International Journal of Mathematical Archive-3(5), 2012, 2130-2134
Available online through www.ijma.info ISSN 2229 – 5046

A STOCHASTIC MODEL FOR IDENTIFYING THE THRESHOLD SHOCK WHICH FOLLOWS THREE PARAMETER GENERALIZED PARETO DISTRIBUTION

A. Loganathan[1], Pandiyan.P*[2], Vinoth. R[2] and Kannadasan. K[2]

[1]Department of Statistics, Manonmaniam Sundranar University, Tirunelveli, Tamil Nadu, India

[2]Department of Statistics, Annamalai University, Annamalai nagar-608 002, Tamil Nadu, India

(Received on: 12-05-12; Revised & Accepted on: 31-05-12)

ABSTRACT

Stochastic process model for approximating the underlying component fails when the total amount of damage exceeds a threshold level. Survival measures of this model are derived, using the theory of cumulative processes. However, most of the behaviour in this stochastic model for the expected time strongly depends on initial conditions. Numerical examples are given to illustrate various aspects of the model considered for the expected time to threshold.

Keywords: *Cumulative damage, Component, Three parameter Generalized Pareto distribution, Threshold.*

INTRODUCTION

The generalized Pareto (GP) distribution was introduced by Pickands (1975) and has since been applied to a number of areas including socio-economic phenomena, physical and biological processes (Saksena & Johnson, 1984), the GP distribution suitable for modelling flood magnitudes exceeding a fixed threshold. Generalized models being more flexible than ordinary single models are usually preferred in analysing most data sets. This has prompted several authors to embark upon investigating the properties and applications of generalized models.

Sexual contacts are the only source of HIV infection and the threshold of any individual is a random variable**.** The inter-arrival times between successive contacts, the sequence of damage and the threshold are mutually independent. If the total damage crosses a threshold level Y which itself is a random variable, the seroconversion occurs and aperson is recognized as an infected. One can see for more detail about the expected time to cross the threshold level of seroconversion period in Esary et al. (1973), Sathiyamoorthi (1980), Pandiyan et al., (2010), Pandiyan and Bhuvana (2012).

NOTATIONS

X_i **:** a continuous random variable denoting the amount of contribution to the threshold due to the HIV transmitted in the i^{th} contact, in other words the damage caused to the immune system in the i^{th} contact, with p.d.f g (.) and c.d.f G (.).

Y **:** a continuous random variable denoting the threshold which follows three parameter generalized Pareto distribution.

U_i **:** a random variable denoting the inter-arrival times between contact with c.d.f. $F_i(.), i = 1,2,3 \dots k.$

$g(.)$: The probability density functions of X_i ; $g^*(.)$: Laplace transform of g (.);

$g_k(.)$:The k- fold convolution of g (.) i.e., p.d.f. of $\sum_{j=1}^{k} X_i$

$f(.)$: p.d.f. of random variable denoting between successive contacts with the corresponding c.d.f. F (.)

$V_k(t)$: Probability of exactly k successive contact;

$F_k(.)$: k-fold convolution of F (.)

$S(.)$: Survival function, i.e., $[T > t]$; $L(t)$: $1 - S(t)$

Corresponding author: Pandiyan.P*[2]
[2]Department of Statistics, Annamalai University, Annamalai nagar-608 002, Tamil Nadu, India

MODEL DESCRIPTION

Let Y be the random which can be expressed the cumulative density function defined as

$$F(x) = 1 - \left[1 - \frac{a\,(x-d)}{b}\right]^{\frac{1}{\alpha}}, \qquad a \neq 0$$

$$put\ \alpha = \ F(x) = 1 - e^{\left(\frac{d-x}{b}\right)} a = 0 \tag{1}$$

Where c is a location parameter, b is a scale parameter, a is a shape parameter

The probability density function (PDF) of the GP distribution is given by

$$f(x) = \frac{1}{b}\left[1 - \frac{a\,(x-d)}{b}\right]^{\frac{1}{\alpha}-1} \qquad a \neq 0$$

