

**RADIATIVE FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE
WITH VARIABLE TEMPERATURE AND MASS DIFFUSION
IN THE PRESENCE OF MAGNETIC FIELD**

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ABSTRACT

Thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion, in the presence of magnetic field has been considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised linearly with time and the concentration level near the plate is also raised linearly with time. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is exponentially accelerated with a velocity $u = u_0 \exp(at')$ in its own plane against gravitational field. The effects of velocity, temperature and concentration are studied for different physical parameters like magnetic field parameter, thermal radiation parameter, thermal Grashof number, mass Grashof number and Schmidt number. It is observed that the velocity increases with decreasing magnetic field parameter or radiation parameter. But the trend is just reversed with respect to a or t .

Keywords: radiation, exponentially, accelerated, vertical plate, magnetic field.

1. INTRODUCTION

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. England and Emery[4] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Raptis and Perdiki [6] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Radiation effect on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar[5]. The governing equations were solved analytically. Das et al.[3] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

Magnetoconvection plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al.[8]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al. [9]. The dimensionless governing equations were solved using Laplace transform technique.

Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [7]. Basant Kumar Jha et al [1] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Basanth Kumar Jha and Ravindra Prasad [2] studied pure heat transfer and hydromagnetic convection effects on flow past an exponentially accelerated vertical plate.

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However the combined study of hydromagnetic effects on infinite vertical plate with variable temperature and mass diffusion in the presence of thermal radiation is not available in the literature. It is proposed to study the thermal radiation effects on unsteady flow past an exponentially accelerated vertical plate, in the presence of transverse magnetic field. The dimensionless governing equations are tackled using the Laplace transform technique and the resultant solutions are in terms of exponential and complementary error function.

2. BASIC EQUATIONS AND ANALYSIS

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞ . Here, the x -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(at')$ in its own plane and the temperature of the plate is raised linearly with respect to time and the concentration level near the plate is also raised linearly with time. The plate is subjected to a uniform magnetic field of strength B_0 and the fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following:

$$\rho \frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$D \frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \quad (3)$$

With the following initial and boundary conditions:

$$\begin{aligned} t' \leq 0: & \quad u = 0, & \quad T = T_\infty, & \quad C' = C'_\infty \text{ for all } y \\ t' > 0: & \quad u = u_0 \exp(at'), & \quad T = T_\infty + (T_w - T_\infty) At', & \quad C' = C'_\infty + (C'_w - C'_\infty) At' \text{ at } y = 0 \\ & \quad u = 0, & \quad T \rightarrow T_\infty, & \quad C' \rightarrow C'_\infty \text{ as } y \rightarrow \infty \end{aligned} \quad (4)$$

Where, $A = \frac{u_0^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is defined as

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma (T_\infty^4 - T^4) \quad (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 (T_\infty - T) \quad (7)$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, t = \frac{t' u_0^2}{\nu}, Y = \frac{y u_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$Gr = \frac{g\beta v(T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = \frac{vg\beta^*(C'_w - C'_\infty)}{u_0^3}, a = \frac{a'v}{u_0^2}, \quad (8)$$

$$R = \frac{16a^*v^2\sigma T_\infty^3}{ku_0^2}, Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2 v}{\rho u_0^2}.$$

in equations (1), (3) and (7), reduces to

$$\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} - M U \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr} \theta \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \quad (11)$$

The initial and boundary conditions in non-dimensional form are

$$\begin{aligned} U = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all } Y, t \leq 0 \\ t > 0: U = \exp(at), \quad \theta = t, \quad C = t, \quad \text{at } Y = 0 \\ U = 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } Y \rightarrow \infty \end{aligned} \quad (12)$$

