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SOME DIFFERENCE SEQUENCE SPACES DEFINED BY A SEQUENCE OF MODULUS FUNCTIONS

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ABSTRACT

In the present paper we study difference sequence spaces defined by a sequence of modulus functions and examine some topological properties of these spaces.
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1. Introduction and Preliminaries:

A modulus function is a function $f : [0,\infty) \to [0,\infty)$ such that

(1) $f(x) = 0$ if and only if $x = 0$,
(2) $f(x + y) \leq f(x) + f(y)$ for all $x \geq 0, y \geq 0$,
(3) f is increasing
(4) f is continuous from right at 0.

It follows that f must be continuous everywhere on$[0,\infty)$. The modulus function may be bounded or unbounded. For example, if we take$f(x) = \frac{x}{x+1}$, then $f(x)$ is bounded. If $f(x) = x^p, 0 < p < 1$, then the modulus $f(x)$ is unbounded. Subsequentially, modulus function has been discussed in ([1], [7], [8]) and many others.

Let X be a linear metric space. A function $p: X \to \mathbb{R}$ is called paranorm, if

(1) $p(x) \geq 0$, for all $x \in X$,
(2) $p(-x) = p(x)$, for all $x \in X$,
(3) $p(x + y) \leq p(x) + p(y)$, for all $x, y \in X$,
(4) if (λ_n) is a sequence of scalars with $\lambda_n \to \lambda_n$ as $n \to \infty$ and (x_n) is a sequence of vectors with $p(x_n - x) \to 0$ as $n \to \infty$, then $p(\lambda_n x_n - \lambda x) \to 0$ as $n \to \infty$.

A paranorm p for which $p(x) = 0$ implies $x = 0$ is called total paranorm and the pair (X, p) is called a total paranormd space. It is well known that the metric of any linear metric space is given by some total paranorm (see [9], Theorem 10.4.2, p-183).

Let ω be the set of all sequences, real or complex numbers and l_∞, c and c_0 be respectively the Banach spaces of bounded, convergent and null sequences $x = (x_k)$, normed by

$||x|| = sup_k|x_k|$, where $k \in \mathbb{N}$, the set of positive integers.

Let $\Lambda = (\lambda_n)$ be a non decreasing sequence of positive reals tending to infinity and $\lambda_1 = 1$ and $\lambda_{n+1} \leq \lambda_n + 1$. The generalized de la vallee-Poussin means is defined by

$$t_n(x) = \frac{1}{\lambda_n}\sum_{k \epsilon I_n} x_k$$

where $I_n = [n - \lambda_n + 1, n]$. A sequence $x = (x_k)$ is said to be $(V, \lambda) -$ summable to a number l if $t_n(x) \to l \; as \; n \to \infty$ (see[4]). If $\lambda_n = n$, $(V, \lambda) -$ summability and strong $(V, \lambda) -$ summability are reduced to$(C, 1) -$ summability and $[C, 1] -$ summability, respectively.

The idea of difference sequence spaces were introduced by Kizmaz. In [3], Kizmaz defined the sequence space

$$X(\Delta) = \{x = (x_k) : (\Delta x_k) \in X\}$$

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for $X = l_\infty$, c or c_0, where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$ for all $k \in \mathbb{N}$.
Later, these difference sequence spaces were generalized by Et and Colak [2]. In [2] Et and Colak generalized the above sequence spaces to the sequence space as follows:

$$X(\Delta^m) = \{x = (x_k) : (\Delta^m x_k) \in X\}$$

for $X = l_\infty, c$ or c_0, where $m \in \mathbb{N}$, $\Delta^o x = (x_k)$, $\Delta x = (x_k - x_{k+1})$,
$\Delta^m x = (\Delta^m x_k) = (\Delta^{m-1} x_k - \Delta^{m-1} x_{k+1})$ for all $k \in \mathbb{N}$.

The generalized difference has the following binomial representation,

$$\Delta^m x_k = \sum_{v=0}^{m} (-1)^v \binom{m}{v} x_{k+v}$$

For all $k \in \mathbb{N}$.