The corresponding survival function is given by

$$\bar{H}(x) = e^{\left(\frac{d-x}{b}\right)}$$

$$P(X_i < Y) = \int_0^\infty g^*(x)\,\bar{H}(x)dx$$

$$= \int_0^\infty g^*(x) e^{\left(\frac{d-x}{b}\right)}\,dx\,; \ = \left[g^*\left(\frac{1-d}{b}\right)\right]^k$$

Therefore $S(t) = P[T > t]$is the survival function which gives the probability that the cumulative antigenic diversity will fail only after time

$$t = \sum_{k=0}^{\infty} P\{there\ are\ exactly\ k\ contacts\ in\ (0,t] * P\{the\ total\ cumulative\ threshold\ (0,t]\}$$

It is also known from renewal process that

$$P(\text{exactly k policy decesions in } (0,t]) = F_k(t) - F_{K+1}(t) \text{with } F_0(t) = 1$$

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t)\,P(X_i < Y) \tag{2}$$

$L(t) = 1 - s(t)$, taking laplace transformation of $l(t)$, we get

$$= 1 - \left\{\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[g^*\left(\frac{1-d}{b}\right)\right]^k\right\}$$

$$= 1 - 1 + \left[1 - g^*\left(\frac{1-d}{b}\right)\right]\sum_{k=1}^{\infty} F_k(t)\left[g^*\left(\frac{1-d}{b}\right)\right]^{k-1}$$

$$l^*(s) = \frac{\left[1 - g^*\left(\frac{1-d}{b}\right)\right]f^*(s)}{\left[1 - g^*\left(\frac{1-d}{b}\right)f^*(s)\right]}$$

$$= \frac{\left[1-g^*\left(\frac{1-d}{b}\right)\right]\left(\frac{c}{c+s}\right)}{\left[1-g^*\left(\frac{1-d}{b}\right)\left(\frac{c}{c+s}\right)\right]}$$

$$= \frac{\left[1-g^*\left(\frac{1-d}{b}\right)\right]c}{\left[c+s-g^*\left(\frac{1-d}{b}\right)c\right]} \tag{3}$$

$$E(T) = \frac{-d}{ds} l^*(s) \; given \; s = 0$$

$$= -\frac{c\left[1-g^*\left(\frac{1-d}{b}\right)\right]}{\left[c+s-g^*\left(\frac{1-d}{b}\right)c\right]^2}$$

$$= \frac{c\left[1-g^*\left(\frac{1-d}{b}\right)\right]}{c^2\left[1-g^*\left(\frac{1-d}{b}\right)\right]^2}$$

$$E(T) = \frac{1}{c\left[1-g^*\left(\frac{1-d}{b}\right)\right]} \tag{4}$$

$$E(T^2) = \frac{d^2}{ds^2} l^*(s) \; given \; s = 0$$

$$= \frac{2}{\left[c+s-g^*\left(\frac{1-d}{b}\right)c\right]^2}$$

$$E(T^2) = \frac{2}{c^2\left[1-g^*\left(\frac{1-d}{b}\right)\right]^2} \tag{5}$$

$$g^*(.) \sim Mittag \; Leffler \; Distribution \; \frac{1}{1+\lambda^\alpha}$$

RESULT

$$E(T) = \frac{b+1-d}{c\,[2b+1-d]}$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$= \frac{[b+1-d]^2}{c^2[2b+1-d]^2}$$

CONCLUSION

When *b* is kept fixed the inter-arrival time $'c'$ which follows Mittag leffler distribution, is an increasing case by the process of renewal theory. Therefore, the value of the expected time $E\,(T)$ to cross the threshold is found to be decreasing, in all the cases of the parameter value. *d = 0.5, 1, 1.5, 2.* When the value of the parameter *b* increases, the expected time is found decreasing, this is observed in Figure 1a. The same case is found in Variance V (T) which is observed in Figure 1b.

When *d* is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value. *b = 0.5, 1, 1.5, 2.* When the value of the parameter *d* increases, the expected time is found increasing; this is indicated in Figure 2a. The same case is observed in the threshold of Variance V (T) which is observed in Figure 2b.

