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HEAT AND MASS TRANSFER EFFECTS ON UNSTEADY MHD FREE CONVECTIVE FLOW ALONG A VERTICAL POROUS PLATE WITH INTERNAL HEAT GENERATION AND VARIABLE SUCTION

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ABSTRACT

Aim of this paper is to investigate the flow of a viscous incompressible electrically conducting fluid along a porous vertical isothermal non-conducting plate with variable suction and internal heat generation in the presence of transverse magnetic field. The governing equations of motion, energy and concentration are transformed into ordinary differential equations using similarity parameters and then solved numerically using fourth ordered Runge-Kutta method along with shooting technique. The effects of various physical parameters on velocity field, temperature field and concentration distribution have been studied and results are presented graphically and discussed quantitatively. The values of skin friction coefficient, Nusselt number and Sherwood number for various values of parameters are presented through tables.

***Keywords**: Unsteady; Free convection; Mass transfer; MHD; Porous plate; Internal heat generation.*

***AMS Number:** 76D10; 76S05; 80A20.*

1. INTRODUCTION:

The study of convective fluid flow with mass transfer along a vertical porous plate in the presence of magnetic field and internal heat generation receiving considerable attention due to its useful applications in different branches of Science and Technology such as cosmical and geophysical science, fire engineering, combustion modeling etc. Ostrach [14] obtained similarity solution of laminar free convective flow and heat transfer about a flat plate parallel to direction of the generating body force. Soundalgekar [21] investigated unsteady free convection flow along vertical porous plate with different boundary conditions and viscous dissipation effect. Vajravelu [22] studied natural convection flow along a heated semi-infinite vertical plate with internal heat generation. Pop and Soundalgekar [15] investigated the free convection flow past an accelerated infinite plate. Singh [20] analyzed the MHD free convective flow past an accelerated vertical porous plate by finite difference method. Hossain and Begum [10] obtained the effects of mass transfer and free convection past a vertical porous plate. Crpeau and Clarksean [8] obtained similarity solution of natural convection with internal heat generation, which decays exponentially. Kim [11] investigated unsteady MHD convective flow and heat transfer past a semi-infinite vertical porous moving plate with variable suction. Sattar et al. [16] obtained analytical and numerical results for free convection flow along a porous plate with variable suction in porous medium. Cookey et al. [7] studied influence of viscous dissipation and radiation on unsteady MHD free-convective flow past an infinite heated vertical plate in a porous medium with time dependent suction. Makinde et al. [13] investigated unsteady free convection flow with suction on an accelerating porous plate. Chamkha [5] discussed unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Ferdows et al. [9] obtained Numerical approach on parameters of the thermal radiation interaction with convection in boundary layer flow at a vertical plate with variable suction. Ali and Mehmood [6] investigated unsteady boundary layer flow adjacent to permeable stretching surface in a porous medium using Homotopy analysis method. Ahmed [1] studied effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Sharma and Singh [18] discussed Unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Attia and Ewis [4] discussed unsteady MHD Couette flow with heat transfer of a viscoelastic fluid under

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exponential decaying pressure gradient. Recently, Al-Odat and Al-Ghamdi [3] investigated numerically Dufour and Soret effects on unsteady MHD natural convection flow past vertical plate embedded in non-Darcy porous medium.

Aim of this paper is to investigate the flow of a viscous incompressible electrically conducting fluid along a porous vertical isothermal non-conducting plate with variable suction and internal heat generation in the presence of transverse magnetic field.

2. FORMULATION OF THE PROBLEM

Consider the unsteady laminar two-dimensional free convection boundary layer flow of an incompressible viscous electrically conducting fluid along a vertical non-conducting porous plate. Let x-axis is taken along the plate and y-axis is normal to the plate. Magnetic field of intensity B_o is applied in y-direction. Let u and v be the velocity components along x-and y- directions, respectively. Then unsteady boundary layer equations with the incorporation of Boussinesq approximation, the governing equations of continuity, momentum, energy and concentration, respectively are given by

$$\frac{\partial v}{\partial y} = 0 \Rightarrow v \text{ is independent of } y \Rightarrow v = v(t), \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta_c(C - C_\infty) - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} + Q, \tag{3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

where t is time variable, ν is kinematic viscosity, ρ is fluid density, g is acceleration due to gravity of the earth, σ is electrical conductivity, β is volumetric expansion coefficient for heat transfer, β_c is volumetric expansion coefficient for mass transfer, C_p is specific heat at constant pressure, T is temperature of fluid in boundary layer, κ is thermal conductivity, Q is volumetric rate of heat generation, C is the mass concentration, C_∞ is the mass concentration far away from the plate and D is molecular diffusivity.

