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# On pgrα- closed sets in topological spaces

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***ABSTRACT***

*In this paper a new class of sets called pgrα- closed sets is introduced and its properties are studied. Further the notion of pgrα-$T_{1/2}$ space and pgrα-continuity are introduced.*

***Mathematics subject classification:*** *54C05.*

***Key words:*** *pgrα- closed set, pgrα- open set, pgrα- continous function, pgrα- $T_{1/2}$ spaces.*

## 1. Introduction

N. Levine [12] introduced generalized closed sets in general topology as a generalization of closed sets and a class of topological spaces called $T_{1/2}$ spaces. This concept was found to be useful and many results in general topology were improved. Since the advent of these notions, several research papers with interesting results in different respects came to existence ([2, 3, 4, 6, 7, 8, 9, 16, 18, 22, 23, 26]). In this paper, we define and study the properties of pgrα- closed sets which is the weaker form of the above mentioned generalization. Moreover in this paper we define pgrα- $T_{1/2}$ spaces and pgrα- continuity and study their properties.

## 2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) represent nonempty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no chance of confusion.

Let us recall the following definition which we shall require later.

**Definition: 2.1** A subset A of a space (X, τ) is called
(1) a pre open set[17] if A⊂ int(cl(A)) and a pre closed set if cl(int(A)) ⊂ A
(2) a semi- open set [11] if A⊂ cl(int(A)) and semi – closed set if int(cl(A)) ⊂ A
(3) a α- open set [20] if A⊂ int(cl(int(A))) and a α- closed set if cl(int(cl(A))) ⊂A
(4) a semi- pre open set [1] if A⊂ cl(int(cl(A))) and a semi- pre closed set if int(cl(int(A))) ⊂ A
(5) a regular open set if A= int(cl(A)) and a regular closed set if A = cl(int(A))
(6) a regular α- open set (briefly rα- open) [25] if there is a regular open set U such that U ⊂ A⊂ αcl(A)

The union of all pre open sets of X contained in A is called pre-interior of A and is denoted by pint(A). Also the intersection of all pre closed subsets of X containing A is called pre- closure of A and is denoted by pcl(A). Note that pcl(A) = A∪ cl(int(A)) and pint(A)= A∩int(cl(A)). The family of all α- open (resp. semi open, pre open, α- closed, pre closed, regular open, regular α-open) subsets of a space X is denoted by αO(X) (resp. SO(X), PO(X), αC(X),PC(X), RO(X), RαO(X)).

**Definition: 2.2** A subset A of a space (X, τ) is called
(1) a generalized closed set (briefly g- closed) [11] if cl(A)⊂ U whenever A ⊂ U and U is open.
(2) a semi – generalized closed set ( briefly sg- closed) [6] if scl(A) ⊆ U whenever A ⊆ U and U is semi open in X.
(3) a generalized α – closed set (briefly gα- closed) [13] if αcl(A) ⊆ U whenever A⊆ U and U is α-open in X.
(4) a α- generalized closed set (briefly αg- closed) [14] if αcl(A) ⊆ U whenever A⊆ U and U is open in X.
(5) a regular generalized closed set (briefly rg-closed) [22] if cl(A) ⊂ U whenever A ⊂ U and U is regular open.
(6) a generalized pre closed set (briefly gp- closed) [15] if pcl(A)⊂ U whenever A ⊂ U and U is open.

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(7) a generalized pre regular closed set (briefly gpr- closed) [9] if pcl(A) ⊆U whenever A⊆U and U is regular open in X.
(8) a weakly closed set (briefly w-closed) [24] if cl(A) ⊆ U whenever A ⊆ U and U is semi open in X.
(9) a weakly generalized closed set ( briefly wg- closed) [19] if cl(int(A)) ⊆ U whenever A ⊆ U and U is open in X.
(10) a regular weakly closed set (briefly rw- closed)[5] if cl(A) ⊆U whenever A⊆U and U is regular open in X.
(11) a regular weakly generalized closed set (briefly rwg- closed)[19] if cl(int(A))⊆ U whenever A⊆U and U is regular open in X.
(12) a regular generalized α- closed set (briefly rgα- closed) [24] if αcl(A)⊆ U whenever A ⊆ U and U is regular α-open in X.

The complement of the above mentioned closed sets are their respective open sets.

## 3. pgrα-closed set

**Definition: 3.1** A subset A of a space X is called pgrα – closed set if pcl(A) ⊂ U whenever A⊂ U and U is regular α-open. By PGRαC(τ), we mean the family of all pgrα- closed subsets of the space (X, τ).