3. SOLUTION PROCEDURE

The solutions are obtained for hydrodynamic flow field in the presence of thermal radiation. The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned} \theta = \frac{t}{2} \left[\exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{at}) \right] \\ - \frac{\eta Pr \sqrt{t}}{2\sqrt{R}} \left[\exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{at}) - \exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{at}) \right] \end{aligned} \quad (13)$$

$$C = t \left[(1 + 2\eta^2 Sc) \operatorname{erfc}(\eta\sqrt{Sc}) - 2\eta \frac{\sqrt{Sc}}{\sqrt{\pi}} \exp(-\eta^2 Sc) \right] \quad (14)$$

$$\begin{aligned} U = \frac{\exp(at)}{2} \left[\exp(2\eta\sqrt{(M+a)t}) \operatorname{erfc}(\eta + \sqrt{(M+a)t}) + \exp(-2\eta\sqrt{(M+a)t}) \operatorname{erfc}(\eta - \sqrt{(M+a)t}) \right] \\ + (e(1+ct) + f(1+dt)) \left[\exp(2\eta\sqrt{Mt}) \operatorname{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta\sqrt{Mt}) \operatorname{erfc}(\eta - \sqrt{Mt}) \right] \\ - \frac{f(c+d)\eta\sqrt{t}}{\sqrt{M}} \left[\exp(-2\eta\sqrt{Mt}) \operatorname{erfc}(\eta - \sqrt{Mt}) - \exp(2\eta\sqrt{Mt}) \operatorname{erfc}(\eta + \sqrt{Mt}) \right] \\ - e \exp(ct) \left[\exp(-2\eta\sqrt{(M+c)t}) \operatorname{erfc}(\eta - \sqrt{(M+c)t}) \right] \\ + \exp(2\eta\sqrt{(M+c)t}) \operatorname{erfc}(\eta + \sqrt{(M+c)t}) \\ - f \exp(dt) \left[\exp(-2\eta\sqrt{(M+d)t}) \operatorname{erfc}(\eta - \sqrt{(M+d)t}) \right] \\ + \exp(2\eta\sqrt{(M+d)t}) \operatorname{erfc}(\eta + \sqrt{(M+d)t}) \\ - e(1+bt) \left[\exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{bt}) + \exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{bt}) \right] \end{aligned}$$

$$\begin{aligned}
 & -\frac{e(1+bt)\eta \text{Pr} \sqrt{t}}{\sqrt{R}} \left[\exp(-2\eta\sqrt{Rt}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} - \sqrt{bt}) - \exp(2\eta\sqrt{Rt}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} + \sqrt{bt}) \right] \\
 & + e \cdot \exp(bt) \left[\exp(-2\eta\sqrt{\text{Pr}(b+c)t}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} - \sqrt{(b+c)t}) \right. \\
 & \qquad \qquad \qquad \left. + \exp(2\eta\sqrt{\text{Pr}(b+c)t}) \cdot \text{erfc}(\eta\sqrt{\text{Pr}} + \sqrt{(b+c)t}) \right] \\
 & + f \cdot \exp(dt) \left[\exp(-2\eta\sqrt{Sc \cdot dt}) \cdot \text{erfc}(\eta\sqrt{Sc} - \sqrt{dt}) \right. \\
 & \left. + \exp(2\eta\sqrt{Sc \cdot dt}) \cdot \text{erfc}(\eta\sqrt{Sc} + \sqrt{dt}) \right] \\
 & - 2fdt \left[(1 + 2\eta^2 \cdot Sc) \cdot \text{erfc}(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}} \cdot \exp(-\eta^2 Sc) \right] - 2f \cdot \text{erfc}(\eta\sqrt{Sc}) \tag{15}
 \end{aligned}$$

Where, $b = \frac{R}{Pr}, c = \frac{M-R}{Pr-1}, d = \frac{M}{Sc-1}, e = \frac{Gr}{2c^2(1-Pr)}, f = \frac{Gc}{2d^2(1-Sc)},$

$\eta = Y/2\sqrt{t}$ and erfc is called complementary error function.

