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MIXED CONVECTIVE FLOW OVER A NON–ISOTHERMAL VERTICAL SURFACE IN A POROUS MEDIUM WITH APPLIED MAGNETIC FIELD AND THE ANALOGY OF A SECOND ORDER FLUID BY CREATING SINUSOIDAL DISTURBANCES

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ABSTRACT

In this paper mixed convective flow over a non–isothermal vertical surface in a porous medium with applied magnetic field (part-a) and the problem of visco elastic fluid flow disturbances on a flat porous bed when the bounding surface is subjected to sinusoidal disturbances have been examined (part-b). The salient features in both the cases have been compared wherever possible. Surprisingly, it has been noticed that some of the parameters in both the cases causes similar effect. It is observed that, the porosity factor in the case of second order fluid is almost similar to that of the magnetic field and the visco elasticity parameter is identical to that of radiation parameter. Further, there exists a close analogy between the porosity of the medium when examined on the flow by creating the sinusoidal disturbances which is identical to the parameter λ in case of mixed convective flow. Interestingly, it is found that for the constant visco elasticity and porosity parameter, the frequency of excitation parameter behaves exactly similar to that of mixed convection parameter. Also frequency of excitation (σ) and time (t) were found to have the same effect as that of mixed convection parameter. However, it is noticed that, the investigations stated above are applicable in the boundary layer region.

Keywords: *Mixed convection, Variable temperature, Porous media, Elastico viscous fluid, Sinusoidal disturbances.*

Nomenclature in part-a:

A	:	Constant
a	:	Absorption coefficient
B	:	Constant
B_0	:	Magnetic field
c_p	:	Specific heat at constant pressure
Ec	:	Eckert number
F	:	Radiation parameter
g	:	Gravitational acceleration
h	:	Heat transfer coefficient
K	:	Porous parameter
k	:	Permeability
k_m	:	Thermal conductivity of medium
M^2	:	Non dimensional magnetic parameter
Pe	:	Peclet number
Pr	:	Prandtl number
q	:	Heat flux
Ra	:	Rayleigh number
Re	:	Reynolds number
S	:	Span of the plate
T	:	Temperature
u, v	:	Velocity components in the direction of x and y directions

Greek symbols:

α	:	Thermal diffusivity
α_m	:	Effective thermal diffusivity
β	:	Coefficient of volume expansion
ε or e	:	Mixed convection parameter
η	:	Dimensionless distance
μ	:	Viscosity
υ	:	Kinematic viscosity
ρ	:	Density
θ	:	Dimensionless temperature
σ	:	Electrical conductivity
σ_R	:	Stefan – Boltzmann constant

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ψ : Stream function

Subscripts:

x : Local
∞ : Free stream conditions
w : Wall

Nomenclature in part-b:

A_i : Acceleration component in i th direction
a_i : Non dimensional acceleration in i th direction
$E_{ij}^{(1)}, E_{ij}^{(2)}$: Strain tensor in the dimensional form
$e_{ij}^{(1)}, e_{ij}^{(2)}$: Strain tensor in the non-dimensional form
F_X, F_Y, F_Z : External forces applied along X, Y and Z directions
$g(s)$: Given history
$g_\alpha(s)$: Retarded History
K : Permeability of the porous bed
k : Non dimensionalised porosity factory
L : Characteristic length
P : Indeterminate hydrostatic pressure
p : Non dimensional indeterminate pressure
r : Polar coordinate
S_{ij} : Stress tensor
T : Dimensional time parameter
t : Non-dimensional time parameter
U_i : Dimensional velocity component in i th direction
u : Non-dimensional velocity
u_i : Non-dimensional velocity component along the ith coordinate
X_i, Y_i : Co-ordinate axes (dimensional form)
x_i, y_i : Co-ordinate axes (non-dimensional form)

Greek symbols

α : Retardation factor
β : Visco elasticity parameter
δ : Polar coordinate
ϕ_1 : Coefficient of viscosity
ϕ_2 : Coefficient of elastico viscosity
ϕ_3 : Coefficient of cross viscosity
ϑ : Non dimensionalised cross viscosity factor
υ_c : Non dimensionalised cross viscosity parameter
μ : Coefficient of viscosity
ρ : Density of fluid
σ : Non-dimensional frequency of excitation

INTRODUCTION:

Mixed convection in porous medium has important applications in soil physics, geothermal energy extraction, chemical engineering, oil reservoir modeling and in biological systems. Radiation in heat transfer accounts in high temperature applications viz., plasma physics, nuclear reactions, liquid metal flows, magneto hydrodynamic accelerators and in power generation systems. In all the above applications, understanding the boundary layer development and convective heat transfer characteristics are of primary requirement to further investigate the concerned problem.

Several authors have studied the effects of magnetic field on convection in porous medium. Cheng and Minkowyiz [1] studied the free convective heat transfer characteristics for vertical plate in porous medium and obtained analytical expressions for boundary layer thickness, local and overall heat flux. Later Ramanarao, *et al.* [2] discussed the effects of transverse magnetic field on the heat transfer characteristics in a porous medium and brought out the effects of porous parameter on temperature and Nusselt number. Subsequently Cheng [3] has investigated the mixed convection on inclined surface using boundary layer approximations, while Sobha and Ramakrishna [4] presented the effects of magnetic field on heat transfer characteristics in porous medium under natural convection. Aly *et al.* [5] has numerically investigated the existence and uniqueness of a vertical flowing fluid past a vertical surface in porous medium by considering variable wall temperature under mixed convection conditions.

Ramakrishna and Sobha [6] examined the effect of magnetic field on mixed convective flow over a heated vertical plate in porous medium for buoyancy aiding and opposing flow conditions. Rabhi A. Damseh [7] investigated the effects of MHD mixed convection heat transfer from a vertical surface with radiation conduction interaction for both transverse and induced magnetic field under isothermal surface conditions. Inclusion of radiation component in the conservation energy equation makes it complicated due to non-linear partial differential equations. Chamkha [8] recently studied the effects of thermal radiation and magnetic field on natural convection heat transfer from an inclined plate embedded in variable porosity medium, while Merkin and Pop [9] obtained similarity solutions for mixed convective conditions considering wall temperature variation.

It is a common experience that, the thermal effects influence the fluid property to some extent. So as to study the thermal effects on fluid entities, sometimes it might be possible that, similar effects could be obtained by selecting appropriate elastic, visco elastic fluid with non thermal effects, but in different situations and an analogy in the fluid entities could be established. Here in the present situation, a visco elastic fluid of second order type has been selected as an object of study (of course in a different context) and a suitable mathematical model has been employed. Interestingly, the results are found to be same as that of the problem of mixed convective flow and an analogy could be drawn to some extent.