The following inequality will be used throughout the paper. If

$$0 \le p_k \le \sup p_k = H,\ D = \max(1, 2^{H-1}) \text{ then}$$
$$|a_k + b_k|^{p_k} \le D\{|a_k|^{p_k} + |b_k|^{p_k}\} \qquad (1)$$

For all k and $a_k, b_k \in \mathbb{C}$. Also $|a|^{p_k} \le \max(1, |a|^H)$ for all $a \in \mathbb{C}$.

Throughout E will represent a seminormed space, seminormed by q. We define ω(E) to be the vector space of all E-valued sequences. Let $F = (f_k)$ be a sequence of strictly positive real numbers, $A = a_{jk}$ be a non negative matrix such that

$$\sup_j \sum_{k=1}^{\infty} a_{jk} < \infty$$

and $s, m \in \mathbb{N}$. Then we define the following sequence spaces :

$$[V_\lambda^E, A, \Delta_m^s, F, p]_0 = \left\{x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k))]^{p_k} = 0, uniformly\ in\ j\right\},$$

$$[V_\lambda^E, A, \Delta_m^s, F, p]_1 = \left\{x \in \omega(E): \lim_{n\to\infty} \tfrac{1}{\lambda_n} \textstyle\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k - L))]^{p_k} = 0, uniformly\ in\ j\ for\ some\ L\right\}$$

and

$$[V_\lambda^E, A, \Delta_m^s, F, p]_\infty = \left\{x \in \omega(E): \sup_j \sup_j \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k))]^{p_k} < \infty\right\}.$$

For $f_k(x) = x$, we have

$$[V_\lambda^E, A, \Delta_m^s, p]_0 = \left\{x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[q(\Delta_m^s x_k)]^{p_k} = 0, uniformly\ in\ j\right\}$$

$$[V_\lambda^E, A, \Delta_m^s, p]_1 = \left\{x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[q(\Delta_m^s x_k - L)]^{p_k} = 0, uniformly\ in\ j\ for\ some\ L\right\},$$

and

$$[V_\lambda^E, A, \Delta_m^s, p]_\infty = \left\{x \in \omega(E): \sup_j \sup_j \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[q(\Delta_m^s x_k)]^{p_k} < \infty\right\}.$$

For $p_k = 1$, we have

$$[V_\lambda^E, A, \Delta_m^s, F]_0 = \left\{x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}\left[f_k\left(q(\Delta_m^s x_k)\right)\right] = 0, uniformly\ in\ j\right\},$$

$$[V_\lambda^E, A, \Delta_m^s, F, p]_1 = \left\{x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k - L))] = 0, uniformly\ in\ j\ for\ some\ L\right\}$$

and

$$[V_\lambda^E, A, \Delta_m^s, F, p]_\infty = \left\{x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}\left[f_k\left(q(\Delta_m^s x_k)\right)\right] < \infty\right\}.$$

For $f_k(x) = x$ and $p_k = 1$ for all $k \epsilon \mathbb{N}$, we have

$$[V_\lambda^E, A, \Delta_m^s]_0 = \left\{ x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[q(\Delta_m^s x_k)] = 0, uniformly\ in\ j \right\},$$

$$[V_\lambda^E, A, \Delta_m^s]_1 = \left\{ x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[q(\Delta_m^s x_k - L)] = 0, uniformly\ in\ j\ for\ some\ L \right\}$$

and

$$[V_\lambda^E, A, \Delta_m^s]_\infty = \left\{ x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[q(\Delta_m^s x_k)] < \infty \right\}$$

For $m = 1$, we have

$$[V_\lambda^E, A, \Delta^s, F, p]_0 = \left\{ x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[f_k(q(\Delta^s x_k))]^{p_k} = 0, uniformly\ in\ j \right\},$$

$$[V_\lambda^E, A, \Delta^s, F, p]_1 = \left\{ x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[f_k(q(\Delta^s x_k - L))]^{p_k} = 0, uniformly\ in\ j\ for\ some\ L \right\}$$

and

$$[V_\lambda^E, A, \Delta^s, F, p]_\infty = \left\{ x \in \omega(E): \sup_j \sup_j \frac{1}{\lambda_n} \sum_{k\in I_n} a_{jk}[f_k(q(\Delta^s x_k))]^{p_k} < \infty \right\}.$$