4. RESULTS AND DISCUSSION

The numerical values of the velocity and skin-friction are computed for different parameters like Magnetic field parameter, radiation parameter, Schmidt number, thermal Grashof number and mass Grashof number. The purpose of the calculations given here is to assess the effects of the parameters M, R, Gr, Gc and Sc upon the nature of the flow and transport. The value of Prandtl number Pr is chosen such that they represent air ($Pr = 0.71$). The solutions are in terms of exponential and complementary error function.

Figure 1 represents the effect of concentration profiles for different values of time ($t = 0.2, 0.4, 0.6, 0.8, 1$) and $Sc = 2.01$. The trend shows that the wall concentration increases with increasing values of time t . The effect of concentration profiles at time $t = 1$ for different Schmidt number ($Sc = 0.16, 0.3, 0.6, 2.01$) are presented in figure 2. The effect of Schmidt number plays an important role in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

The temperature profiles are calculated for different values of the time ($t = 0.2, 0.4, 0.6, 1$) at time $R = 0.2$ and these are shown in figure 3. The trend shows that the temperature increases with increasing values of time t . The temperature profiles are calculated for different values of thermal radiation parameter ($R = 0.2, 2, 5, 10$) at time $t = 1$ and these are shown in figure 4. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

The effect of velocity for different values of ($a = 0, 0.5, 1$), $R = 2, Gr = Gc = 5$ at $t = 0.2$ are studied and presented in figure 5. It is observed that the velocity increases with increasing values of a . When $a=0$, then the value of $U=1$. This case is quite agree with moving vertical plate.

Figure 4. demonstrates the velocity profiles for different thermal radiation parameter ($R = 2, 5, 15$), $Gr = 5, Gc = 10, a = 1, M = 0.2$ and $t = 0.6$. It is observed that the velocity increases with decreasing thermal radiation parameter. This shows that there is dip in velocity in the presence of radiation. The effect of velocity for different values of ($a = 0, 0.5, 1$), $R = 2, Gr = Gc = 5$ at $t = 0.2$ are studied and presented in figure 5. It is observed that the velocity increases with increasing values of a .

Figure 6. illustrates the velocity profiled for different values of the magnetic field parameter ($M=0.2, 5, 10$), $Gr=5, Gc=10, a=1, R=15$ and $t=0.2$. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that there is fall in velocity in the presence of higher magnetic intensity. This agrees with the expectations, since the magnetic field exerts retarding force on the free convective flow. Velocity profiles for different values of time ($t=0.2, 0.4, 0.6$), $a=1, Gr=5, Gc=10, m=0.2$ and $R=5$ are presented in figure 6. It is observed that the velocity increases with increasing values of time t . The effect of velocity for different values of thermal Grashof number

(Gr=5,10), mass Grashof number(Gc=5,10), R=2, a=1, M=0.2 and t=0.6 are presented in figure 7. It is observed that the velocity increases with increasing thermal Grashof number or mass Grashof number.

5. CONCLUDING REMARKS

Theoretical solution of hydromagnetic and thermal radiation effects on unsteady flow past an exponentially accelerated infinite isothermal vertical plate with variable mass diffusion has been studied in detail. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different parameters like M , R , Gr , Gc , Sc and t are studied. The study concludes that the velocity increases with decreasing magnetic field parameter or radiation parameter. It is also observed that the velocity increases with increasing thermal Grashof number or mass Grashof number.

NOMENCLATURE

A	constant	t'	time s
C'	species concentration in the fluid	t	dimensionless time
C	dimensionless concentration	u	velocity of the fluid in the x-direction
C _p	specific heat at constant pressure	u ₀	velocity of the plate
C' _w	concentration of the plate	U	dimensionless velocity
C' _∞	concentration of the fluid far away from the plate	x	spatial coordinate along the plate
D	mass diffusion coefficient	y'	coordinate axis normal to the plate m
Gr	mass Grashof number	y	dimensionless coordinate axis normal to the plate
Gc	thermal Grashof number	β	volumetric coefficient of thermal expansion
g	accelerated due to gravity	β*	volumetric coefficient of expansion with concentration
k	thermal conductivity	μ	coefficient of viscosity
R	Radiation parameter	ν	kinematic viscosity
Pr	Prandtl number	ρ	density of the fluid
Sc	Schmidt number	θ	dimensionless temperature
T	temperature of the fluid near the plate	η	similarity parameter
T _w	concentration of the plate	erfc	complementary error function
T _∞	concentration of the fluid far away from the plate		