Ramakrishna, Sobha *et al.* [10] studied mixed convective flow over a non-isothermal radiative vertical surface embedded in a porous medium with applied magnetic field and found that, the magnetic effect increases the boundary layer thickness while reducing the local heat transfer. Further, increasing radiation parameter also increases the rate of heat transfer and also observed that, it decreases while enhancing the rate of heat transfer. In addition to the above under high radiative conditions the effect of magnetic field and mixed convective parameter is found to be negligible

Ramana Murthy and Kulkarni [11] studied on the class of exact solutions of an incompressible second order fluid flow by creating the sinusoidal disturbances. In course of analysis it is found that, as the porosity effects the velocity profiles and the frequency of excitation shows the decreasing trend on the velocity, also it is noted that, the elastico viscosity parameter has profound effect on the magnification factor.

Thus certain common effects are found in both the above situations. Therefore, the situations are found to be almost similar in global and different in context.

Noll [12] defined a simple material as a substance for which stress can be determined with the entire knowledge of the history of the strain. This is called a simple fluid, having the property that at all local states, with the same mass density, intrinsically equal in response, with all observable differences in response being due to definite differences in the history. For any given history g(s), a retarded history $g_\alpha(s)$ can be defined as:

$$g_\alpha(s) = g(\alpha s); \qquad 0 < s < \infty, \qquad 0 < \alpha \le 1 \tag{1}$$

α being termed as a retardation factor. Assuming that, the stress is more sensitive to recent deformation that to the deformations at distant past, Coleman and Noll [13] proved that, the theory of simple fluids yields the theory of perfect fluids as $\alpha \to 0$ and that of Newtonian Fluids as a correction (up to the order of α) to the theory of the perfect fluids. Neglecting all the terms of the order of higher than two in α, we have incompressible elastic viscous fluid of second order type whose constitutive relation is governed by:

$$S_{ij} = -P\delta_{ij} + \phi_1 E_{ij}^{(1)} + \phi_2 E_{ij}^{(2)} + \phi_3 E_{ij}^{(1)^2} \tag{2}$$

where

$$E_{ij}^{(1)} = U_{i,j} + U_{j,i} \tag{3}$$

and

$$E_{ij}^{(2)} = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \tag{4}$$

In the above equations, S_{ij} is the stress-tensor, U_i, A_i are the components of velocity and acceleration in the direction of the i-th coordinate X_i , P is indeterminate hydrostatic pressure and the coefficients ϕ_1, ϕ_2 and ϕ_3 are material constants.

The constitutive relation for general Rivlin-Ericksen [14] fluids also reduces to equation (2) when the squares and higher orders of E^2 are neglected, the coefficients being constants. Also the non-Newtonian models considered by Reiner [15] could be obtained from equation (2) when $\phi_2 = 0$, naming ϕ_3 as the coefficient of cross viscosity. With reference to the Rivlin – Ericksen fluids, ϕ_2 may be called as the coefficient of viscosity. It has been reported that, a solution of poly-iso-butylene in cetane behaves as a second order fluid and Markovitz [16] determined the constants ϕ_1, ϕ_2 and ϕ_3.

Viscous fluid flow over wavy wall had attracted the attention of relatively few researchers although the analysis of such flows finds application in different areas such as transpiration cooling of re-entry vehicles and rocket boosters, cross hatching ablative surfaces and film vaporization in combustion chambers. Especially the stream, where the heat and mass transfer takes place in the chemical processing industry. The problem by considering the permeability of the bounding surface in the reactors assumes greater significance.

In view of several industrial and technological importances, Ramacharyulu [17] studied the problem of the exact solutions of two dimensional flows of a second order incompressible fluid by considering the rigid boundaries. Later, Lekoudis *et al* [18] presented a linear analysis of the compressible boundary layer flow over a wall. Subsequently, Shankar and Sinha [19] studied the problem of Rayleigh for wavy wall. The effect of small amplitude wall waviness upon the stability of the laminar boundary layer had been studied by Lessen and Gangwani [20]. Further, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a vertical flat wall was examined by Vajravelu and Shastri [21] and thereafter by Das and Ahmed [22]. The free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls was investigated by Patidar and Purohit [23]. Rajeev Taneja and Jain [24] had examined the problem of MHD flow with slip effects and temperature dependent heat in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate.

In this chapter two problems (i.e.) namely Mixed convective flow over a non-isothermal radiative vertical surface embedded in a porous medium with applied magnetic field in (part-a) and the problem of a visco elastic fluid flow of fluid disturbances on a flat bed in (part-b) has been examined when the fluid bed is subjected to sinusoidal disturbances. The salient features in both the cases have been compared and studied wherever possible.

MATHEMATICAL FORMULATION AND SOLUTION:

Part – a:

Consider a semi infinite vertical flat plate subjected to convective environment as shown in Figure 1. The fluid surrounding the plate is considered as gray, emitting and absorbing medium subjected to transverse applied magnetic field. The variation of the plate wall temperature is considered as proportional to x^{λ}. The velocity and radiation effect in the x-direction is considered negligible. Further, it is assumed that the convective fluid in the surrounding porous medium is isotropic in nature and has constant physical properties.

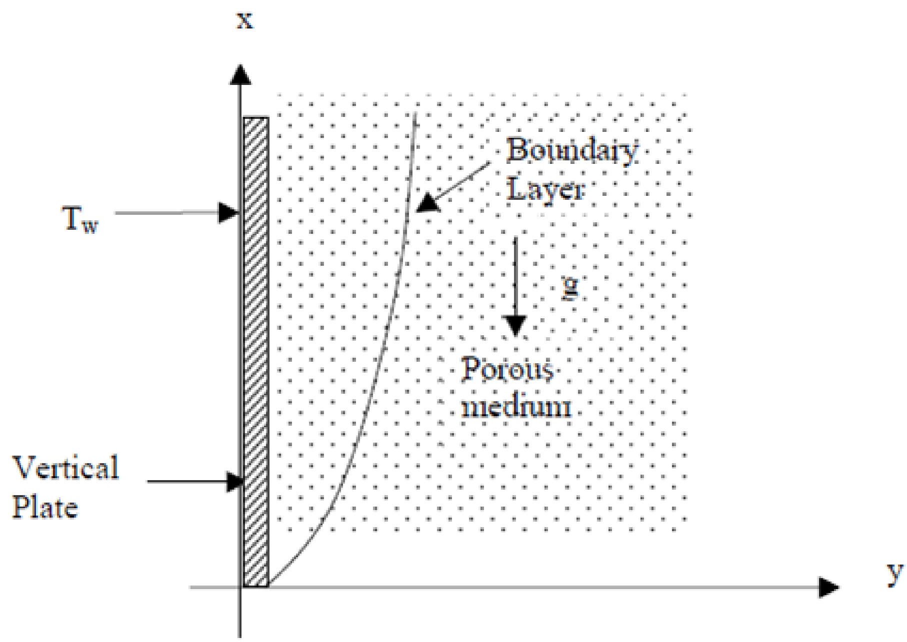

Figure 1: Physical model of the problem

Under boundary layer approximations

The continuity equation is:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5}$$