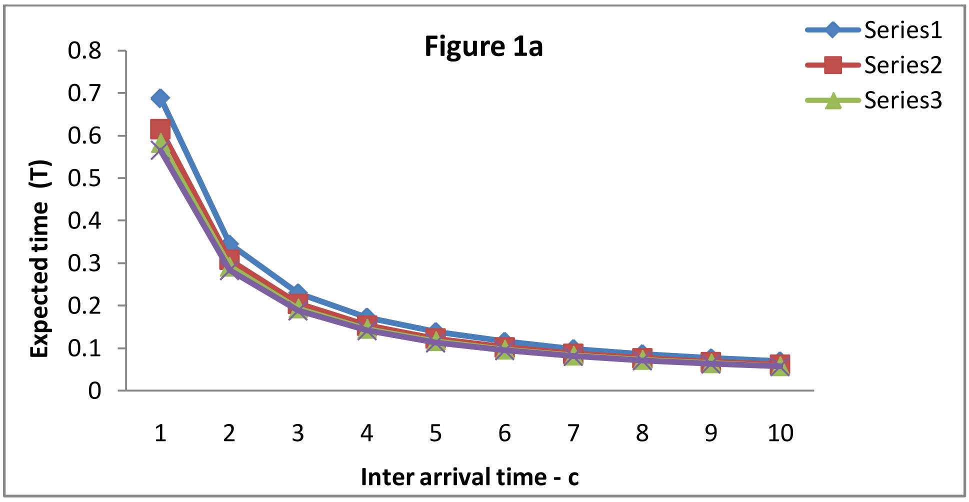


Figure 1a
Expected time (T)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10
Inter arrival time - c
Series1
Series2
Series3

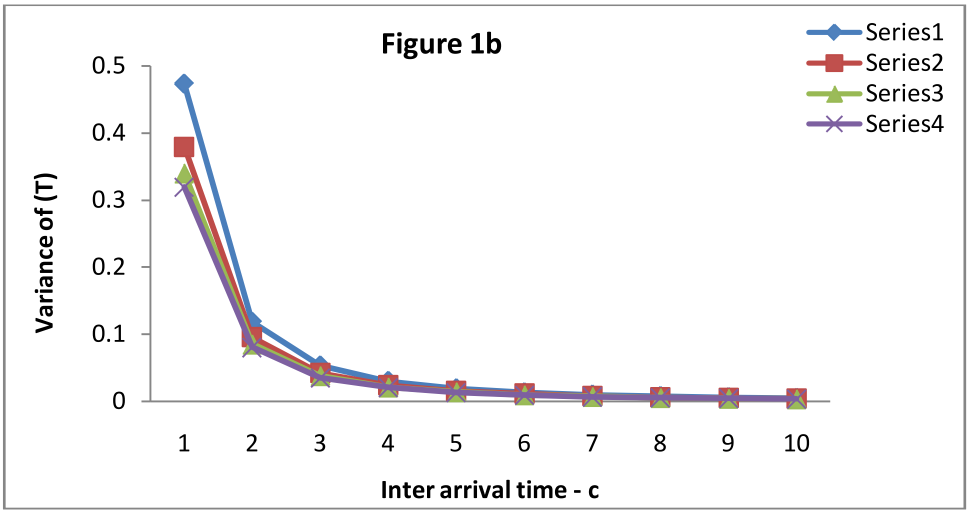


Figure 1b
Variance of (T)
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10
Inter arrival time - c
Series1
Series2
Series3
Series4