The boundary conditions are

$$\begin{aligned} &u = 0,\ v = v(t),\ T = T_0,\ C = C_0 \quad \text{at} \quad y = 0, \\ &u \to 0,\ T \to T_\infty,\ C \to C_\infty \text{ as} \quad y \to \infty;\ t \to 0 \end{aligned} \tag{5}$$

where T_0 and T_∞ are fluid temperatures at plate and far away from the plate, respectively.

3. METHOD OF SOLUTION

Defining time dependent similarity parameter h {Schlichting and Gersten [17]} having length scale as

$$h = \{= h(t)\} = 2\sqrt{\nu t}, \tag{6}$$

Specially used for unsteady boundary layer problems. In terms of h (t), a convenient solution of (1) is given by

$$v = v(t) = -V_0\frac{\nu}{h(t)}, \tag{7}$$

where V_0 is suction parameter

Introducing the following dimensionless variables for a similarity solution

$$\eta = \frac{y}{h}\ ,\ u = Uf(\eta)\ ,\ \theta(\eta) = \frac{T - T_\infty}{T_0 - T_\infty}\ ,\ \phi(\eta) = \frac{C - C_\infty}{C_0 - C_\infty}\ , \tag{8}$$

into the equations (2) to (4), we get

$$f'' + (2\eta + V_0) f' + Gr\theta + Gc\phi - Mf = 0, \tag{9}$$

$$\theta'' + (2\eta + V_0)\Pr\theta' + S\theta = 0, \tag{10}$$

$$\phi'' + (2\eta + V_0)Sc\phi' = 0, \tag{11}$$

where η is similarity variable, U is uniform characteristic velocity, f is dimensionless stream function, $Gr\left\{= \frac{g\beta h^2 (T_0 - T_\infty)}{\nu U}\right\}$ is the Grashof number for heat transfer, $Gc\left\{= \frac{g\beta_c h^2 (C_0 - C_\infty)}{\nu U}\right\}$ is the Grashof number for mass transfer, $M\left\{= \frac{\sigma B_0^2 h^2}{\nu\rho}\right\}$ is magnetic parameter, $\Pr\left\{= \frac{\nu\rho C_p}{\kappa}\right\}$ is the Prandtl number, $Sc\left\{= \frac{\nu}{D}\right\}$ is the Schmidt number and $S = \left\{\frac{h^2 \mu C_p Q}{\kappa(T_0 - T_\infty)}\right\}$ is heat generation parameter.

The reduced corresponding boundary conditions are

$$f(0) = 0\ ,\ \theta(0) = 1\ ,\ \phi(0) = 1\ ,\ f(\infty) = 0\ ,\ \theta(\infty) = 0\ ,\ \phi(\infty) = 0. \tag{12}$$

The governing equations (9) to (11) are second ordered linear differential equations and solved under the boundary conditions (12) using Runge-Kutta fourth order method {Krishnamurthy and Sen [12]} along with shooting technique {Conte and Boor [6] and Sharma and Singh [19]].

4. SKIN-FRICTION COEFFICIENT

Skin-friction coefficient at the plate is given by

$$C_f = \frac{2\nu}{Uh} f'(0) \tag{13}$$

5. NUSSELT NUMBER

The rate of heat transfer in terms of Nusselt number is given by

$$Nu = \frac{2q\sqrt{\nu t}}{\kappa(T_0 - T_\infty)} = -\theta'(0) \tag{14}$$

, where q is heat flux per unit area.

Sherwood Number

$$Sh = \frac{J\nu}{(C_0 - C_\infty)D\nu} = -\phi'(0) \tag{15}$$

, where J mass transfer coefficient.

6. RESULTS AND DISCUSSION

The governing equations (9) to (11) with the boundary conditions (12) are solved using Runge-Kutta fourth order method along with shooting technique for different values of the parameters taking step size 0.005. The numerical calculations are presented in the form of table and graphs for different values of parameters.