The momentum equation is:

$$-\rho_{\infty} g\beta\frac{\partial T}{\partial y}+\frac{\mu}{k}\frac{\partial u}{\partial y}+\sigma B_0^{\,2}\frac{\partial u}{\partial y}=0 \tag{6}$$

While the energy equation is:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^{\,2}u^2}{\rho c_p}-\frac{1}{\rho c_p}\frac{\partial q}{\partial y} \tag{7}$$

where
$$\frac{\partial q}{\partial y}=-16a\sigma_R T_{\infty}^{\,3}\left(T_{\infty}-T\right). \tag{8}$$

The momentum equation includes both the inertial forces and magnetic influence while the energy equation contains the terms of radiation heat transfer effect with Joule heating. The induced magnetic forces in the free stream, which effects the external pressure gradient, or the free stream velocity especially in mixed convection boundary layers are also taken care while framing the governing equations.

Introducing the stream functions

$$u=\frac{\partial\psi}{\partial y}\ and\ v=-\frac{\partial\psi}{\partial x} \tag{9}$$

The above equations (6) and (7) reduce to:

$$-\rho_\infty g\beta\frac{\partial T}{\partial y}+\frac{\mu}{k}\frac{\partial^2\psi}{\partial y^2}+\sigma B_0{}^2\frac{\partial^2\psi}{\partial y^2}=0 \tag{10}$$

$$\frac{\partial\psi}{\partial y}.\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}.\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\frac{\sigma B_0{}^2}{\rho c_p}\left(\frac{\partial\psi}{\partial y}\right)^2+\frac{1}{\rho c_p}16a\sigma_R T_\infty{}^3\left(T_\infty-T\right) \tag{3.11}$$

The boundary conditions are

$$\left.\begin{array}{l} at\ y=0, T=T_w=T_\infty+Ax^\lambda\ \ and\ \ \psi=0 \\ as\ y\to\infty, T=T_\infty,\ and\ \ u=U_\infty \end{array}\right\} \tag{12}$$

Introducing the similarity variables as

$$\psi=f\left(\eta\right)\left(\alpha_m U_\infty x\right)^{1/2},\quad \eta=\left(\frac{U_\infty x}{\alpha_m}\right)^{1/2}\left(\frac{y}{x}\right),\quad U_\infty=Bx^\lambda, \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$

and $T_w=T_\infty+Ax^\lambda$

Equations (10) and (11) in the non dimensional form will be

$$f''(\eta)=\varepsilon C\theta'(\eta) \tag{13}$$

$$\lambda f'\theta-\left(\frac{1+\lambda}{2}\right)f\theta'=\theta''+\Pr Ec(f'')^2+\frac{M^2 Ec}{\mathrm{Re}_x}(f')^2-\mathrm{Re}_x F\theta \tag{14}$$

where
$$Ra_x=\frac{Ax^{\lambda+1}}{\upsilon\alpha}g\beta k,\ C=\frac{k^2}{k^2+M^2}, \mathrm{Re}_x=\frac{U_\infty x}{\upsilon}, \varepsilon=\frac{Ra_x}{Pe_x}, Pe_x=\frac{U_\infty x}{\alpha_m}$$

$$M^2=\frac{\sigma B_0{}^2x^2}{\rho\upsilon},\quad \Pr=\frac{\upsilon}{\alpha},\quad Ec=\frac{U_\infty^2}{c_p(T_w-T_\infty)},\ F=\frac{16a\upsilon\sigma_R T_\infty^3}{\rho c_p U_\infty^2}$$

The boundary conditions (12) in the non dimensional form are now redefined as

$$\left.\begin{array}{l} at\,\eta=0,\ \theta=1\ and\ \ f(0)=0 \\ as\,\eta\to\infty,\ \theta=0\ and\ \ f^1\left(\infty\right)=1 \end{array}\right\} \tag{15}$$

Applying boundary conditions in equation (13), we get

$$f'=\varepsilon C\theta \tag{16}$$

Substituting (16) in (14) we get

$$f'''+\varepsilon C\Pr Ec(f'')^2+\left[\varepsilon C\frac{M^2}{\mathrm{Re}}Ec-\lambda\right]f'^2+\left(\frac{1+\lambda}{2}\right)ff''+f'(\lambda-\mathrm{Re}_x F)+\mathrm{Re}_x F=0 \tag{17}$$

The above equation (17) is split into system of first order ordinary differential equations and then solved by using Runge-Kutta method using the shooting technique.

Part – b:

In this part the aim of the problem is to investigate a class of exact solutions for the flow of incompressible second order fluid by taking into account the porosity factor of the bounding surfaces when it is subjected to sinusoidal disturbances and then to compare the results with those of in Newtonian case and also to find the analogy of the thermal effects as examined earlier. The effects of the disturbance due to sinusoidal oscillation of the bottom of a semi infinite depth are examined. The results are expressed in terms of a non-dimensional porosity parameter K, which depends on the non-Newtonian coefficient ϕ_2 and the frequency of excitation σ. It is noticed, that the flow properties are identical with those of in the Newtonian case $\left(K=\infty\right)$.

