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# EFFECTS OF THERMAL RADIATION AND HEAT SOURCE / SINK ON THE NATURAL CONVECTION IN UNSTEADY HYDROMAGNETIC COUETTE FLOW BETWEEN TWO VERTICAL PARALLEL PLATES WITH CONSTANT HEAT FLUX AT ONE BOUNDARY

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### ABSTRACT

**T**he effects of thermal radiation and heat source/sink on the natural convection in unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid confined between two vertical parallel plates with constant heat flux at one boundary are analyzed here. The magnetic lines of force are assumed to be fixed relative to the moving plate. In deriving the governing equations, a temperature dependent heat source/sink term is employed and the Rosseland approximation for the thermal radiation term is assumed to be valid. The non-dimensional governing equations involved in the present analysis are solved analytically, to the best possible extent. Velocity and temperature profiles are shown graphically, whereas the numerical value of the skin-friction has been tabulated. The effects of various parameters associated with flow like thermal Grashof number Gr, magnetic parameter H, radiation parameter R, Prandtl number  $\Pr$ , heat generation/absorption parameter Q, accelerating parameter a and time t are studied with the help of graphs and tables.

Keywords: MHD, Thermal radiation, Natural convection, Couette flow, Vertical channel, Constant heat flux.

### **1- INTRODUCTION:**

Fluid flow, induced by the relative movement of the bounding plates with unsteady natural convection form of heat transfer, continues to receive attention due to their industrial and technological applications. On the other hand, free convection in vertical channels has been studied widely in the last few decades under different physical effects due to its importance in many engineering applications such as cooling of electronic equipments, design of passive solar systems for energy conversion, cooling of nuclear reactors, design of heat exchangers, chemical devices and process equipments, geothermal systems, and others. However, sufficient attention has not been paid on the problems dealing with free convection in Couette motion between vertical parallel plates. Singh [1] studied the effect of free convection in unsteady Couette motion between two vertical parallel plates. This problem was further extended for magnetohydrodynamic case by Jha [2]. The problem of the transient natural convection flow between two vertical parallel plates with heat sources/sinks has been examined by Jha [3], and Jha and Ajibade [4]. Singh and paul [5] has presented transient free convection flow of a viscous and incompressible fluid between two vertical walls as a result of asymmetric heating or cooling of the walls. Mebine [6] studied the effect of thermal radiation on MHD Couette flow with heat transfer between two parallel plates. Narahari [7] has studied the transient free convection flow of a viscous incompressible fluid between two infinite vertical parallel plates in the presence of constant temperature and mass diffusion. Chaudhary et at. [8] have presented free convection flow of a viscous incompressible fluid past an infinite vertical accelerated plate embedded in a porous medium with constant heat flux in the presence of transverse magnetic field. The natural convection in unsteady Couette flow of a viscous incompressible fluid confined between two vertical parallel plates in the presence of thermal radiation has been studied by Narahari [9]. Jha and Ajibade [10] studied unsteady free convection Couette flow of heat generating/absorbing fluid. Deka and Bhattacharya [11] obtained an exact solution of unsteady free convective Couette flow of a viscous incompressible heat generating/absorbing fluid confined between two vertical plates in porous medium. Seth et al. [12] have studied unsteady MHD Couette flow of a viscous incompressible electrically conducting fluid, in the presence of a transverse magnetic field, between two parallel porous plates. Uwanta et al. [13] have presented transient convection fluid flow with heat and mass flux in a fixed vertical plate with radiation. We [14] have studied natural convection in unsteady hydromagnetic Couette flow of

a viscous incompressible electrically conducting fluid through a vertical channel in the presence of thermal radiation. Jha and Musa [15] studied unsteady natural convection Couette flow of a viscous incompressible heat generating/absorbing fluid in a vertical channel (formed by two infinite vertical and parallel plates) filled with the fluid-saturated porous medium.

The main objective of this paper is to study the effects of heat generation/absorption and radiation on the natural convection in unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting between two vertical parallel plates with constant heat flux at one boundary.