**Theorem: 3.2**
(1) Every rgα- closed set in X is pgrα- closed set in X.
(2) Every pgrα- closed set in X is gpr closed set in X.
(3) Every w- closed set in X is pgrα- closed set in X.
(4) Every rw- closed set in X is pgrα- closed set in X.
(5) Every closed set in X is pgrα- closed set in X.
(6) Every regular closed set in X is pgrα-closed in X.
(7) Every pre closed set in X is pgrα- closed in X.
(8) Every pgrα- closed set in X is rwg closed set in X.

**Proof:** Straight forward. Converse of the above need not be true as in the following examples.

**Example: 3.3** Let X= **{**a, b, c, d, e} and τ= {∅, {a, b}, {c, d}, {a, b, c, d}, X}. Let A = {a} then A is pgrα- closed set. But A is not rgα- closed set, w-closed set, rw-closed set, closed set, regular closed set, pre closed set.

**Example: 3.4** Every rwg closed and gpr closed set in X need not be pgrα- closed set.

Let X= {a, b, c, d, e} and τ = { X,∅,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}}. The only regular open set containing {a, b} is set X. Now cl(int({a, b})={a, b, c}⊂ X. This implies {a, b} is rwg closed and gpr closed set in X. But {a, b, e} is regular α- open set and {a, b}⊂ {a,b,e}. Then pcl{a, b}= {a, b, c}⊈ {a, b, e}. Therefore {a, b} is not pgrα –closed set.

**Remark: 3.5** The above discussions are summarized in the following diagram.


**Remark: 3.6** rg- closed and pgrα- closed sets are independent concept.

**Example: 3.7**
(i) Let X={a, b, c, d, e} and τ= {X,∅,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}}.Consider A = {a,b}, the only regular set which contain A is X. Then cl(A) = {a, b, c}⊂ X, which implies that A is rg- closed set. Since {a, b, e} is a regular α-open set containing {a, b}. But pcl(A) = {a, b, c}. Therefore A is not pgrα- closed set.

(ii) Let X= {a, b, c, d, e} and τ= {X,∅ ,{a},{d},{e}{a,d},{a,e}{d,e},{a,d,e}}. Consider {d}⊂ {b, c, d}, then pcl{d}= {b, c, d}⊆ {b, c, d}. Therefore {d} is pgrα- closed set.

But only {d}⊂ {a, d}, cl{d}= {b, c, d}⊈{a,d}. this implies {d} is not rg- closed set.

**Remark: 3.8** Intersection of two pgrα- closed sets need not be pgrα- closed set.

**Example: 3.9** Let X= {a, b, c, d, e} and d τ= {X,∅,{a},{d},{e}{a,d},{a,e},{d,e},{a,d,e}}.

 

If A = {a, b, c} and B= {a, d, e}. Consider A⊂{a, b, c, d} then pcl(A)= {a, b, c, d}⊆ {a, b, c, d}. This implies that {a, b, c}is pgrα-closed set. The only regular α- open set containing B is X then pcl(B) = X. This implies that {a, d, e}is pgrα-closed set. But A∩B= {a}.Consider {a}⊂{a,b}, then pcl{a}= {a,b,c}⊄ {a,b}. Therefore A∩ B is not pgrα- closed set.

**Remark: 3.10** Union of two pgrα- closed sets need not be pgrα- closed.

**Example: 3.11** Let X= {a, b, c, d, e}and τ={X, ∅,{a},{d},{e}{a,d},{a,e}{d,e},{a,d,e}}.

Let A= {a, b}, B= {b, e} are pgrα-closed set. But A∪B = {a, b, e}. Consider {a, b, e}⊆ {a, b, e}then pcl({a, b, e})={a, b, c, e}⊄ {a, b, e}. This implies that A∪B is not pgrα- closed.

**Definition: 3.12** [9] Let (X, τ) be a topological space , A⊂X and x∈ X, x is said to be a pre limit point of A if and only if every pre open set containing x contains a point of A different from x.

**Definition: 3.13** [9] Let (X, τ) be a topological space and A ⊂ X .The set of all pre limit points of A is said to be the pre- derived set of A and is denoted by $D_p[A]$.

**Theorem: 3.14** Let A and B be pgrα- closed sets in (X, τ) such that D[A]⊂$D_p$[A] and D[B]⊂ $D_p$[B], then A ∪ B is pgrα- closed set.

**Proof:** For any set E ⊂(X, τ), $D_p$[E]⊂D[E]. Therefore $D_p$[A]= D[A] and $D_p$[B]=D[B]. That is cl(A)= pcl(A) and cl(B) = pcl(B). Let A ∪B ⊂ U, where U is regular $\alpha$-open then A ⊂ U and B⊂U. Since A and B are pgrα-closed then pcl(A) ⊂U and pcl(B) ⊂U. Now cl(A∪B) = cl(A) ∪ cl(B)= pcl(A) ∪ pcl(B) ⊂U.