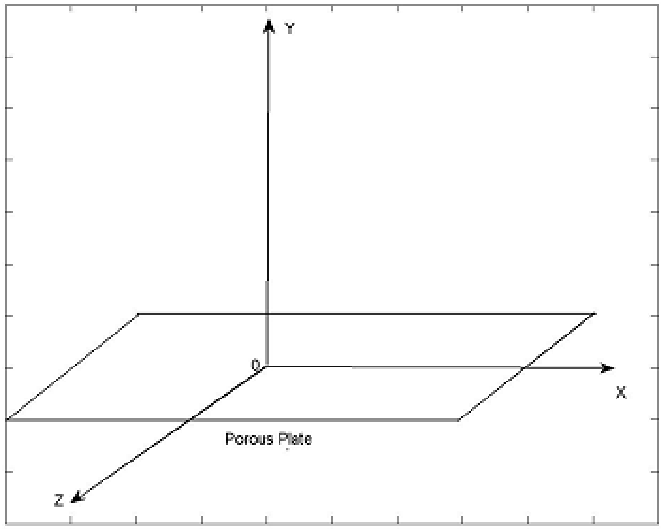

Figure 2: Geometry of the fluid over porous bed

In general, the equations (in the dimensional form) of motions in the X, Y and Z directions are

$$\rho\frac{DU_1}{DT}=\rho F_X+\frac{\partial S_{XX}}{\partial X}+\frac{\partial S_{XY}}{\partial Y}+\frac{\partial S_{XZ}}{\partial Z}-\frac{\mu}{k}U_1 \tag{18}$$

$$\rho\frac{DU_2}{DT}=\rho F_Y+\frac{\partial S_{YX}}{\partial X}+\frac{\partial S_{YY}}{\partial Y}+\frac{\partial S_{YZ}}{\partial Z}-\frac{\mu}{k}U_2 \tag{19}$$

$$\rho\frac{DU_3}{DT}=\rho F_Z+\frac{\partial S_{ZX}}{\partial X}+\frac{\partial S_{ZY}}{\partial Y}+\frac{\partial S_{ZZ}}{\partial Z}-\frac{\mu}{k}U_3 \tag{20}$$

Introducing the following non-dimensional variables as:

$$U_i=\frac{\phi_1 u_i}{\rho L}\qquad T=\frac{\rho L^2 t}{\phi_1}\qquad \phi_2=\rho L^2\beta\qquad P=\frac{\phi_1^2 p}{\rho L^2}$$

$$\frac{X_i}{L}=x_i\qquad \frac{Y_i}{L}=y_i\qquad \phi_3=\rho L^2 \upsilon_c\qquad A_i=\frac{\phi_1^2 a_i}{\rho^2 L^3}$$

$$S_{ij}=\frac{\phi_1^2 s_{ij}}{\rho L^2}\qquad E_{ij}^{(1)}=\frac{\phi_1 e_{ij}^{(1)}}{\rho L^2}\qquad E_{ij}^{(2)}=\frac{\phi_1^2 e_{ij}^{(2)}}{\rho^2 L^4}\qquad k=\frac{\rho L^3}{\phi_1^2 K}$$

Where T is the (dimensional) time variable, ρ is the mass density and L is a characteristic length. We consider a class of plane flows given by the velocity components

$$u_1=u(y,t)\ \text{and}\ \ u_2=0\ \text{while}\ \ u_3=0 \tag{21}$$

The flow characterized by the velocity in the non-dimensional form is given by:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+\beta\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{1}{K}u \tag{22}$$

where K is the non-dimensional porosity constant. It may be noted that the presence of β changes the order of differential from two to three.

The oscillations of a classical viscous liquid on the upper half of the plane $y\geq 0$ with the bottom oscillating with a velocity $\alpha e^{i\sigma t}$ then

$$u(0,t) = \alpha e^{i\sigma t} \tag{23}$$

$$u(\infty,t) = 0 \tag{24}$$

Assuming the trial solution as

$$u(y,t) = \alpha e^{i\sigma t} f(y) \tag{25}$$

$$f''(y) = p^2 f(y) \tag{26}$$

where $$p^2 = \frac{\frac{1}{K} + i\sigma}{1 + i\beta\sigma} = \frac{\left(\beta\sigma^2 + \frac{1}{K}\right) + i\left(\sigma - \frac{\beta\sigma}{K}\right)}{\left(1+\beta^2\sigma^2\right)} \tag{27}$$

When expressed in the polar form

$$p = r\left(\cos\left(\frac{\pi}{4} - \frac{\delta}{2}\right) + i\sin\left(\frac{\pi}{4} - \frac{\delta}{2}\right)\right) \tag{28}$$

where $$r = \frac{\left[\left(\beta\sigma^2 + \frac{1}{K}\right)^2 + \left(\sigma - \frac{\beta\sigma}{K}\right)^2\right]^{\frac{1}{4}}}{\sqrt{\left(1+\beta^2\sigma^2\right)}}, \quad \delta = \tan^{-1}(Q) \; and \; Q = \frac{\left(\beta\sigma^2 + \frac{1}{K}\right)}{\left(\sigma - \frac{\beta\sigma}{K}\right)}$$

Also the conditions satisfied are:

$$f(0) = 1, \qquad f(\infty) = 0 \tag{29}$$

This yields the solution

$$f(y) = e^{-yr\left(\cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right) + i\sin\left(\frac{\pi}{4}-\frac{\delta}{2}\right)\right)} \tag{30}$$

and hence

$$u(y,t) = \alpha e^{\left(i\sigma t - yr\left(\cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right) + i\sin\left(\frac{\pi}{4}-\frac{\delta}{2}\right)\right)\right)} \tag{31}$$

The flow is thus represented by standing transverse wave with its amplitude rapidly diminishing with increasing distance from the plane. This phenomenon is independent of v_c as is noticed for all two-dimensional flows.

The magnification factor A^* of the amplitude this wave, with respect to the amplitude of the disturbance (α), may be written as

$$A^* = \left(real\ part\ of\ u(y,t)\right)^2 + \left(\mathrm{Im}aginary\ part\ of\ u(y,t)\right)^2$$

$$A^* = \alpha e^{-y\sqrt{r}\cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right)} \tag{32}$$

which is in the form of $A^* = e^{-\chi y^*}$ where

$$\chi y^* = \frac{y\sqrt{r}}{\sqrt{2}}\left[\cos\left(\frac{\delta}{2}\right) + i\sin\left(\frac{\delta}{2}\right)\right] \tag{33}$$

where $$\chi = \frac{1}{\left(1+\beta^2\sigma^2\right)^{\frac{1}{4}}}\sqrt{\frac{Q + \sqrt{\left(1+Q^2\right)}}{1+Q^2}} \tag{34}$$

and $$y^* = \frac{y\left(1+\beta^2\sigma^2\right)^{\frac{1}{4}}}{\sqrt{2}} \frac{\left[\left(\beta\sigma^2 + \frac{1}{K}\right)^2 + \left(\sigma - \frac{\beta\sigma}{K}\right)^2\right]^{\frac{1}{4}}}{\sqrt{\left(1+\beta^2\sigma^2\right)}} \tag{35}$$

RESULTS AND DISCUSSIONS:

1. The effect of temperature with respect to magnetic and radiation effects within the boundary layer have been discussed in Figure 3. It is observed that, for constant radiation parameter, the increase in the magnetic field the temperature increases. However, with increase in the radiation parameter, the temperature decreases within boundary at a given location which indicates that, the rate of heat transfer will be higher under high radiative conditions. Also the profiles show the fact that, the variation in temperature with magnetic field is significant only when there is no radiation effect. This indicates that, at high radiative conditions the effect of magnetic field on temperature distribution with in the boundary region is negligible.