Particular Case:

It is observed from the table 1 the numerical values of $-\theta^{/}(0)$ for S = 0 and $h\{=h(t)\}=\sqrt{\nu t}$ presented in present paper are in good agreement with those obtained by Sattar et al. [6] and Sharma and Singh [18].

From figure 1, it is observed that the temperature field increases as heat generation parameter *S* increases. Figure 2 depicts that the temperature field decreases as Prandtl number increases. From figure 3, it is seen that more suction implies the decrement in temperature field i.e. as the suction parameter V_0 increases, the fluid temperature decreases. Figure 4 and figure 5, respectively show that the concentration profiles decrease as the Schmidt number Sc or suction parameter V_0 increases. It is observed easily from the figure 6 and figure 8 respectively that the velocity profiles decrease as the magnetic parameter or Prandtl number increases. Figure 7 depicts that velocity profiles increase as heat generation parameter *S* increases. Figure 9 shows that the velocity profiles decrease as the suction parameter V_0 increases. It is noted from the figure 10 and figure 11 respectively that the velocity profiles increase Grashof number for heat transfer *Gr* or Grashof number for mass transfer *Gc* increases.

It is seen from table 2 that the rate of heat transfer {-$\theta^{/}(0)$ } increases with the increase in Prandtl number or suction parameter irrespective of absence (i.e. S = 0.0) or presence (i.e. S = 1.0) of heat generation. Further, it is observed from table 3 that the value of $-\phi^{/}(0)$ increases with the increase in Schmidt number or suction parameter. Table 4 depicts that $f^{/}(0)$ decreases with the increase in Prandtl number, magnetic parameter or suction parameter, but it increases with the increase in Grashof number for heat transfer or Grashof number for mass transfer. Also, the increase in $f^{/}(0)$ is seen with the increase in heat generation parameter, as seen from table 4.