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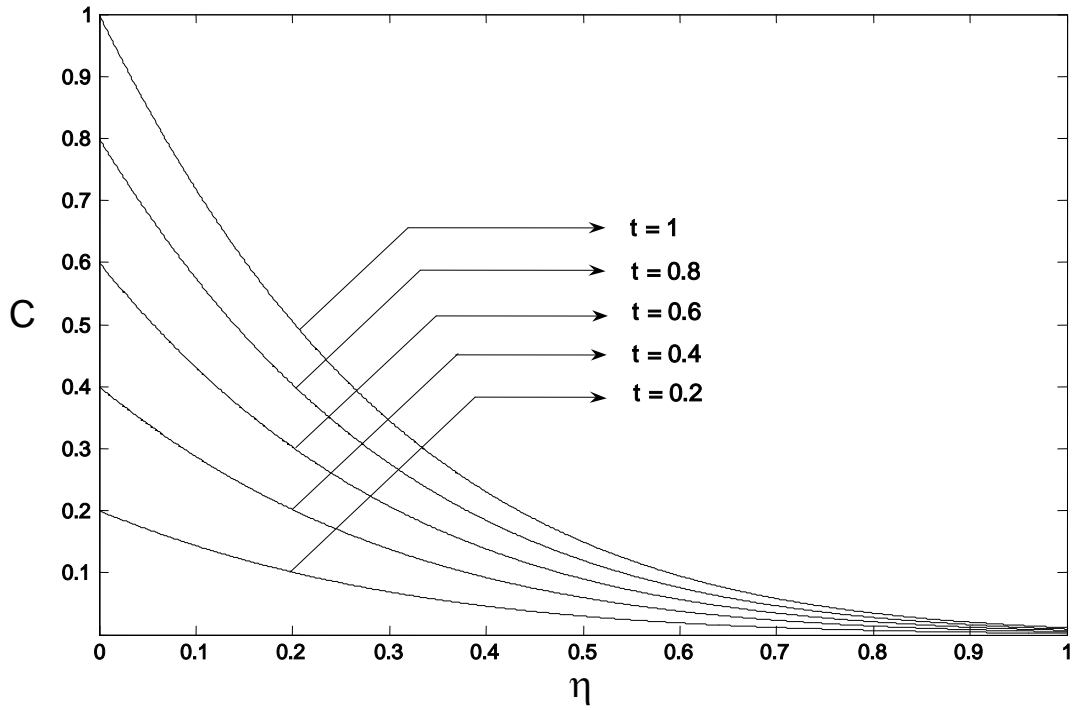