For $A = 1$, we have

$$[V_\lambda^E, \Delta_m^s, F, p]_0 = \left\{ x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} [f_k(q(\Delta_m^s x_k))]^{p_k} = 0, uniformly\ in\ j \right\},$$

$$[V_\lambda^E, \Delta_m^s, F, p]_1 = \left\{ x \in \omega(E): \lim_{n\to\infty} \frac{1}{\lambda_n} \sum_{k\in I_n} [f_k(q(\Delta_m^s x_k - L))]^{p_k} = 0, uniformly\ in\ j\ for\ some\ L \right\}$$

and

$$[V_\lambda^E, \Delta_m^s, F, p]_\infty = \left\{ x \in \omega(E): \sup_j \sup_j \frac{1}{\lambda_n} \sum_{k\in I_n} [f_k(q(\Delta_m^s x_k))]^{p_k} < \infty \right\}.$$

For $E = \mathbb{C}, q(x) = |x|, f_k(x) = x, p_k = 1$, for all $k \in \mathbb{N}$, $s = 0$, $m = 0$ the spaces $[V_\lambda^E, A, \Delta_m^s, F, p]_0$, $[V_\lambda^E, A, \Delta_m^s, F, p]_1$ and $[V_\lambda^E, A, \Delta_m^s, F, p]_\infty$ reduces to $[V, \lambda]_0, [V, \lambda]_1$ and $[V, \lambda]_\infty$ respectively. These spaces are called as λ-strongly summable to zero, λ-strongly summable and $\lambda -$ strongly bounded by the de la Vallee-Poussin method. When $\lambda_n = n$, for all $n = 1, 2, 3, \ldots$ the sets $[V, \lambda]_0, [V, \lambda]$ and $[V, \lambda]_\infty$ reduce to the set ω_0, ω and ω_∞ introduced and studied by Maddox [5]. Throughout this paper, we will denote any one of the notations 0, 1 or ∞ by X.

In this paper we study some topological properties and inclusion relations between above defined sequence spaces.

2. MAIN RESULTS:

Theorem: 2.1 Let $F = (f_k)$ be a sequence of modulus functions and $p = (p_k)$ be a bounded sequence of strictly positive real numbers. Then the sequence spaces $[V_\lambda^E, \Delta_m^s, A, F, p]_0$, $[V_\lambda^E, \Delta_m^s, A, F, p]_1$ and $[V_\lambda^E, \Delta_m^s, A, F, p]_\infty$ are linear spaces.

Proof: Let $x, y \in [V_\lambda^E, \Delta_m^s, A, F, p]_0$ and $\alpha, \beta \in \mathbb{C}$. Then there exist positive number M_α and N_β such that $|\alpha| \le M_\alpha$ and $|\beta| \le N_\beta$. Since f_k is subadditive and Δ^m is linear, we have

$$\frac{1}{\lambda_n} \sum_{k \in I_n} a_{jk}[f_k(q(\Delta^m(\alpha u_k x_k + \beta u_k y_k)))]^{p_k} \le \frac{1}{\lambda_n} \sum_{k \in I_n} a_{jk}[f_k(|\alpha| q(\Delta_m^s(x_k)) + f_k(|\beta| q(\Delta_m^s(y_k))]^{p_k}$$

$$\le D(M_\alpha)^H \frac{1}{\lambda_n} \sum_{k \in I_n} a_{jk}[f_k(q(\Delta_m^s x_k))]^{p_k} + D(N_\beta)^H \frac{1}{\lambda_n} \sum_{k \in I_n} a_{jk}[f_k(q(\Delta_m^s y_k))]^{p_k} \to 0\ as\ n \to \infty.$$

This proves that $[V_\lambda^E, \Delta_m^s, A, F, p]_0$ is linear space. Similarly we can prove that $[V_\lambda^E, \Delta_m^s, A, F, p]_1$ and $[V_\lambda^E, \Delta_m^s, A, F, p]_\infty$ are linear spaces in view of the above proof.