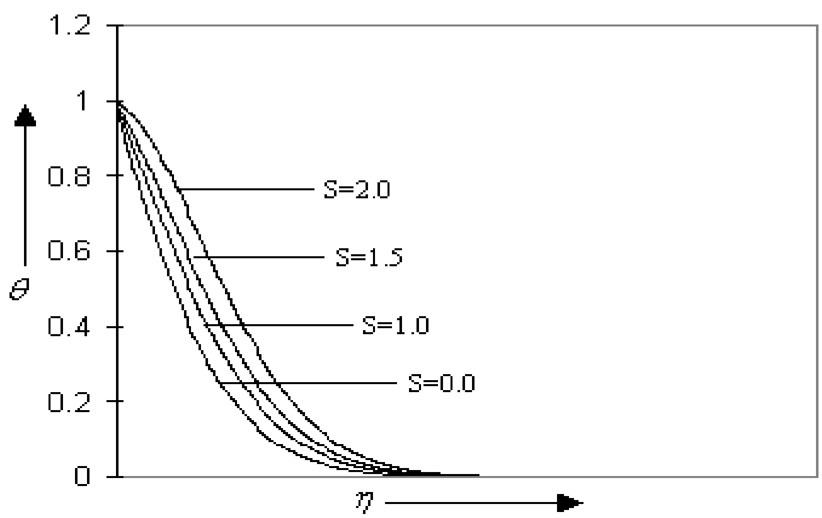

Figure 1. Temperature distribution versus η when *Pr* = 0.71 and $V_0 = 1.0$

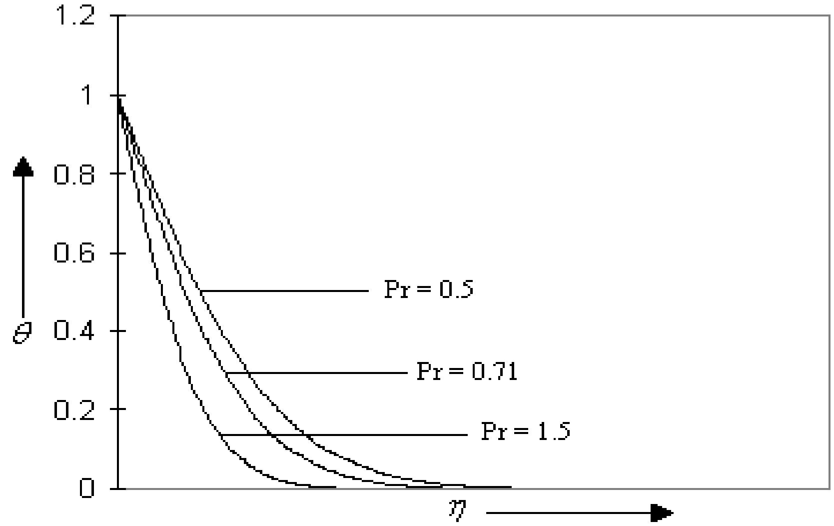

Figure 2. Temperature distribution versus η when S = 0.0 and $V_0 = 0.5$

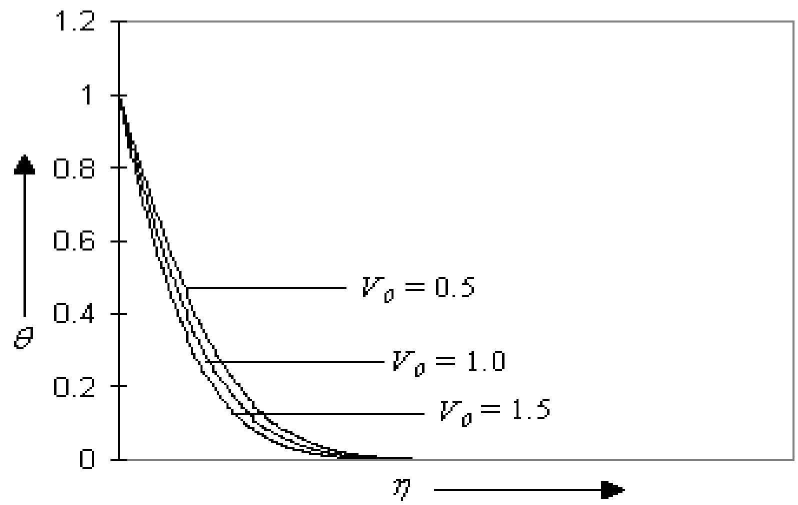

Figure 3. Temperature distribution versus η when Pr = 0.71 and $S = 0.0$

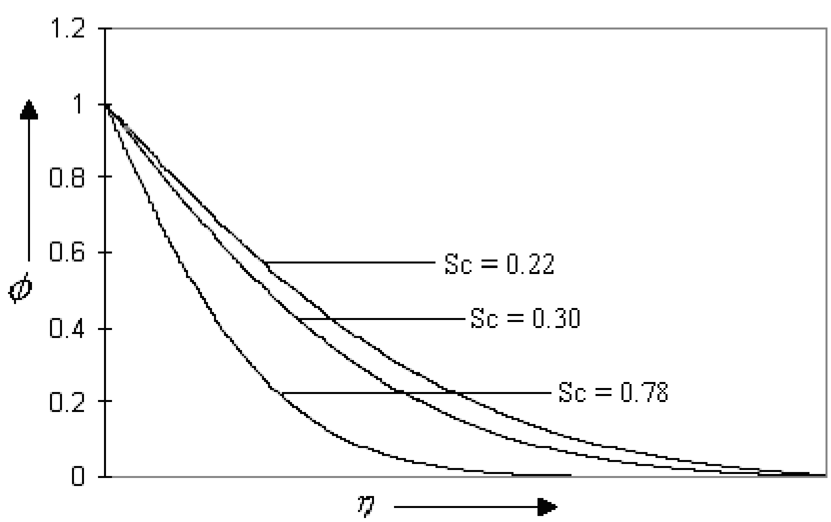

Figure 4. Concentration profiles versus η when V_0 = 0.5.