Figure 1. Concentration profiles for different values of t

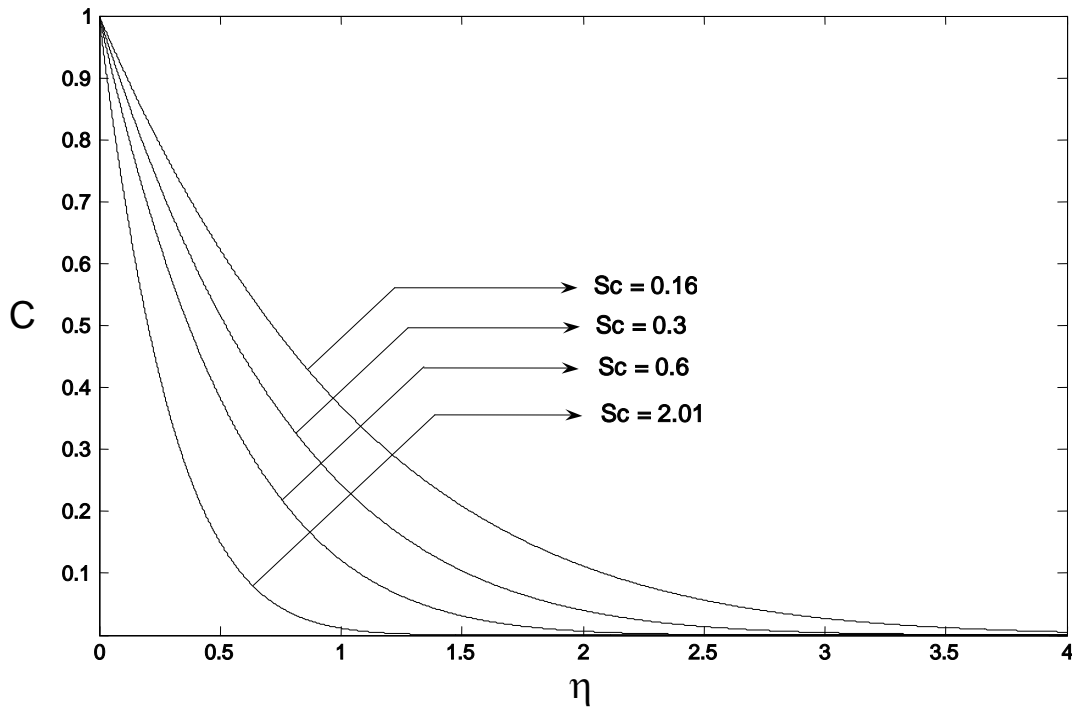


Figure 2. Concentration profiles for different values of Sc