Theorem: 2.2 Let $F = (f_k)$ be a sequence of modulus functions. Then

$$[V_\lambda^E, \Delta_m^s, A, F, p]_0 \subset [V_\lambda^E, \Delta_m^s, A, F, p]_1 \subset [V_\lambda^E, \Delta_m^s, A, F, p]_\infty.$$

Proof: The first inclusion is obvious. For the second inclusion, let $x \in [V_\lambda^E, \Delta_m^s, A, F, p]_1$. Then by definition, we have

$$\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k))]^{p_k} = \frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k - L + L))]^{p_k}$$
$$\leq D\,\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k - L))]^{p_k} + D\,\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(L)]^{p_k}\ .$$

Now, there exists a positive number A such that $L \leq A$. Hence we have

$$\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k))]^{p_k} \leq\ D\,\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k - L))]^{p_k} + D\,\frac{1}{\lambda_n}\sum_{k\in I_n}[Af_k(1)]^H\ \lambda_n \sum_{k\in I_n} a_{jk}.$$

Since $x \in [V_\lambda^E, \Delta_m^s, A, F, p]_1$ we have $x \in [V_\lambda^E, \Delta_m^s, A, F, p]_\infty$. Therefore,
$[V_\lambda^E, \Delta_m^s, A, F, p]_1 \subset [V_\lambda^E, \Delta_m^s, A, F, p]_\infty$.
This completes the proof.

Theorem: 2.3 Let $F = (f_k)$ be a sequence of modulus functions and $p = (p_k)$ be a bounded sequence of strictly positive real numbers. Then $[V_\lambda^E, \Delta_m^s, A, F, p]_0$ is a paranormed space with

$$g(x) = \sup_n\left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k))]^{p_k}\right)^{\frac{1}{K}}$$

where $K = \max(1, \sup p_k)$.

Proof: Cleary $g(x) = g(-x)$. It is trivial that $\Delta_m^s x_k = 0$ for $x = 0$. Since$f(0) = 0$, we get $g(x) = 0$ for $x = 0$. Since $\frac{p_k}{K} \leq 1$, using the Minkowski's inequality, for each n, we have

$$\left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k + \Delta_m^s y_k))]^{p_k}\right)^{\frac{1}{p_k}} \leq \left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k)) + f_k(q(\Delta_m^s y_k))]^{p_k}\right)^{\frac{1}{K}}$$
$$\leq \left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s x_k))]^{p_k}\right)^{\frac{1}{K}} + \left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s y_k))]^{p_k}\right)^{\frac{1}{K}}\ .$$

Hence $g(x)$ is subadditive. For, the continuity of multiplication, let us take any complex number α. By definition, we have

$$g(x) = \sup_n\left(\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta_m^s \alpha x_k))]^{p_k}\right)^{\frac{1}{K}} \leq C_\alpha^{H/K} g(x),$$

where C_α is a positive integer such that $|\alpha| \leq C_\alpha$. Now, let $\alpha \to 0$ for any fixed x with $g \neq 0$. By definition for $|\alpha| < 1$, we have

$$\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\alpha\Delta_m^s x_k))]^{p_k} < \epsilon \text{ for } n > n_0(\epsilon) \tag{2}$$

Also, for $1 \leq n \leq n_0$, taking α small enough, since f is continuous, we have

$$\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\alpha\Delta_m^s x_k))]^{p_k} < \epsilon. \tag{3}$$

Now, eqn. (2) and (3) together imply that $g(\alpha x) \to 0\ as\ \alpha \to 0$.

Theorem: 2.4 Let $F = (f_k)$ be a sequence of modulus functions and $m \geq 1$, then the inclusion $[V_\lambda^E, \Delta_m^{s-1}, A, F]_X \subset [V_\lambda^E, \Delta_m^s, A, F]_X$ is strict. In general $[V_\lambda^E, \Delta_m^i, A, F]_X \subset [V_\lambda^E, \Delta_m^s, A, F]_X$ for all $i = 1, 2, \ldots, s-1$ and the inclusion is strict.