But pcl(A∪B) ⊂ cl(A ∪B).Therefore pcl(A ∪ B)⊂ cl(A ∪B) ⊂ U. Hence A∪B is pgrα- closed set.

**Theorem 3.15** Let A be pgrα- closed in (X, τ). Then pcl(A)- A does not contain any nonempty regular α- open set in X.

**Proof:** Suppose that A is pgrα- closed set in X. Let U be a nonempty regular α-open set such that U⊂ pcl(A) – A. Then U⊂X- A implies A ⊂ X-U. Since A is pgrα- closed and X-U is regular α-open then pcl(A)⊂ X-U. That is U ⊂ X-pcl(A). Hence U⊂ pcl(A)∩(X-pcl(A))= ∅. Hence pcl(A)- A does not contain any nonempty regular α-open set in X.

**Corollary 3.16** If a subset A of X is pgrα- closed in X then pcl(A)-A does not contain any non empty regular closed set in X, but not conversely.

**Proof:** Let F be a regular closed set such that F ⊂ pcl(A)- A then F⊂X-A. Therefore A⊂X-F. Since A is pgrα-closed and X- F is regular open then pcl(A)⊂ X-F. That is F⊂ X-pcl(A). Hence F⊂pcl(A) ∩(X- pcl(A)) = ∅.

This shows that F=∅.

Converse implication does not hold.

**Example: 3.17** If pcl(A)- A contains no nonempty regular closed set in X, then A need not be pgrα- closed set in X.

Let A= {a, b}. Then pcl(A)= {a, b, c}. pcl(A)-A= {c} which does not contain any nonempty regular closed set.

Also A is not pgrα- closed.

**Theorem: 3.18** For an element x∈ X, the set X-{x} is pgrα- closed or regular α-open.

**Proof:** Suppose X- {x}is not regular α- open set then X is the only regular α- open set containing X- {x}. This implies pcl(X- {x}) ⊂ X. Hence X- {x}is pgrα- closed.

**Theorem: 3.19** If A is regular open and pgrα- closed then A is pre closed and hence pre- clopen.

**Proof:** Suppose A is regular open and pgrα- closed. As every regular open is regular α-open and A⊆A.

We have pcl(A) ⊂ A . Also A⊂ pcl(A).Therefore pcl(A)= A, then A is pre closed . Since every pre closed (regular) open set is (regular) closed. Hence A is pre- clopen

**Theorem: 3.20** If A is pgrα- closed subset of X such that A⊂ B⊂pcl(A ), then B is pgrα- closed.

 

**Proof:** Let B⊂U, where U is regular α-open. Since A⊂B⊂U and A is pgrα- closed, pcl(A)⊂U. Now B⊂pcl(A), this implies pcl(B)⊂pcl(pcl(A)) = pcl(A) ⊂U. That is pcl(B)⊂U. Hence B is pgrα- closed set in X.

**Theorem: 3.21** Let A be pgrα- closed in X then A is pre closed if and only if pcl(A)- A is a regular α- open.

**Proof:** Suppose A is pre closed in X then pcl(A)= A and so pcl(A) –A = ∅, which is regular α-open in X.

Conversely, suppose pcl(A)-A is a regular α- open in X. Since A is pgrα-closed and by Theorem 3.15, pcl(A)-A does not contain any nonempty regular α-open set in X. Then pcl(A)-A =∅. Therefore pcl(A)= A. Hence A is pre closed.

**Theorem: 3.22** If A is regular open and rg- closed, then A is pgrα- closed set in X.

**Proof:** Let U be any regular α-open set in X such that A⊂U. Since A is regular open and rg-closed then cl(A)⊆A⊆U whenever A⊂U and U is regular α-open. Therefore pcl(A)⊂cl(A)⊂U. Hence A is pgrα-closed set in X.

**Theorem: 3.23** If a subset A of topological space X is both regular α-open and pgrα- closed, then it is pre closed.

**Proof:** Suppose a subset A of topological space X is both regular α- open and pgrα- closed. Now A⊆A then pcl(A)⊂A. Hence A is pre closed.

**Theorem: 3.24** If A is both open and g-closed set in X then it is pgrα-closed set in X.

**Proof:** Let A be an open and g-closed set in X. Let A⊂U and U be regular α-open set in X. By hypothesis pcl(A)⊂cl(A)⊂A⊂U, then A is pgrα- closed in X.