#### 2- MATHEMATICAL ANALYSIS:

Consider the unsteady hydromagnetic Couette flow of an incompressible viscous electrically conducting fluid between two infinite vertical parallel plates with constant heat flux at one boundary in the presence of thermal radiation and heat source/sink. A magnetic field (fixed relative to the moving plates) of uniform strength  $H_0$  is assumed to be applied transversely to the plates. The x' - axis is taken along one of the vertical plate (at y' = 0) and the y' - axis is taken normal to the plate. Let the plates are separated by a distance h. At time t' > 0, the plate (at y' = 0) starts moving with time dependent velocity  $U_0 t'^n$  ( $U_0$  being a constant and n being a non-negative integer) in its own plane and is heated by supplying heat at constant rate whereas the plate (at y' = h) is stationary and maintained at a constant temperature  $T'_h$ . It is also assumed that the radiative heat flux in the x' - direction is negligible as compared to that in the y' - direction. As the plates are infinite in length, the velocity and temperature fields are functions of y' and t' only.

Under the above assumptions and Boussinesq approximation, the equations of motion and energy equation become:

$$\frac{\partial u'}{\partial t'} = v \frac{\partial^2 u'}{\partial {y'}^2} + g \beta (T' - T'_h) - \frac{\sigma \mu_e^2 H_0^2}{\rho} (u' - U_0 t'^n) , \qquad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial {y'}^2} - \frac{\partial q_r}{\partial {y'}} - Q_0 (T' - T'_h) . \qquad (2)$$

Under the following boundary conditions:

$$u' = U_0 t'^n \quad \frac{\partial T'}{\partial y'} = -\frac{q}{k} \quad \text{at} \qquad y' = 0, \\ u' = 0, \qquad T' = T'_h \qquad \text{at} \qquad y' = h. \end{cases}$$
(3)

The radiative heat flux term is simplified by making use of the Rosseland approximation [16] as

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T'^4}{\partial y'} , \qquad (4)$$

where  $\sigma$  is the Stefan-Boltzmann constant and  $k^*$ - the mean absorption coefficient. It is assumed that the temperature difference within the flow is sufficiently small such that  $T'^4$  can be expressed as a linear function of temperature. This is accomplished by expanding  $T'^4$  in a Taylor series about  $T'_h$  and neglecting higher-order terms. This results in the following approximation:

$$T'^{4} \cong 4T_{h}'^{3}T' - 3T_{h}'^{4} .$$
<sup>(5)</sup>

Using equations (4) and (5) in the second last term of equation (2), we obtain:

$$\frac{\partial q_r}{\partial y'} = -\frac{16\sigma T_h^{\prime 3}}{3k^*} \frac{\partial^2 T'}{\partial y'^2} .$$
<sup>(6)</sup>

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Using (6) in equation (2), the energy equation becomes:

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial {y'}^2} + \frac{16\sigma T_h^{\prime 3}}{3k^*} \frac{\partial^2 T'}{\partial {y'}^2} - Q_0 (T' - T_h') .$$
<sup>(7)</sup>

Introducing the following non-dimensional quantities in equations (1) and (7):

$$u = \frac{u'}{U_0}, \quad y = \frac{y'}{h}, \quad t = \frac{t'\nu}{h^2}, \quad \Pr = \frac{\mu C_p}{k}, \quad \theta = \frac{T' - T'_h}{(hq/k)}, \quad Gr = \frac{g\beta h^3 q}{U_0 \nu k},$$

$$a = \frac{h^2}{\nu}, \quad Q = \frac{Q_0 h^2}{\rho \nu C_p}, \quad R = \frac{kk^*}{4\sigma T'^3} \quad and \quad M^2 = \frac{\sigma \mu_e^2 H_0^2 h^2}{\rho \nu}.$$
(8)

The equations (1) and (7) reduce to the following non-dimensional form of equations:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - M^2 \left( u - a^n t^n \right) ,$$

$$\frac{\partial \theta}{\partial t} = \frac{(3R+4)}{2} \frac{\partial^2 \theta}{\partial t^2} - \theta Q$$
(9)

$$\frac{\partial t}{\partial t} = \frac{3R \operatorname{Pr}}{3R \operatorname{Pr}} \frac{\partial y^2}{\partial y^2} = \partial Q \,. \tag{10}$$

The boundary conditions (3) reduce to:

$$u = a^{n}t^{n} \qquad \frac{\partial\theta}{\partial y} = -1 \qquad \text{at} \qquad y = 0, \\ u = 0, \qquad \theta = 0 \qquad \text{at} \qquad y = 1. \end{cases}$$
(11)

Let the solution of the equations (9) and (10) be of the form:

$$u(y,t) = u_0(y)e^{iwt},$$
  

$$\theta(y,t) = \theta_0(y)e^{iwt}.$$
(12)