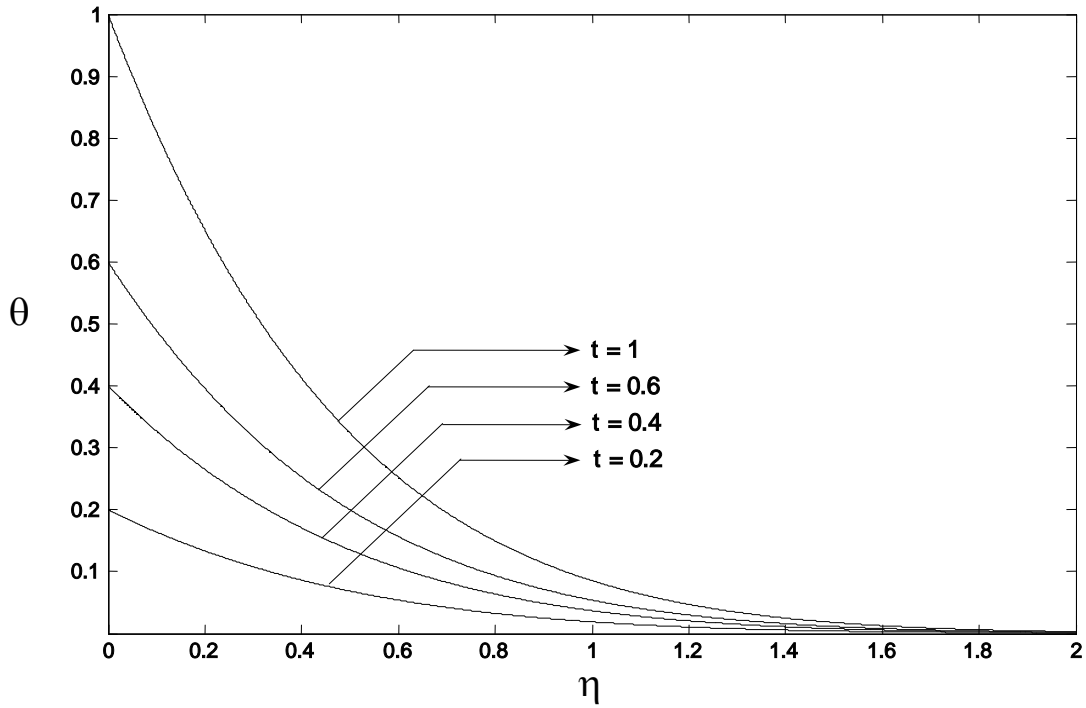


Figure 3. Temperature profiles for different values of t

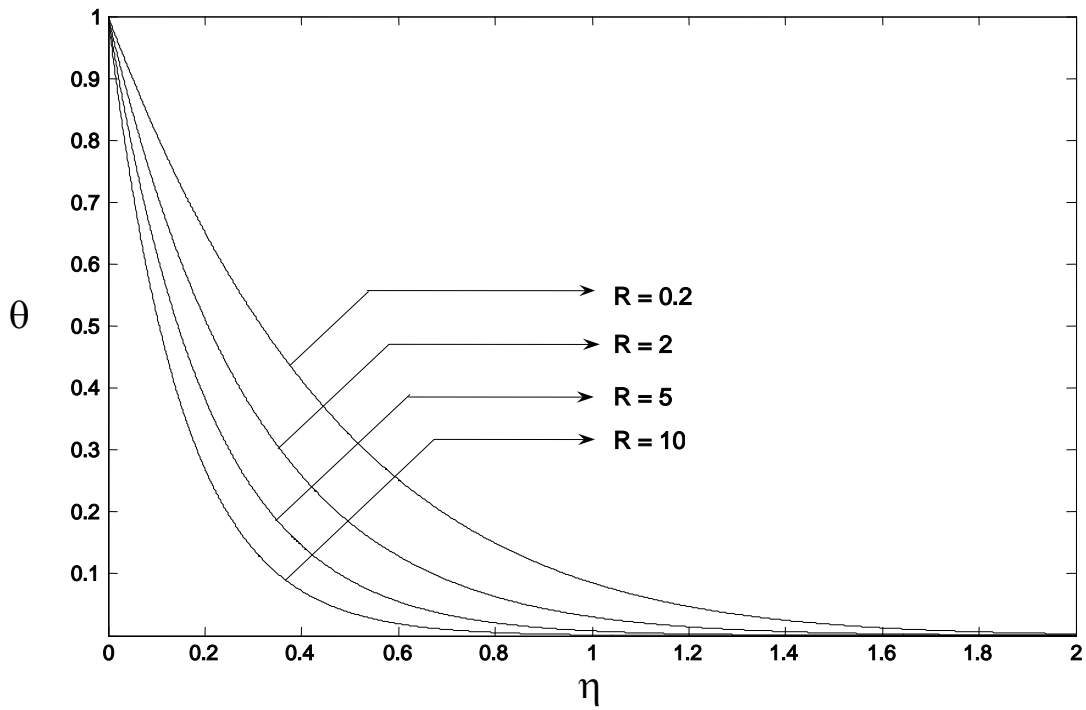


Figure 4. Temperature profiles for different values of R