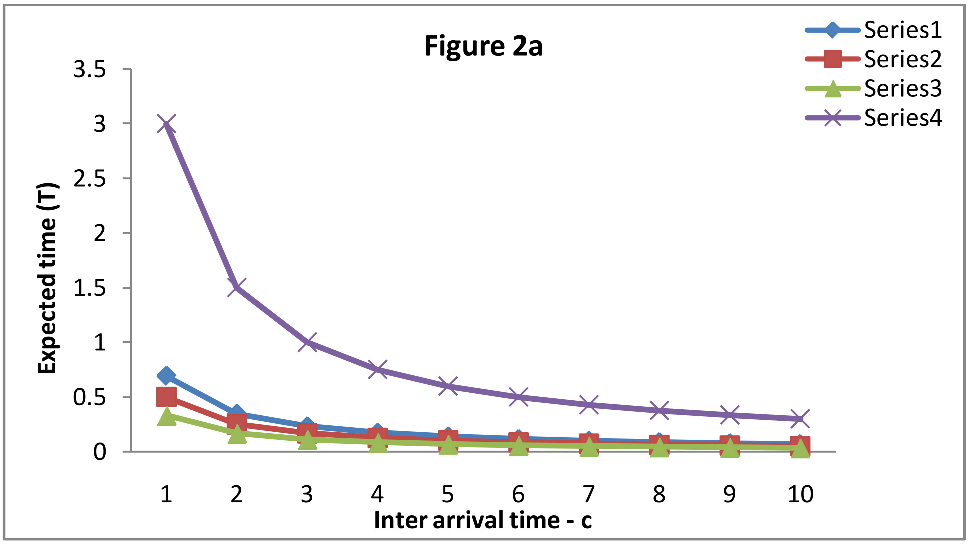
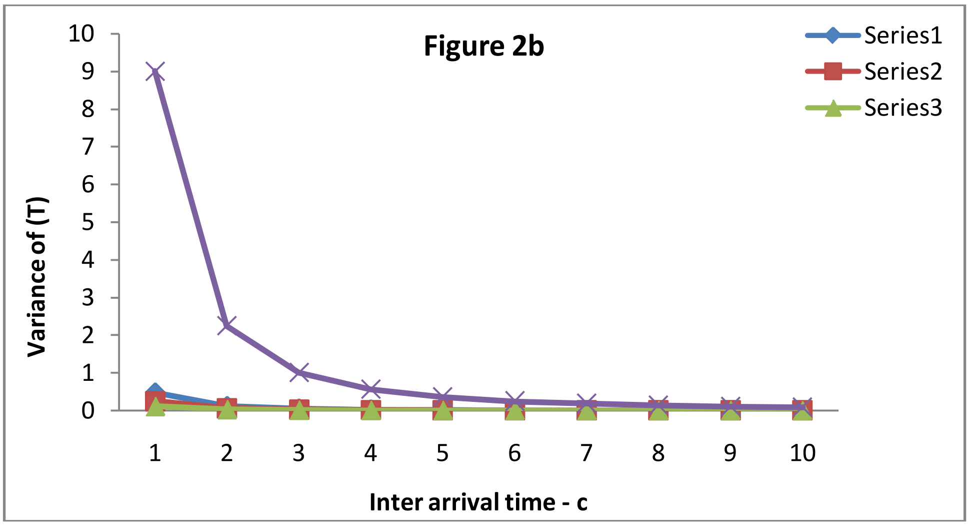


Figure 2a
Expected time (T)
3.5
3
2.5
2
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9 10
Inter arrival time - c
Series1
Series2
Series3
Series4



REFERENCE

[1] Bhuvana, V.S andPandiyan, P. (2012). Reduce the Spread of HIV through Estimating the Mean and Variance using Two Components, Advances in Applied Mathematical Biosciences. Vol. 3, No 1, pp. 25-30.

[2] Esary, J.D., A.W. Marshall and F. Proschan. (1973). Shock models and wear processes, Ann. Probability, 1(4), 627-649.

[3] Pandiyan, P., R. Agasthiya., R.M. Palanivel., K. Kannadasan and R. Vinoth (2010). Expected Time to Attain the Threshold Level Using Multisource of HIV Transmission – Shock Model Approach, Journal of Pharm Tech Research. No.2, pp 1088-1096.

[4] Pickands, J. (1975). Statistical inference using extreme order statistics. Ann. Statist. Vol. 3, 119-131.

[5] Saksena, S. K. and Johnson, A. M. (1984). Best unbiased estimators for the parameters of a two-parameterPareto distribution. Biometrika Vol. 31, 77-83.

[6] Sathiyamoorthi, R. (1980). Cumulative Damage model with Correlated Inter arrival Time of Shocks. IEEE Transactions on Reliability, R-29, No.3.

Source of support: Nil, Conflict of interest: None Declared

