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PARTIAL BALANCED INCOMPLETE BLOCK DESIGNS ARISING FROM SOME MINIMAL DOMINATING SETS OF SRNT GRAPHS

Anwar Alwardi and N. D. Soner

Department of Studies in Mathematics, University of Mysore, Mysore 570 006, India
E-mails*: *a_wardi@hotmail.com, ndsoner@yahoo.co.in

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ABSTRACT

In this paper we prove that the set of all minimal dominating sets which induced from the neighbourhoods $N(v)$ of Clepsch graph with parameters $(16,5,0,2)$ are blocks of partial balanced incomplete block designs, and we generalize this result for minimal dominating sets $N(v)$ for all strongly regular graphs without triangles.

***Keywords**: Clebsch graph, Strongly regular graph without triangles, Minimal dominating set, Partial balanced incomplete block design.*

***Mathematics Subject Classification (2000):05C15**.*

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1. INTRODUCTION:

The strongly regular graphs and its relation with partial balanced incomplete block designs (PBIBD's) were studied in [1], [2]and [3] it was shown that the strongly regular graphs are emerged from PBIBD with two association schemes. In this paper, we consider different way to establish a link between PBIBD and strongly regular graph through the number of some minimal dominating sets, in the folded 5-cube $(16,5,0,2)$ graph and we generalize this result to all SRNT graphs.

We refer to [4], [6] and [8] for the necessary background about strongly regular graphs, dominating set, and PBIBD's.

2. DEFINITIONS AND NOTATIONS:

Definition: 2.1

A strongly regular graph with no triangles (SRNT graph) G with the parameters $(n,k,0,\mu)$ is k - regular graph with n vertices such that for any two adjacent vertices have no common neighbours, and any two non- adjacent vertices have μ common neighbours.

Definition: 2.2

Given v objects a relation satisfying the following conditions is said to be an association scheme with m classes:

(i) any two objects are either first associates, second associates,..., m^{th} associates, the relation of association being symmetric.

(ii) each object has n_i i^{th} associates $(i=1,2)$.

(iii) if two objects α and β are i^{th} associates, then the number of objects common to the j^{th} associates of the first and k^{th} associates of the second is p^i_{jk} and is independent of the pair of i^{th} associates α and β. Also $p^i_{jk} = p^i_{kj}$.

If we have association scheme for the v objects we can define a PBIBD as the following definition.

Definition: 2.3

The PBIB design is arrangement of v objects into b sets of size k where $k<v$ such that:

(i) every object is contained in exactly r blocks.

(ii) each block contains k distinct objects.

(iii) any two objects which are i^{th} associates occur together in exactly λ_i blocks.

The numbers $v,b,r,k,\lambda_i (i=1,2,...m)$ are called the parameters of PBIBD with m association.

Definition: 2.4

Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. A subset D of $V(G)$ is said to be dominating set if every vertex not in D, is adjacent to at least one vertex in D. The domination number $\gamma(G)$ of a graph G is defined to be the minimum cardinalities taken over all dominating sets of G, and the set D in G is said to be minimum dominating set if $|D| = \gamma(\Gamma)$.

****Corresponding author: N.D. Soner***
E-mail: ndsoner@yahoo.co.in

Definition: 2.5
A dominating set D is called minimal dominating set of a graph G if no proper subset of D is dominating set.

3. RESULTS:

Proposition: 3.1
The minimal dominating sets which induced from the neighbourhood of vertices of Clebsch graph in Figure. 1 are blocks of PBIB design.

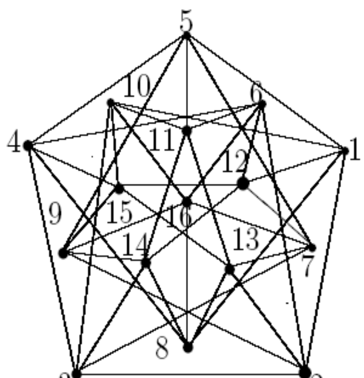

Figure. 1

Proof: Each vertex of the Clebsch graph belongs to exactly 5 minimal dominating sets $N(v)$, and the list of minimal dominating sets given below:

$\{2, , 5, 8, 10, 12\}$ $\{1, 3, 6, 9, 13\}$
'$\{2, 4, 7, 10, 14\}$ $\{3, 5, 6, 8, 15\}$
$\{,1, 4, 7, 9, 11\}$ $\{2, 4, 11, 12, 16\}$
$\{3, 5, 12, 13, 16\}$ $\{1, 4, 13, 14, 16\}$
$\{2, 5, 14, 15, 16\}$ $\{1, 3, 11, 15, 16\}$
$\{5, 6, 10, 13, 14\}$,
$\{1, 6, 7, 14, 15\}$ $\{2, 7, 8, 11, 15\}$
$\{3, 8, 9, 11, 12\}$ $\{4, 9, 10, 12, 13\}$
$\{6, 7, 8, 9, 10\}$

By consider the minimal dominating sets above as blocks, and the two association scheme can be defined as the two elements α and β are 1^{st} associates if they are adjacent vertices in G and they are second associates otherwise. Thus we have

Second Associates	First Associates	Elements
3,4,6,7,9,11,13,14,15,16	**2,8,12,10,5**	**1**
4,5,7,8,10,11,12,14,15,16	**1,3,6,9,13**	**2**
1,5,6,8,9,11,12,13,15,16	**2,4,7,10,14**	**3**
1,2,7,9,10,11,12,13,14,16	**3,5,6,8,15**	**4**
2,3,6,8,10,12,13,14,15,16	**1,4,7,9,11**	**5**
1,3,5,7,8,9,10,13,14,15	**2,4,11,12,16**	**6**
1,2,4,6,8,9,10,11,14,15	**3,5,12,13,16**	**7**
2,3,5,6,7,9,10,11,12,15	**1,4,13,14,16**	**8**
1,3,4,6,7,8,10,11,12,13	**2,5,14,15,16**	**9**
2,4,5,6,7,8,9,12,13,14	**1,3,11,15,16**	**10**
1,2,3,4,7,8,9,12,15,16	**5,6,10,13,14**	**11**
2,3,4,5,8,9,10,11,13,16	**1,6,7,14,15**	**12**
1,3,4,5,6,9,10,12,14,16	**2,7,8,11,15**	**13**
1,2,4,5,6,7,10,13,15,16	**3,8,9,11,12**	**14**
1,2,3,5,6,7,8,11,14,16	**4,9,10,12,13**	**15**
1,2,3,4,5,11,12,13,14,15	**6,7,8,9,10**	**16**

From the above association it is easy to verify that the minimal dominating sets of Clebsch graph which induced from the neighbourhood of the vertices forms PBIBD with the parameters

$v = 16, b = 5, r = 5, k = 5$, λ_1 =0 and λ_2 =2,

$$P_1 = \begin{bmatrix} P_{11}^1 & P_{12}^1 \\ P_{21}^1 & P_{22}^1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 6 \end{bmatrix}, \text{ and}$$

$$P_2 = \begin{bmatrix} P_{11}^2 & P_{12}^2 \\ P_{21}^2 & P_{22}^2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}.$$

Thus the sets of minimal dominating sets $N(v)$ of Clebsch graph are the blocks of PBIB design with the above parameters.

Theorem: 3.2
Let G be SRNT graph with the parameters $(n, k, 0, \mu)$ and let v be any vertex of G then $N(v)$ is minimal dominating set of G.

Proof: Let u be any vertex in $V(G) - N(v)$. Then we have two cases, either $u = v$ or u not adjacent to v.
Case 1. If $u = v$ then u has k adjacent vertices in $N(v)$.
Case 2. If u not adjacent to v, then from the definition of SRNT graph u adjacent to μ vertices in $N(v)$. Hence $N(v)$ is dominating set. Now we want to prove that $N(v)$ is minimal dominating. Suppose $N(v)$ is not dominating set of G then there is vertex $w \in N(v)$ such that

$N(v)-w$ is dominating set, hence w adjacent to at least one vertex in $N(v)$, and this is contradiction with the definition of SRNT graph. Therefore $N(v)$ is minimal dominating set of G

.

Theorem: 3.3

The set of minimal dominating set $N(v)$ in SRNT graph G with the parameters $(n, k, 0, \mu)$, where v any vertex of G are blocks of PBIB design.

Proof: We can defined the PBIB design as following:

Point set is the vertices of G, and the **block set** is the set of minimal dominating sets $N(v)$, where v any vertex in G, and for any points α and β are 1^{st} associates if they are adjacent in G and 2^{nd} associates otherwise and it is clear that each vertex of the SRNT graph belong to exactly k minimal dominating sets, so the parameters of PBIB design are $v^{'}, r^{'}, k^{'}, \lambda_1, \lambda_1$ where $v = b = n$, $r^{'} = k^{'} = k$, $\lambda_1 = 0$, $\lambda_2 = \mu$.

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