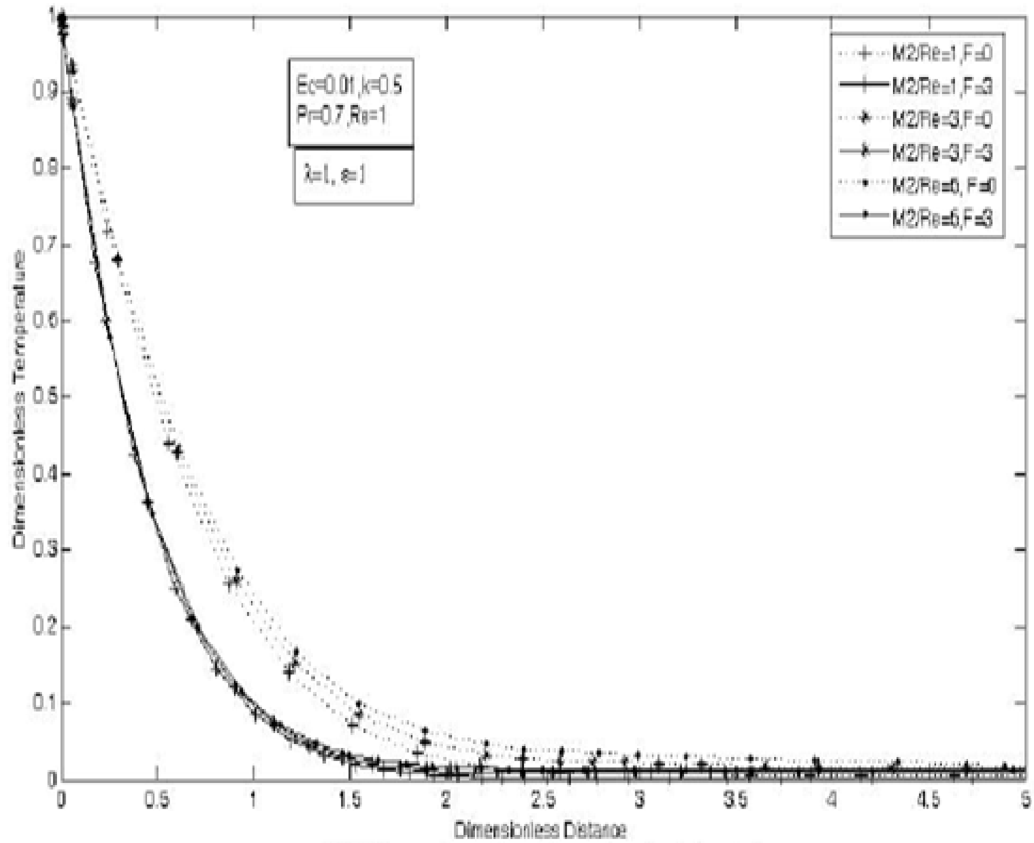

Figure 3: Temperature profiles with magnetic and radiation effect

In comparison with the earlier result, the variations of velocity profiles for different values of porosity factor are illustrated in the Figure 4 shown below. It is noticed that, the stream velocity decreases as we move away from the plate. As the porosity increases the stream velocity increases. This can be attributed due to the term $\left(\frac{1}{K}\right)$ in the governing equation of motion.

From the above observations it can be concluded that, there exists a close analogy in the problem of mixed convective flow over a non-isothermal radiative vertical surface in a porous medium under magnetic field and in the problem of visco elastic fluid of second order bounded by a porous surface and is subjected to sinusoidal disturbances that are created on the porous bed. The porosity factor in case the second order fluid is almost similar to that of the magnetic field and visco elasticity parameter is identical to that of radiation parameter. However, both the investigations stated above are applicable in the boundary layer region.

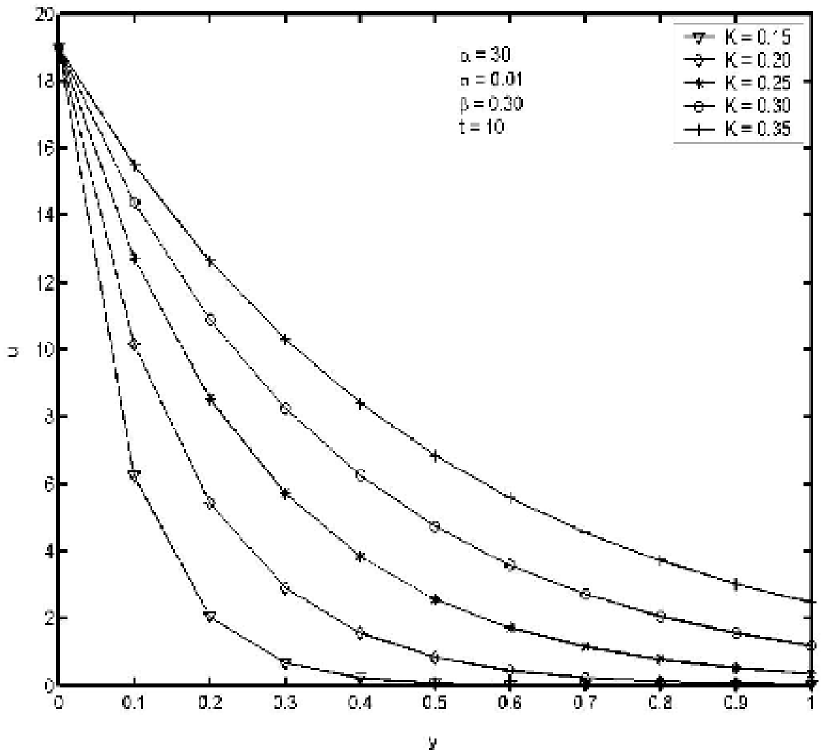

Figure 4: Effect of porosity (K) on the velocity profiles

2. The effect of λ on the temperature distribution along the plate has been investigated in Figure 5. It is observed that, as λ increases, the temperature decreases. This can be attributed to the fact that, high variation in the wall temp, the boundary layer thickness obviously decreases and the free stream temperature is attained within the short distance from the wall. This further enhances the rate heat transfer in the porous medium.