Proof: Let $x \in [V_\lambda^E, \Delta_m^{s-1}, A, F]_\infty$. Then we have

$$\sup_n \frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}[f_k(q(\Delta^{s-1} x_k))]) < \infty\ .$$

By definition, we have

$$\frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}\left[f_k\left(q(\Delta_m^s x_k)\right)\right] = \frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}\left[f_k\left(q(\Delta_m^{s-1} x_k)\right)\right] + \frac{1}{\lambda_n}\sum_{k\in I_n} a_{jk}\left[f_k\left(q(\Delta_m^{s-1} x_{k+1})\right)\right] \leq \infty.$$

Thus $[V_\lambda^E,\ \Delta_m^{s-1},\ A,\ F]_\infty \subset [V_\lambda^E,\ \Delta_m^{s},\ A,\ F]_\infty$. Proceeding in this way, we have

$[V_\lambda^E,\ \Delta_m^{i},\ A,\ F]_\infty \subset [V_\lambda^E,\ \Delta_m^{s},\ A,\ F]_\infty$ for all $i = 1, 2, \dots, m-1$. Let $E = \mathbb{C}$ and $\lambda_n = n$ for each $n \in \mathbb{N}$. Then the sequence $x = (x^m) \in [V_\lambda^E,\ \Delta_m^{s},\ A,\ F]_\infty$ but does not belong to $[V_\lambda^E,\ \Delta_m^{s-1},\ A,\ F]_\infty$ for
$f_k(x) = x$.
Similarly, we can prove for the case $[V_\lambda^E,\ \Delta_m^{s},\ A,\ F]_0$ and $[V_\lambda^E,\ \Delta_m^{s},\ A,\ F]_1$ in view of the above proof.

Corollary: 2.5 Let $F = (f_k)$ be a sequence of modulus functions. Then

$$[V_\lambda^E,\ \Delta_m^{s-1},\ A,\ F,\ p]_1 \subset [V_\lambda^E,\ \Delta_m^{s},\ A,\ F,\ p]_0.$$

Theorem: 2.6 Let $F = f_k$ be a sequence of modulus functions and s be a positive integer. Then we have

$$[V_\lambda^E, \Delta_m^s, A, F, q]_\infty \subset [V_\lambda^E, \Delta_m^s, A, F, p]_\infty.$$

Proof: (i) Let $\epsilon > 0$ and choose δ with $0 < \delta < 1$ such that $f(t) < \epsilon$ for $0 \le t \le \delta$. Write $y_k = f_k^{s-1}(q(\Delta_m^s x_k - L))$ and consider

$$\sum_{k\in I_n} a_{jk}[f_k(y_k)]^{p_k} = \sum_{k\in I_s, y_k\le\delta} a_{jk}[f_k(y_k)]^{p_k} + \sum_{k\in I_n, y_k>\delta} a_{jk}[f_k(y_k)]^{p_k}.$$

Since f_k is continuous, we have

$$\sum_{k\in I_s, y_k\le\delta} a_{jk}[f_k(y_k)]^{p_k} \le \epsilon^H \sum_{k\in I_s, y_k>\delta} a_{jk} \tag{4}$$

and for $y_k > \delta$, we use the fact that

$$y_k < \frac{y_k}{\delta} \le 1 + \frac{y_k}{\delta}.$$

By the definition, we have for $y_k > \delta$,

$$f_k(y_k) < 2f_k(1)\frac{y_k}{\delta}.$$

Hence

$$\frac{1}{\lambda_n}\sum_{k\in I_s, y_k\le\delta} a_{jk}[f_k(y_k)]^{p_k} \le \max(1, (2f_k(1)\delta^{-1})^H)\frac{1}{\lambda_n}\sum_{k\in I_n, y_k\le\delta} a_{jk}[y_k]^{p_k}. \tag{5}$$

From eqn. (4) and (5), we have

$[V_\lambda^E,\ \Delta_m^{s},\ A,\ F,\ q]_\infty \subset [V_\lambda^E,\ \Delta_m^{s},\ A,\ F,\ p]_\infty$.

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