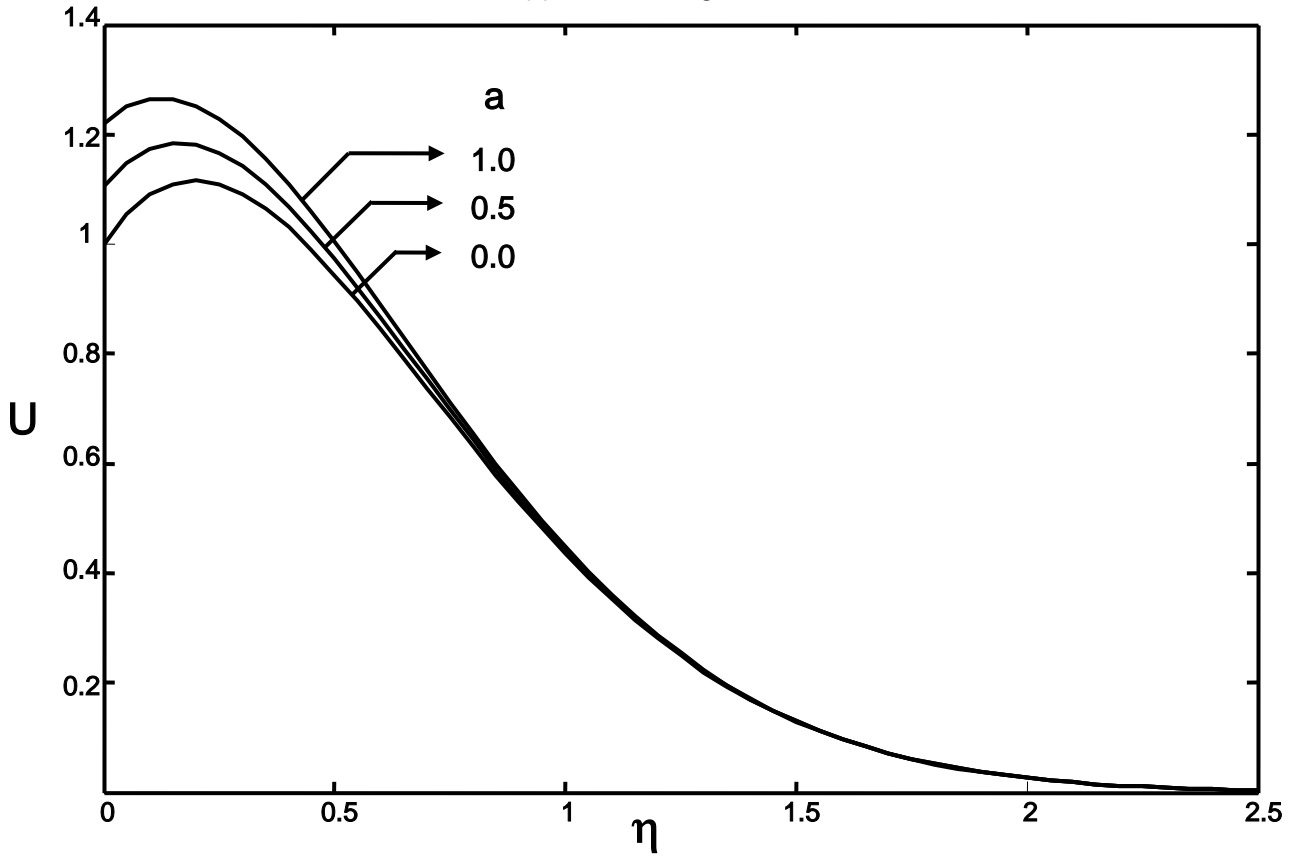


Figure 5. Velocity profiles for different a

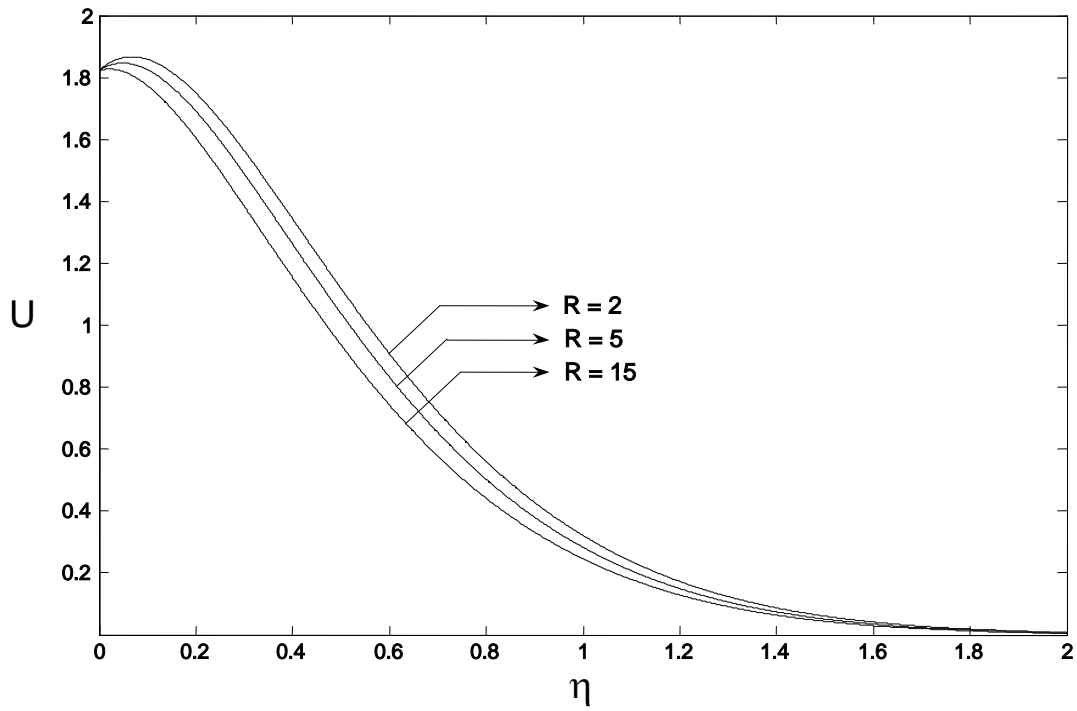


Figure 6. Velocity profiles for different values of R