A further interesting conclusion can also be drawn from Figure 4 shown earlier that, as the porosity of the medium increases the velocity also increases. In other words, there seems to be a close analogy between the porosity of the medium when examined on the flow by creating the sinusoidal disturbances is just similar to the parameter λ in case of mixed convective flow.

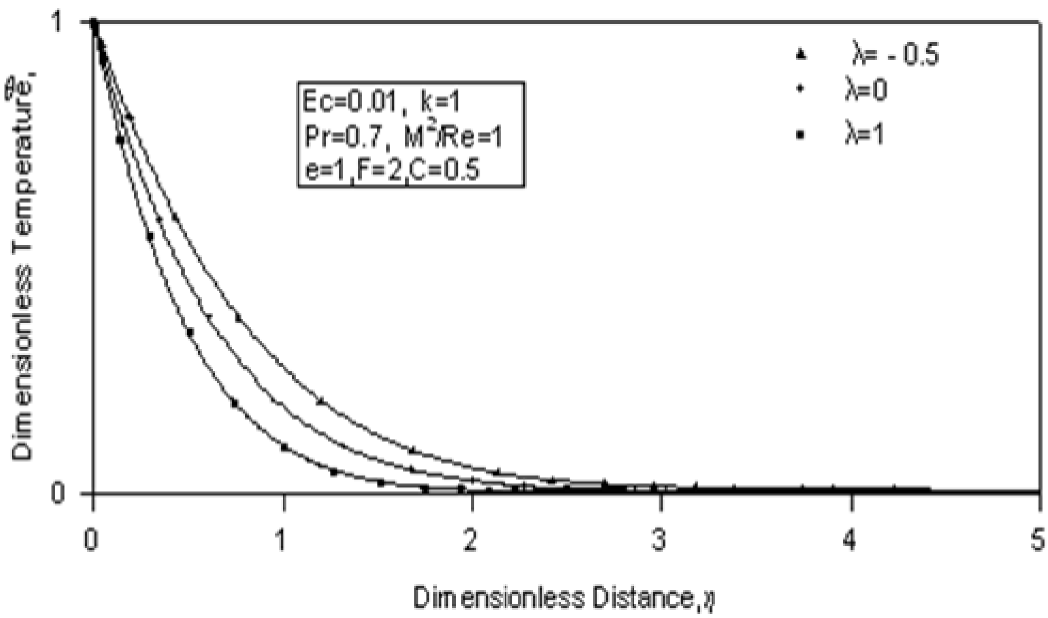

Figure 5: Effect of λ on temperature distribution

3. Figure 6 brings out the effect of mixed convection for both buoyancy aiding and opposing flows on temperature distribution. Increasing ε increases the temperature for the buoyancy aiding flow and increasing ε decreases the temperature for the buoyancy opposing flow. Further, it indicates that, the effect of mixed convective parameter on temperature distribution is considerable only at the middle of the boundary and is negligible at the plate end and at the edge of the boundary.

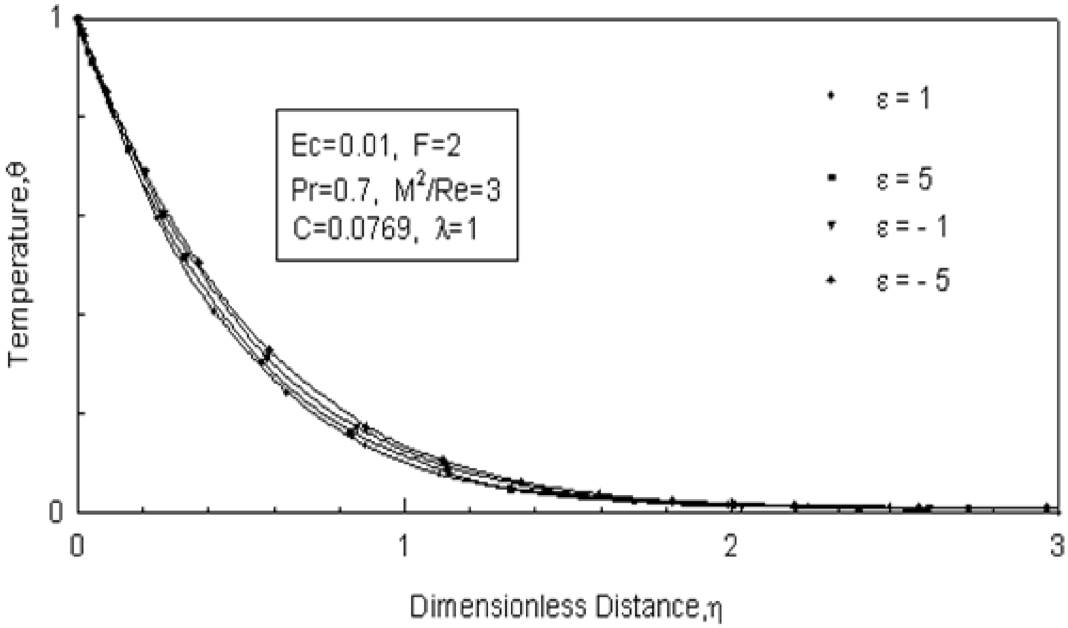

Figure 6: Effect of mixed convection temperature distribution

4. Figure 7 shows the effect of radiation and mixed convection on temperature distribution. With increase in radiation parameter, temperature within the boundary decreases. Further, with increase in mixed convection parameter also the temperature decreases. It is also observed from Figure 7 that, the aiding flow develops higher temperature than opposing flow with in the boundary. However, for a fixed radiation parameter, the trends are found to be reverse for opposing and aided flow. Also the radiation effect dominates the mixed convective parameter as visible in the lower set of curves.