Using equation (12) in equations (9) and (10), we get:

$$u_0'' - (H + iw)u_0 = -Gr\theta_0 - Ha^n t^n e^{-iwt},$$
(13)

$$\theta_0^{\prime\prime} - (NQ + \iota w N)\theta_0 = 0. \tag{14}$$

The modified boundary conditions become:

$$u_{0} = a^{n}t^{n}e^{-iwt}, \quad \frac{\partial\theta_{0}}{\partial y} = -e^{-iwt} \quad \text{at} \quad y = 0, \\ u_{0} = 0, \qquad \theta_{0} = 0 \qquad \text{at} \quad y = 1. \end{cases}$$
(15)

The solutions of equations (13) and (14) with boundary conditions (15) are given by:

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$$u_{0}(y,t) = a^{n}t^{n}e^{m_{2}y}e^{-iwt} + \frac{Gre^{m_{2}y}e^{-iwt}\tanh m_{1}}{m_{1}\left\{m_{1}^{2} - (H + iw)\right\}} - \frac{Gre^{-iwt}\sinh(1 - y)m_{1}}{m_{1}\left\{m_{1}^{2} - (H + iw)\right\}} - \left[\frac{Gre^{-iwt}e^{m_{2}}\tanh m_{1}}{m_{1}\left\{m_{1}^{2} - (H + iw)\right\}} - \frac{Ha^{n}t^{n}e^{-iwt}\left(e^{m_{2}} - 1\right)}{(H + iw)} + a^{n}t^{n}e^{-iwt}e^{m_{2}}\right]\frac{\sinh m_{2}y}{\sinh m_{2}} + \frac{Ha^{n}t^{n}e^{-iwt}}{(H + iw)}\left[1 - e^{m_{2}y}\right],$$
(16)
  
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(16)

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$$\theta_0(y,t) = e^{-iwt} \frac{\sinh(1-y)m_1}{m_1\cosh m_1}.$$
(17)

Hence, the solutions of equations (9) and (10) become:

$$u(y,t) = a^{n}t^{n}e^{m_{2}y} + \frac{Gre^{m_{2}y}\tanh m_{1}}{m_{1}\left\{m_{1}^{2} - (H + iw)\right\}} - \frac{Gr\sinh(1 - y)m_{1}}{m_{1}\left\{m_{1}^{2} - (H + iw)\right\}}\cosh m_{1}$$
$$-\left[\frac{Gre^{m_{2}}\tanh m_{1}}{m_{1}\left\{m_{1}^{2} - (H + iw)\right\}} - \frac{Ha^{n}t^{n}\left(e^{m_{2}} - 1\right)}{(H + iw)} + a^{n}t^{n}e^{m_{2}}\right]\frac{\sinh m_{2}y}{\sinh m_{2}} + \frac{Ha^{n}t^{n}}{(H + iw)}\left[1 - e^{m_{2}y}\right],$$
(18)

$$\theta(y,t) = \frac{\sinh(1-y)m_1}{m_1\cosh m_1}.$$
(19)

#### **3- SKIN-FRICTION:**

Using Equation (18), the skin-friction or the shear stress at the moving plate of the channel in non-dimensional form, is given by:

$$\tau_{0} = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = -a^{n}t^{n}m_{2} - \frac{m_{2}Gr\tanh m_{1}}{m_{1}\left\{m_{1}^{2} - (H + iw)\right\}} - \frac{Gr}{\left\{m_{1}^{2} - (H + iw)\right\}} + \left[\frac{Gre^{m_{2}}\tanh m_{1}}{m_{1}\left\{m_{1}^{2} - (H + iw)\right\}} - \frac{Ha^{n}t^{n}\left(e^{m_{2}} - 1\right)}{(H + iw)} + a^{n}t^{n}e^{m_{2}}\right]\frac{m_{2}}{\sinh m_{2}} + \frac{Ha^{n}t^{n}m_{2}}{(H + iw)},$$
(20)

Where  $H = M^2$ ,  $N = \frac{3R \operatorname{Pr}}{3R + 4}$ ,  $m_1 = \sqrt{NQ + iwN}$  and  $m_2 = \sqrt{H + iw}$ .