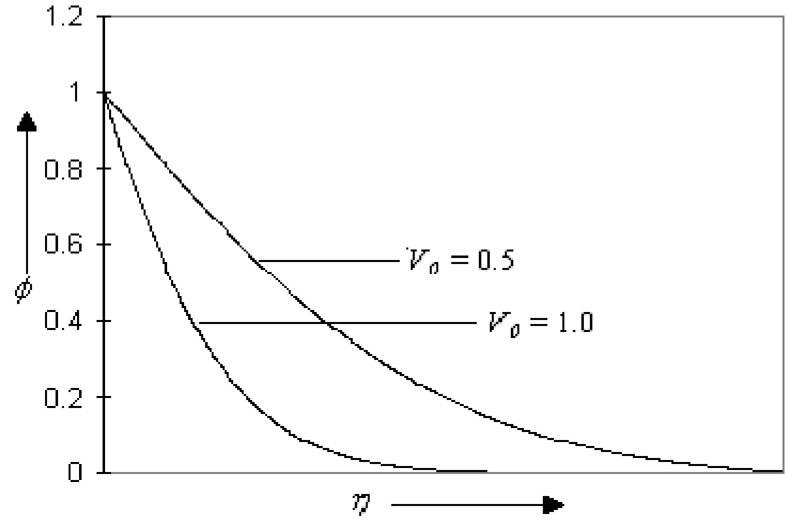

Figure 5. Concentration profiles versus η when Sc = 0.22.

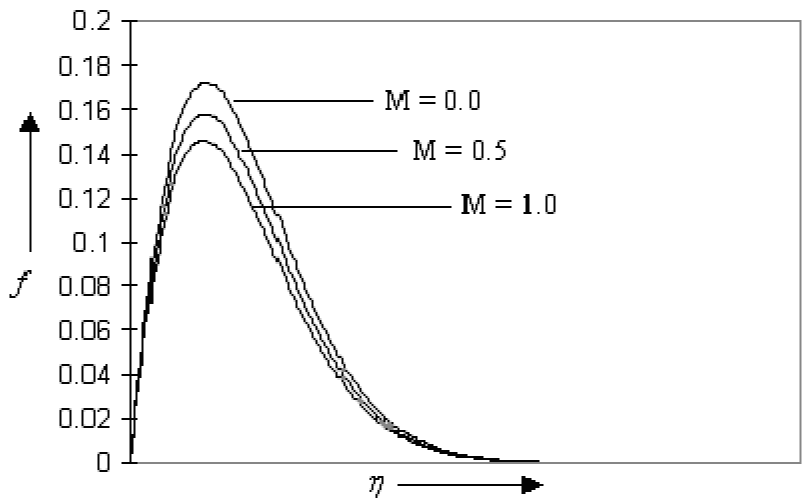

Figure 6. Velocity distribution versus η when Pr=0.71, Gc=0.0, Gr=2.0, V_0=0.5, S=0.0.

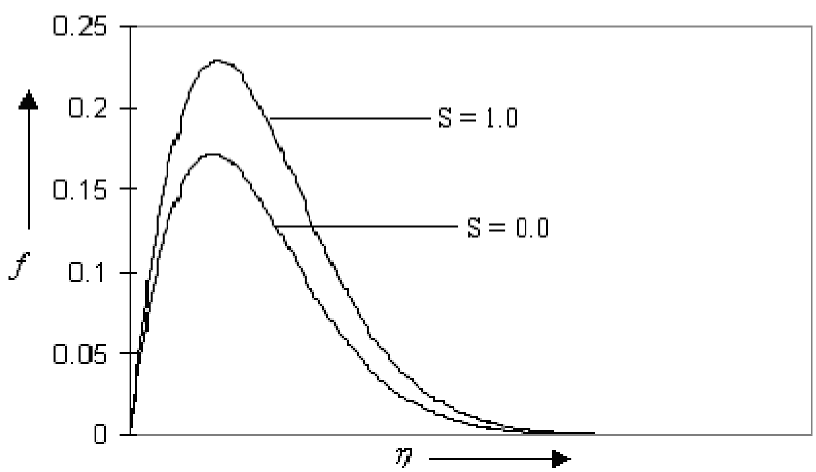

Figure 7. Velocity distribution versus η when Pr=0.71, Gc=0.0, Gr=2.0, V_0=0.5, M=0.0.

