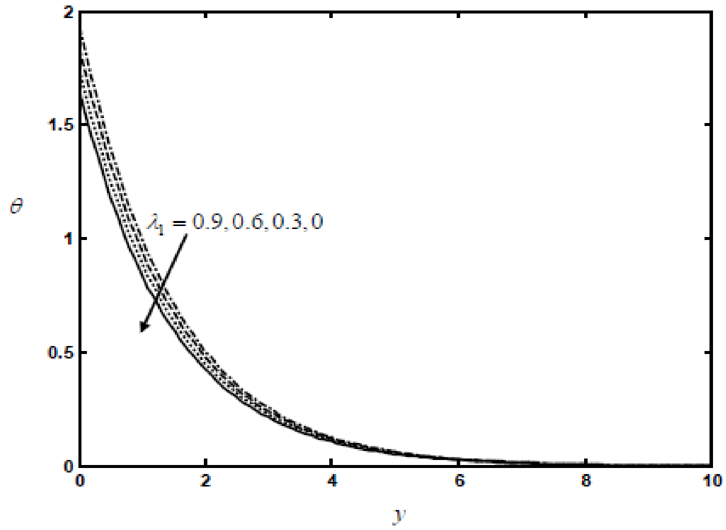

Fig. 7 Effect of material parameter λ_1 on the temperature θ for $M = 1, Gr = 10$, $\Pr = 0.71$ and $Ec = 0.01$.

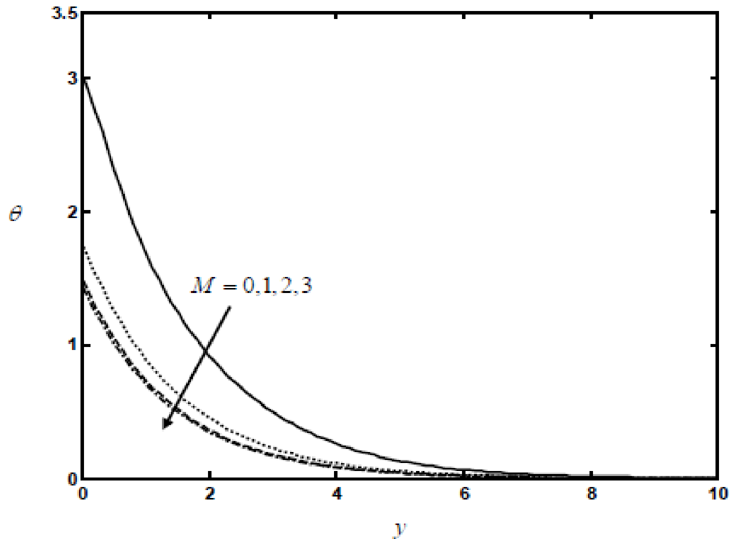

Fig. 8 Effect of Hartmann number M on the temperature θ for $\lambda_1 = 0.3, Gr = 10$, $\Pr = 0.71$ and $Ec = 0.01$.

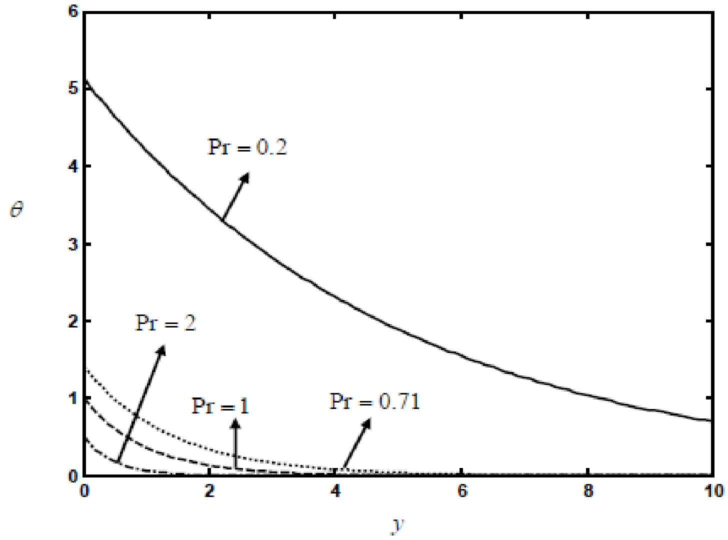

Fig. 9 Effect of Prandtl number Pr on the temperature θ for $\lambda_1 = 0.3, Gr = 1,$ $M = 1$ and $Ec = 0.01$.

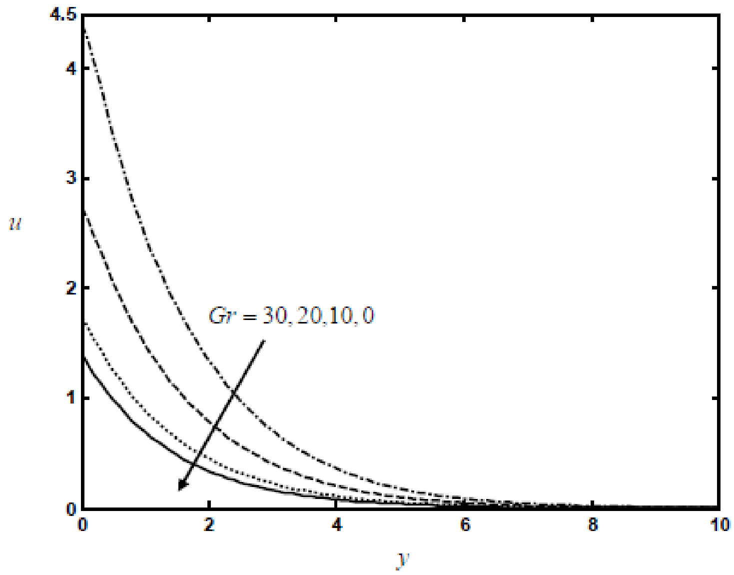

Fig. 10 Effect of Grashof number Gr on the temperature θ for $\lambda_1 = 0.3, M = 1,$ $\Pr = 0.71$ and $Ec = 0.01$.

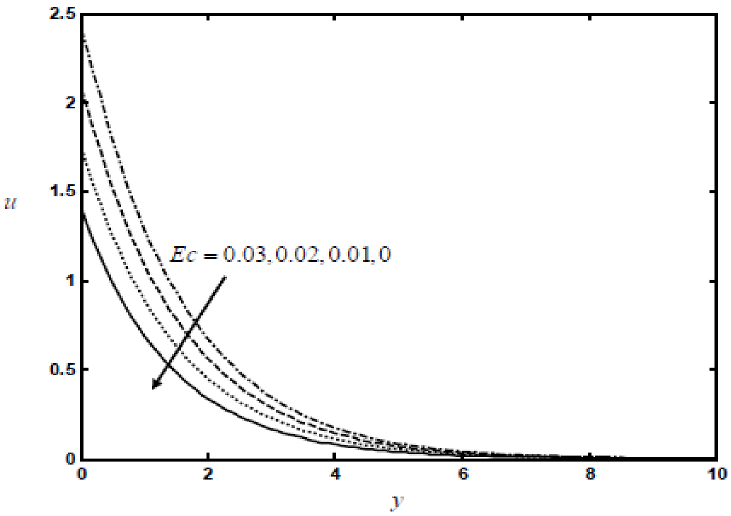

Fig. 11 Effect of Eckert number Ec on the temperature θ for $\lambda_1 = 0.3, Gr = 10,$ $\Pr = 0.71$ and $M = 1$.

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