# 4- RESULTS AND DISCUSSION:

We discuss velocity field, temperature field and skin-friction by assigning numerical values to various parameters like thermal Grashof number Gr, magnetic parameter H, radiation parameter R, Prandtl number Pr, heat generation/absorption coefficient Q, accelerating parameter a and time t. The values of the main parameters considered are: magnetic parameter H = 2.0, 4.0, 6.0; radiation parameter R = 1.0, 2.0, 3.0, 4.0; accelerating parameter a = 0.3, 0.5, 0.7, 0.9; heat generation/absorption parameter Q = -2.0, -1.0, 0.0, 1.0, 2.0; time t = 0.2, 0.4, 0.6; Grashof number Gr = 5.0, 10.0, 15.0; and Prandtl number Pr = 0.71 (for air), 3.0 (for the saturated liquid Freon at 273.3K), 7.0 (for water) and 10.0 (for Gasoline at 1 atm. Pressure at  $20^{0}C$ ). Two cases are considered here: (I) impulsive moment of the plate at y = 0 (i.e. n = 0) and (II) uniformly accelerated movement of the plate at y = 0(i.e. n = 1). Again, the term  $Q_0(T' - T'_h)$  is assumed to be the amount of heat generated or absorbed.  $Q_0$  is a constant, which may take either positive or negative values. So, the heat is generated when Q < 0 and absorbed when Q > 0.

The temperature profiles are illustrated in figures 1 to 4 for different values of Prandtl number, radiation parameter, heat generation/absorption parameter and the frequency of excitation of the plate respectively. In figure 1, it can be seen that the temperature of the fluid is inversely proportional to the value of Pr. Thus increasing Pr reduces the temperature in the system. This trend is generally due to the decrease of the thermal diffusivity at high value of Pr. From figure 2, it is clear that an increase in the radiation parameter results in decreasing the temperature within the boundary layer, as well as a decrease in the thermal boundary layer thickness. Figure 3 shows that fluid temperature increases on increasing Q(<0) and decreases on increasing Q(>0) which imply that thermal source tends to increase fluid temperature, whereas thermal sink has reverse effect on it. The effect of the frequency of excitation on

the temperature profiles is illustrated in figure 4. It is observed that as the frequency of excitation is decreased, the temperature appears to be increasing.

Figures 5 and 6 display the effects of H (magnetic parameter) on the velocity field. It is observed that the velocity of fluid increases with increasing magnetic parameter in case of impulsive movement of the plate (i.e. n = 0), but decreases with increasing magnetic parameter in case of uniformly accelerated movement of the plate (i.e. n = 1). Figures 7 (n=0) and 8 (n=1) illustrate the effects of Gr (thermal Grashof number) on the velocity field. We observe that the velocity of fluid increases with increasing thermal Grashof number in both the cases. It is due to reason that increases in Gr give rise to buoyancy effects resulting in more induced flows. Figures 9 (n = 0) and 10 (n = 1) represent the velocity profiles for different values of Pr (Prandtl number). It is observed that the velocity of fluid decreases as the value of Prandtl number increases in both the cases. This is in agreement with the physical fact that the thermal boundary layer thickness decreases with increasing  $\mathbf{Pr}$ . The effects of radiation parameter R on the velocity profiles are presented in figures 11 (n = 0) and 12 (n = 1). From these figures, we observe that as the value of R (radiation parameter) increases, the velocity decreases in both the cases when the other physical parameters are fixed. Figures 13 (n = 0) and 14 (n = 1) represent the velocity profiles for various values of the accelerating parameter a. From these figures, it is observed that the velocity of fluid increases with increasing accelerating parameter in case of uniformly accelerated movement of the plate (i. e. n = 1), whereas the fluid velocity is unchanged as the value of accelerating parameter increases in case of impulsive movement of the plate (i.e. n = 0). Figure 15 shows that the fluid velocity increases as the value of time t increases in case of uniformly accelerated movement of the plate (i.e. n = 1). Figures 16 (n=0) and 17 (n=1) represent the velocity profiles for various values of heat generation/absorption parameter. From these figures, it is observed that the fluid velocity increases on increasing O(<0) and decreases on increasing O(>0) in both the cases. Figures 18 (n=0) and 19 (n=1) illustrate the effect of the frequency of excitation of the plate on the velocity of fluid. From these figures, it is observed that as the frequency of excitation decreases, the velocity increases.