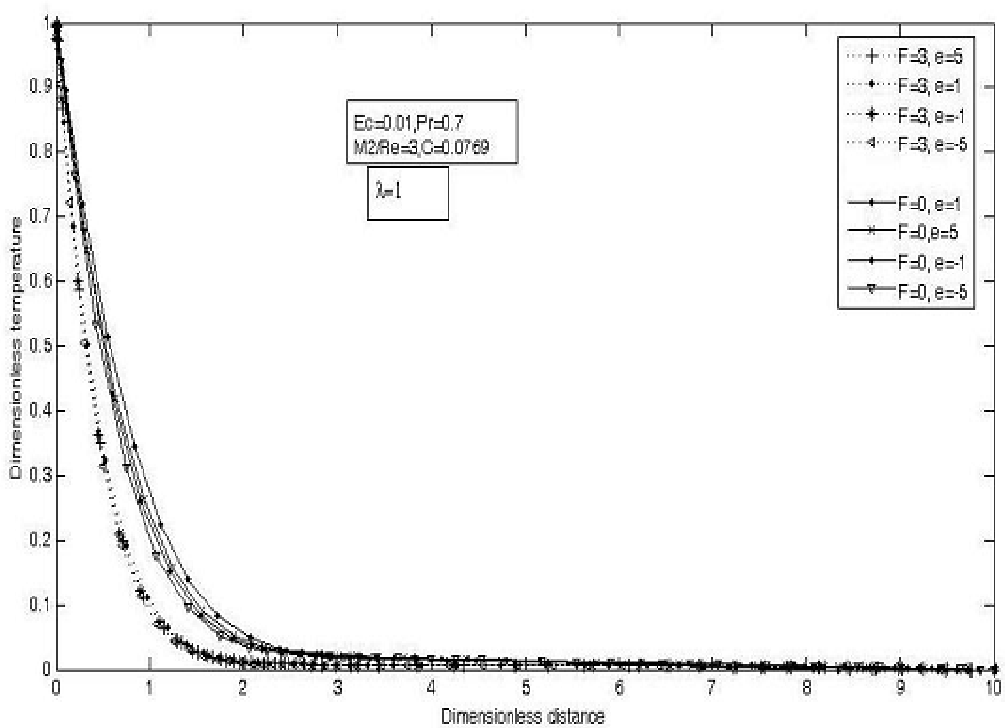

Figure 7: Effect of radiation and mixed convection temperature distribution

5. Figure 8 shows the effect of mixed convection on velocity profile. It is found that, with increase in mixed convective parameter the boundary layer thickness increases both in aiding and opposing flows. The rate of change of velocity with in the boundary layer is high relatively at high convective parameter compared to lower values.

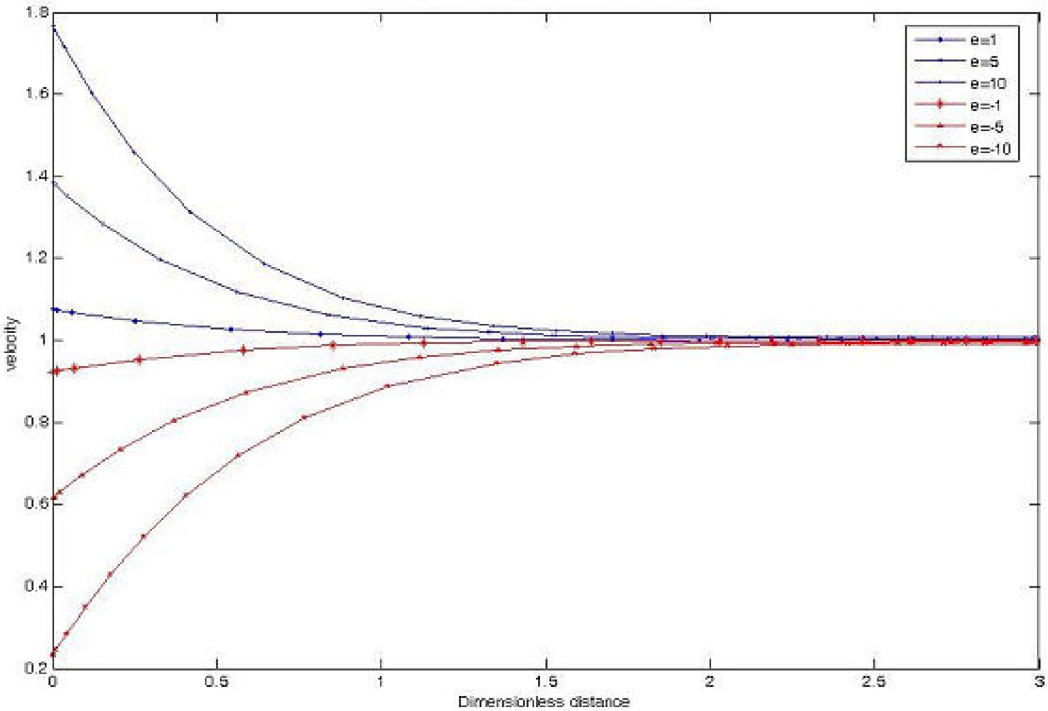

Figure 8: Effect of mixed convection on velocity

In contrast to the situation examined earlier, the effect frequency of excitation (σ) of the fluid bed has been investigated in Figure 9. It is noticed that, the increase in the frequency of excitation, decreases the velocity in the boundary layer region. Further, it is also noticed that, at times there is a back flow in the neighborhood of the plate which subsequently settles down as we move away from the plate. This is in agreement with the real life situation. In view of the above observations, it is noticed that, for the constant visco elasticity and porosity parameter, the frequency excitation parameter behaves exactly similar to that of mixed convection parameter.

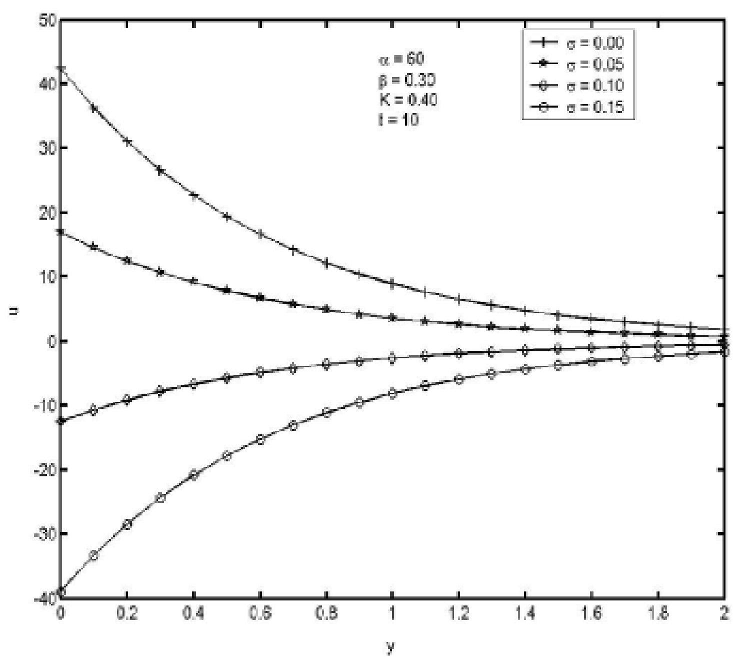

Figure 9: Effect of frequency excitation (σ) on the velocity profiles

6. The effect of mixed convective parameter on velocity profile is illustrated in Figure 10. With increase in radiation it is observed that, the boundary layer thickness decreases. Similarly with increase in mixed convective parameter, it is observed that, the rate of change in velocity with in the boundary layer rapidly increases. Under the aiding and opposed convective flow conditions the velocity profile formation is brought out clearly and represented.