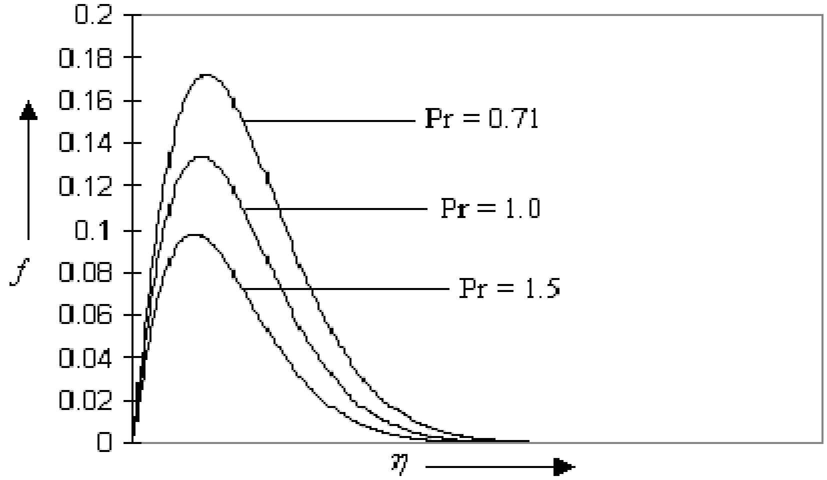

Figure 8. Velocity distribution versus η when *S*=0.0, *Gc*=0.0, *Gr*=2.0, V_0 =0.5, *M*=0.0.

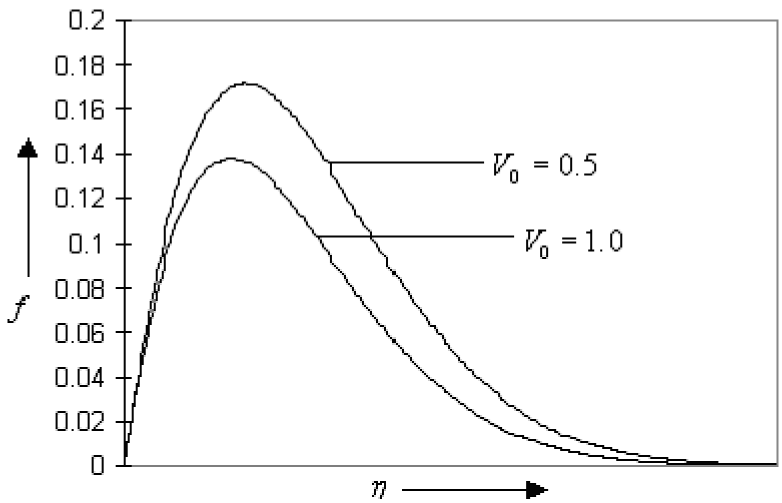

Figure 9. Velocity distribution versus η when *S*=0.0, *Gc*=0.0, *Gr*=2.0, $\Pr$ =0.71, *M*=0.0.

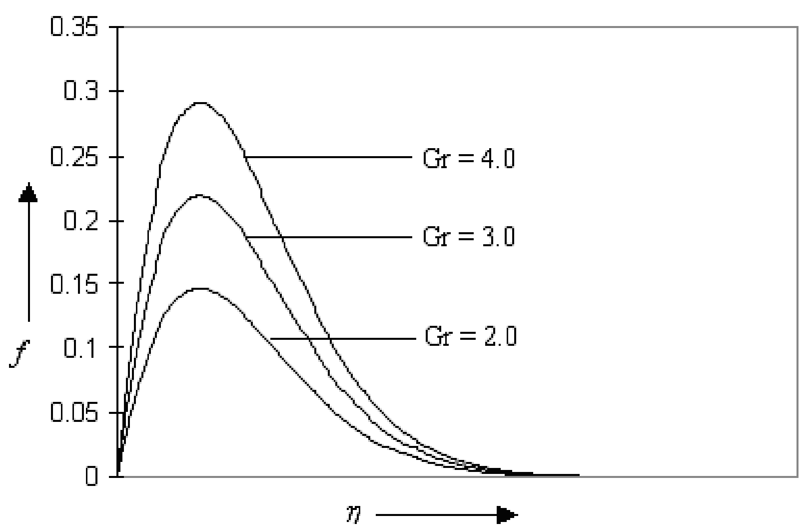

Figure 10. Velocity distribution versus η when *S*=0.0, *Gc* = 0.0, V_0 =0.5, $\Pr$ =0.71, *M*=0.0.