Table 1 and 2 represent the skin-friction in case of uniformly accelerated movement of the plate (i.e. n = 1) and in case of impulsive movement of the plate (i.e. n = 0) respectively. From the two tables, it is clear that in case of uniformly accelerated movement of the plate the skin-friction increases as the value of magnetic field H increases (keeping other parameters constant), but for impulsive movement of the plate, it decreases as the value of magnetic field H increases (keeping other parameters constant). Again, the skin-friction increases as the values of the Prandtl number and radiation parameter increase (keeping other parameters constant) in both the cases (n = 0 and n = 1). When the value of thermal Grashof number is increased (keeping other parameters constant) the value of skin-friction also gets decreased in both the cases (n = 0 and n = 1). Further, the skin-friction increases as the values of time t and accelerating parameter a (keeping other parameters constant) are increased in case of uniformly accelerated movement of the plate, whereas the effects of time t and accelerating parameter a on the skin-friction are negligible in case of impulsive movement of the plate. It is also observed that the skin-friction increases on increasing Q(>0) and decreases on increasing Q(<0) in both the cases (n = 0 and n = 1) which implies that thermal source has tendency to decrease skin friction whereas thermal sink has reverse effect on it.



Figure-1: Temperature profiles



Figure-4: Temperature profiles



Figure-5: Veocilty Profiles in case of impulsive movement of the plate



Figure-6: Velocity Profiles in case of uniformly accelerated movement of the plate



Figure-7: Velocity Profiles in case of impulsive movement of the plate © 2012, IJMA. All Rights Reserved



Figure-8: Velocity Profiles in case of uniformly accelerated movement of the plate



Figure-9: Velocity Profiles in case of impulsive movement of the plate



Figure-10: Velocity Profiles in case of uniformly accelerated movement of the plate



Figure-11: Velocity Profiles in case of impulsive movement of the plate



Figure-12: Velocity Profiles in case of uniformly accelerated movement of the plate



Figure-13: Velocity Profiles in case of impulsive movement of the plate

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Figure-14: Velocity Profiles in case of uniformly accelerated movement of the plate



Figure-15: Velocity Profiles in case of uniformly accelerated movement of the plate



Figure-16: Velocity Profiles in case of impulsive movement of the plate

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Figure-17: Velocity Profiles in case of uniformly accelerated movement of the plate



Figure-18: Velocity Profiles in case of impulsive movement of the plate



Figure-19: Velocity Profiles in case of uniformly accelerated movement of the plate

Н	Pr	R	Q	а	t	Gr	w	$ au_0$
2.0	0.71	1.0	0.0	0.5	0.2	5.0	1.0	-1.36720
4.0	0.71	1.0	0.0	0.5	0.2	5.0	1.0	-1.25589
6.0	0.71	1.0	0.0	0.5	0.2	5.0	1.0	-1.16848
2.0	0.71	1.0	0.0	0.5	0.2	10.0	1.0	-2.80929
2.0	0.71	1.0	0.0	0.5	0.2	15.0	1.0	-4.25138
2.0	3.0	1.0	0.0	0.5	0.2	5.0	1.0	-1.05794
2.0	10.0	1.0	0.0	0.5	0.2	5.0	1.0	-0.28284
2.0	7.0	2.0	0.0	0.5	0.2	5.0	1.0	-0.293483
2.0	7.0	3.0	0.0	0.5	0.2	5.0	1.0	-0.223029
2.0	7.0	4.0	0.0	0.5	0.2	5.0	1.0	-0.188607
2.0	0.71	1.0	0.0	0.7	0.2	5.0	1.0	-1.33725
2.0	0.71	1.0	0.0	0.9	0.2	5.0	1.0	-1.30730
2.0	0.71	1.0	0.0	0.5	0.4	5.0	1.0	-1.29232
2.0	0.71	1.0	0.0	0.5	0.6	5.0	1.0	-1.21744
2.0	0.71	1.0	-2.0	0.5	0.2	5.0	1.0	-1.80304
2.0	0.71	1.0	-1.0	0.5	0.2	5.0	1.0	-1.55662
2.0	0.71	1.0	1.0	0.5	0.2	5.0	1.0	-1.21731
2.0	0.71	1.0	2.0	0.5	0.2	5.0	1.0	-1.09584
2.0	0.71	1.0	0.0	0.5	0.2	5.0	2.0	-1.25677
2.0	0.71	1.0	0.0	0.5	0.2	5.0	3.0	-1.09566
2.0	0.71	1.0	0.0	0.5	0.2	5.0	4.0	-0.90880