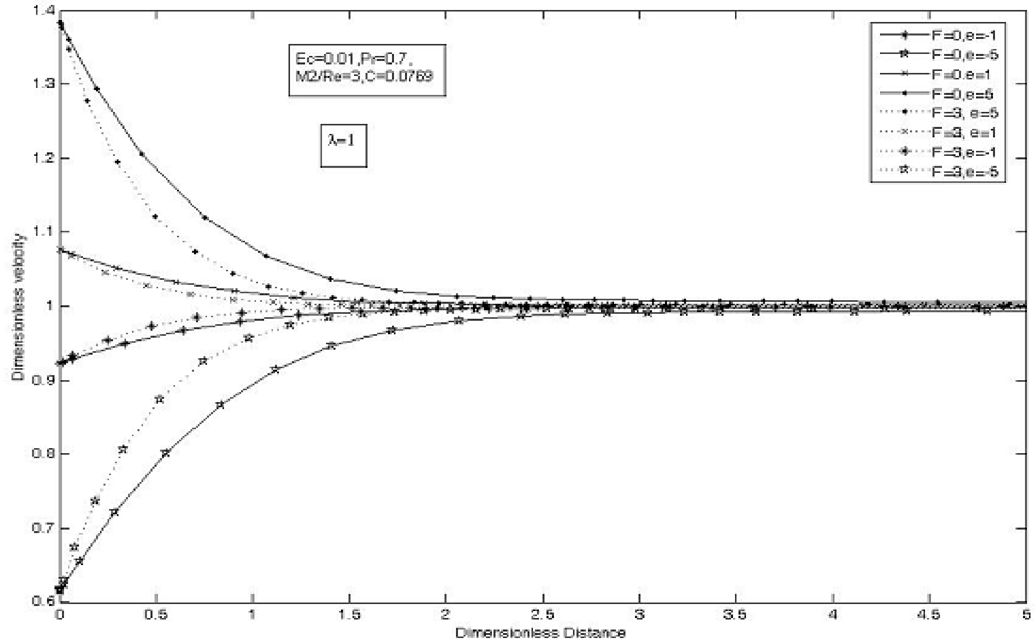

Figure 10: Effect of mixed convective parameter on velocity profile

The above conclusion is in agreement with respect to the conclusion 5 as stated above. The effect of time on the velocity profile over the flat plate in case of second order fluid has been examined by considering the variation in time parameter t . It has been noticed that, as t increases, the velocity profile decreases and also a back flow has been noticed. This can attributed to the fact that, the fluid particles close to the bounding surface though they in the forward direction, due to sinusoidal disturbances of the plate, just before the particle rises up it will be pull down due to the frequency of excitation (partly) and secondly due to the intra molecular viscous forces in the fluid. Also the porosity of the medium plays a significant role in such problems.

However, in either case the frequency of excitation (σ) and (t) has found to have the same effect as that of mixed convection parameter.

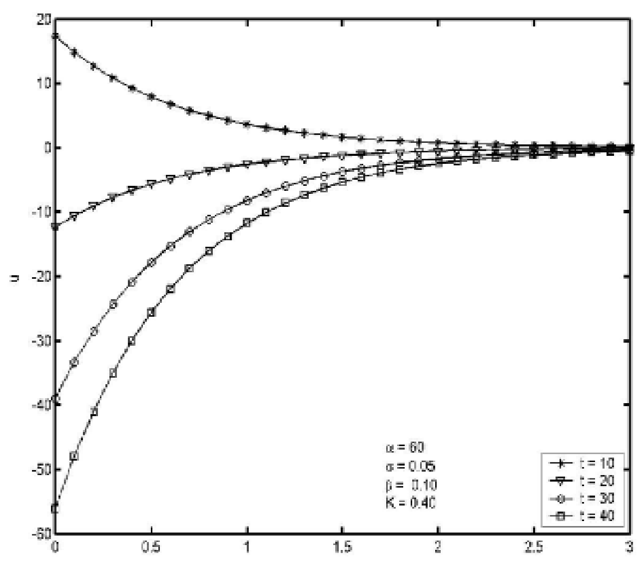

Figure 11: Effect of time (t) on the velocity profiles

7. The effect of time (t) has been studied in Figure 12 the effect of visco elasticity parameter (β) was found to have significant effect on the magnification parameter (A^*).The magnification factor increases as (β) increases.

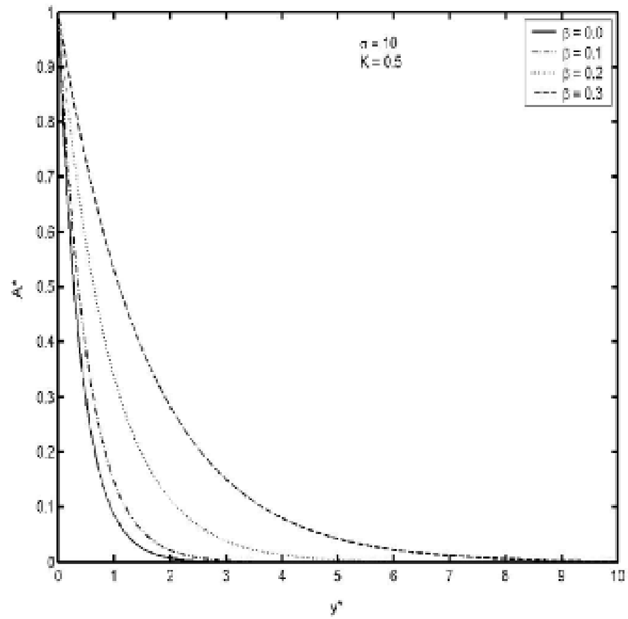

Figure 12: Effect of elastico velocity (β) on the magnification factor.

CONCLUSIONS:

1. Similarities between mixed convective flow over a non-isothermal radiative surface in a porous medium under magnetic field and the visco elastic fluid flow of second order bounded by a porous surface subjected to sinusoidal disturbances are established.
2. The effect of porosity factor in a second order fluid is found similar to that of applied magnetic field.
3. The visco elastic parameter is found similar to that of radiation parameter.
4. The effect of sinusoidal disturbances in a porous medium is found similar to the parameter λ in the mixed convective flow.
5. The behavior of constant visco elasticity parameter and the frequency excitation parameter are found similar to that of mixed convective parameter in a porous medium.
6. The magnification factor is found increasing with increase in visco elasticity parameter.

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