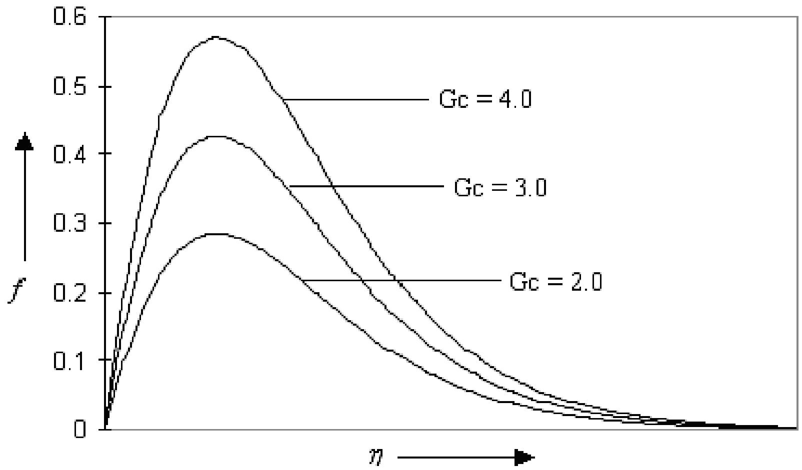

Figure 11. Velocity distribution versus η when S=0.0, V_0=0.5, Gr=2.0, $\Pr$ =0.71, M=0.0.

Table 1. Numerical values of $-\theta'(0)$ for various values of Pr and V_0 are compared with the results obtained by Sattar et al. (2000) and Sharma and Singh (2008).

Pr	V_0	$-\theta'(0)$		
		Sattar et al.	Sharma and Singh	Present work
0.71	0.5	0.9134	0.9134	0.91344
1.0	0.5	1.1411	1.1410	1.14106
1.0	1.0	1.5252	1.5251	1.5253

Table 2. Numerical values of $-\theta'(0)$ for various values of Pr and V_0

Pr	V_0	$-\theta'(0)$	
---		S=0.0	S=1.0
0.5	0.5	0.96355	0.24750
0.71	0.5	1.18772	0.66184
1.5	0.5	1.89217	1.57935
0.5	1.0	1.14107	0.49750
0.71	1.0	1.43224	0.96728
1.5	1.0	2.45764	2.18130
0.5	1.5	1.32878	0.747499
0.71	1.5	1.71679	1.28775
1.5	1.5	3.06491	2.82079

Table 3. Numerical values of $-\phi'(0)$ for various values of *Sc* and V_0

Sc	V_0	$-\phi'(0)$
0.22	0.5	0.45245
0.30	0.5	0.53855
0.78	0.5	0.97028
0.22	1.0	0.52267
0.30	1.0	0.64533
0.78	1.0	1.26305
0.22	1.5	0.60947
0.30	1.5	0.75883
0.78	1.5	1.58174

Table 4. Numerical values of $f^{/}(0)$ for various values of *Sc, Pr, M, S, Gc, Gr* and V_0

	V_0 = 0.5, *Gr* = 2.0					
	S = 0.0			S = 1.0		
Pr	M = 0.0	M = 0.5	M = 1.0	M = 0.0	M = 0.5	M = 1.0
0.71	0.836534	0.789987	0.750730	1.009852	0.946673	0.893779
1.0	0.719755	0.684908	0.655216	0.814740	0.772046	0.735824
1.5	0.595236	0.571052	0.550220	0.643210	0.615725	0.592103
	V_0 = 1.0, *Gr* = 2.0					
0.71	0.784309	0.746194	0.713352	0.920518	0.870953	0.828494
1.0	0.657628	0.630338	0.606588	0.728975	0.696598	0.668512
1.5	0.525279	0.507455	0.491778	0.558903	0.539123	0.521754
	V_0 = 0.5, M = 1.0					
	Gr = 2.0	*Gr* = 3.0	*Gr* = 4.0	*Gr* = 2.0	*Gr* = 3.0	*Gr* = 4.0
0.71	0.750730	1.126095	1.501460	0.893779	1.340669	1.787559
Sc	*Gc* = 2.0	*Gc* = 3.0	*Gc* = 4.0			
0.22	1.113550	1.670325	2.227100			

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