Table -1: Skin-friction in case of uniformly accelerated movement of the plate

**Table 2** Skin-friction in case of impulsive movement of the plate

Н	Pr	R	Q	а	t	Gr	w	$ au_0$
2.0	0.71	1.0	0.0	0.5	0.2	5.0	1.0	-0.69327
4.0	0.71	1.0	0.0	0.5	0.2	5.0	1.0	-0.74638
6.0	0.71	1.0	0.0	0.5	0.2	5.0	1.0	-0.77404
2.0	0.71	1.0	0.0	0.5	0.2	10.0	1.0	-2.13536
2.0	0.71	1.0	0.0	0.5	0.2	15.0	1.0	-3.57745
2.0	3.0	1.0	0.0	0.5	0.2	5.0	1.0	-0.38401
2.0	10.0	1.0	0.0	0.5	0.2	5.0	1.0	0.39108
2.0	7.0	2.0	0.0	0.5	0.2	5.0	1.0	0.38044
2.0	7.0	3.0	0.0	0.5	0.2	5.0	1.0	0.45089
2.0	7.0	4.0	0.0	0.5	0.2	5.0	1.0	0.48532
2.0	0.71	1.0	0.0	0.7	0.2	5.0	1.0	-0.69327
2.0	0.71	1.0	0.0	0.9	0.2	5.0	1.0	-0.69327
2.0	0.71	1.0	0.0	0.5	0.4	5.0	1.0	-0.69327
2.0	0.71	1.0	0.0	0.5	0.6	5.0	1.0	-0.69327
2.0	0.71	1.0	-2.0	0.5	0.2	5.0	1.0	-1.12911
2.0	0.71	1.0	-1.0	0.5	0.2	5.0	1.0	-0.88269
2.0	0.71	1.0	1.0	0.5	0.2	5.0	1.0	-0.54337
2.0	0.71	1.0	2.0	0.5	0.2	5.0	1.0	-0.42191
2.0	0.71	1.0	0.0	0.5	0.2	5.0	2.0	-0.53555
2.0	0.71	1.0	0.0	0.5	0.2	5.0	3.0	-0.299478
2.0	0.71	1.0	0.0	0.5	0.2	5.0	4.0	-0.01504

### **5- CONCLUSION:**

In this paper, a mathematical model has been presented for thermal radiation and heat source/sink effects on the natural convection in unsteady hydromagnetic Couette flow between two vertical parallel plates with constant heat flux at one boundary. The effects of different parameters such as thermal Grashof number Gr, magnetic parameter H, radiation parameter R, Prandtl number  $\Pr$ , heat generation/absorption coefficient Q, accelerating parameter a and time t were studied. The conclusion is as follows:

- The temperature decreases with increase in Pr, R, w and Q(>0) but it increases with increase in Q(<0).
- The fluid velocity increases with increase in Gr and Q(<0) for both the cases.
- The fluid velocity decreases with increase in R,  $\Pr$ , w and Q(>0) for both the cases.
- The fluid velocity increases with increase in H (in case of impulsive movement of the plate) but it decreases with increase in H (in case of uniformly accelerated movement of the plate)
- The skin-friction increases with increase in R, w, Pr and Q(>0) for both the cases.
- The skin-friction decreases with increase in Gr and Q(<0) for both the cases.
- The skin-friction decreases with increase in H (in case of impulsive movement of the plate) but it increases with increase in H (in case of uniformly accelerated movement of the plate).

# **5- NOMENCLATURE:**

<i>x</i> ′	Co-ordinate axis along the moving plate	q	Rate of heat transfer
<i>y</i> ′	Co-ordinate axis normal to the plate	Pr	Prandtl number
y	Dimensionless co-ordinate axis normal to the	Gr	Thermal Grashof number
	plate	H	Magnetic field parameter
u'	Velocity of fluid in the $x'$ direction	R	Radiation parameter
и	Dimensionless velocity	$Q_0$	Heat generation/absorption coefficient
ť	Time	Q	Dimensionless heat generation/absorption
t	Dimensionless time	~	coefficient
W	Frequency of excitation of the plate		
$C_{n}$	Specific heat at constant pressure	Greek	letters:
ν 	Magnatia nama shilita	$\sigma$	Electrical conductivity
$\mu_{_e}$	Magnetic permeability	$\rho$	Fluid density
$H_0$	External magnetic field along $y'$ - axis	ν	Kinematic viscosity
k	Thermal conductivity of the fluid	$\beta$	Co-efficient of thermal expansion
8	Acceleration due to gravity	μ	Co-efficient of viscosity
T'	Temperature of the fluid	$\theta$	Dimensionless temperature
$T_h'$	The temperature of the plate at $y' = h$	$ au_0$	Dimensionless skin-friction at the moving plate

 $q_r$  Radiative heat flux

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