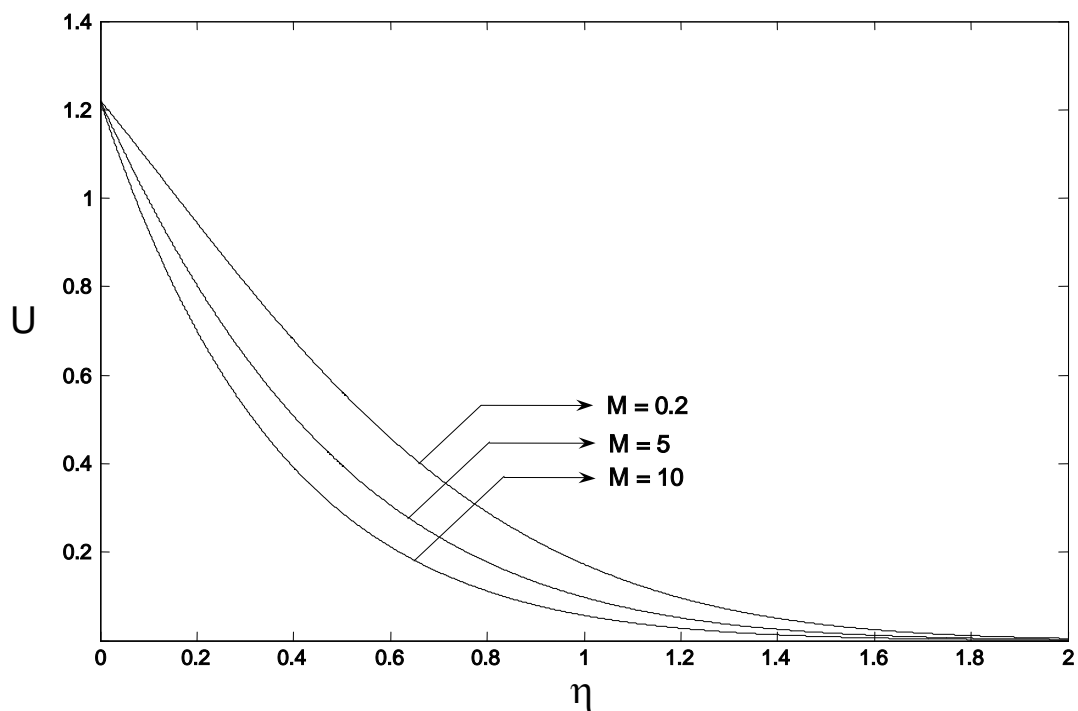


Figure 7. Velocity profiles for different values of M

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