**Remark: 3.25** If A is both open and pgrα- closed in X then A need not be g-closed**.**

**Example: 3.26** Let X= {a, b, c, d, e} and τ= {∅, X,{a},{d},{e},{a, d},{a, e},{d, e},{a, d, e}}. Consider {a, d, e}which is both open and pgrα-closed in X. But {a, d, e}⊆{a,d,e}then cl({a,d,e})= X ⊈{a,d,e}. Therefore {a, d, e} is not g-closed.

**Theorem: 3.27** In a topological space X, if RαO(X) = {X,∅}, then every subset of X is a pgrα-closed set.

**Proof:** Let X be a topological space and RαO(X) = {X,∅}. Let A be any subset of X. Suppose A =∅, then ∅ is pgrα-closed set in X. Suppose A≠ ∅, then X is the only regular α-open set containing A and so pcl(A)⊂X. Hence A is pgrα-closed set in X.

Converse of the theorem need not be true.

**Example: 3.28** Let X= {a, b, c, d} and τ= {∅, X,{a,b},{c,d}}.Every subset of (X,τ) is pgrα- closed set in X. But RαO(X, τ)= {∅,X,{a,b},{c,d}}

**4. pgrα- open set**

We introduce and study pgrα-open sets in topological space and obtain some of their properties.

**Definition: 4.1** A subset A of X is called pgrα- open in X if $A^c$ is pgrα- closed in X.

We denote the family of pgrα- open sets in X by PGRαO(X).

**Theorem: 4.2** If a subset A of a space X is w- open then it is pgrα- open but not conversely.

**Proof:** Let A be a w-open set in a space X then $A^c$ is w-closed set. By theorem 3.2 $A^c$ is pgrα- closed set. That is A is pgrα- open set in X.

The converse of the above theorem need not be true.

**Corollary: 4.3**Every open set is pgrα- open set but not conversely.

**Corollary: 4.4** Every regular open set is pgrα-open set but not conversely.

**Theorem: 4.5** Assume if PO(X,τ) is closed under finite intersection, A and B are pgrα- open sets in a space X then A∩B is also pgrα- open set in X.

 

**Proof:** Let X-(A∩B)= (X-A)∪(X-B)⊂F, where F is regular α-open. Then X-A⊂F and X-B⊂F. Since A and B are pgrα-open then pcl(X-A)⊂F and pcl(X-B)⊂F. By hypothesis pcl((X-A)∪(X-B))⊂pcl(X-A)∪pcl(X-B)⊂F.

That is pcl(X-(A∩B))⊂F. This shows that A∩B is pgrα- open.

**Theorem 4.6** If a set A is pgrα- open in a space X then G=X, whenever G is regular α- open and int(A)∪$A^c$⊂G.

**Proof:** Suppose that A is pgrα- open in X. Let G be regular α- open and int(A)∪A⊂G then $G^c$⊂cl($A^c$)-$A^c$. Now $G^c$ is also regular α- open and $A^c$ is pgrα- closed. By theorem 3.15, it follows that $G^c$ =∅. Hence G= X.

The converse of the above theorem is not true.

**Example: 4.7** Let X={a, b, c, d) and τ= {X, ∅,{a},{b},{a,b},{b,c},{a,b,c},{a,b,d}}.Then PGRαO(X) = {∅,X,{a},{b,c},{a,d},{c,d},{a,c,d},{b,c,d}}.Take A={a,b,c}, then A is not pgrα- open. However int(A)∪$A^c$= {a, b, c, d} ⊂G. This implies G= X. But A is not pgrα-open.

**Theorem: 4.8** Every singleton point set in a space is either pgrα- open or rα-open.

**Proof:** Follows from Theorem 3.18.

**Theorem: 4.9** A⊂X is pgrα- open if and only if F⊂pint(A), whenever F is regular α- closed and F⊂A.

**Proof:** Let A be pgrα-open. Let F be regular α- closed and F⊂A then X- A ⊂X-F, where X- F is regular α- open. Since A is pgrα- open implies pcl(X-A) ⊂X-F. This implies X- pint(A)⊂X-F. Hence F⊂pintA.

Conversely, suppose F is regular α- closed and F⊂A. Let X-A⊂U, where U is regular α-open then X-U⊂A and X-U is regular α- closed. By hypothesis, X- U⊂ pint(A) , this implies pcl(X-A) ⊂U. Then X-A is pgrα- closed. Hence A is pgrα- open.

**Theorem: 4.10** If pint(A) ⊂B⊂A and A is pgrα- open then B is pgrα- open.

**Proof:** pint(A) ⊂B⊂A implies (X-A)⊂(X-B)⊂(X-pint(A)). That is (X-A) ⊂(X-B)⊂pcl(X-A). Since (X-A) is pgrα-closed then by Theorem3.20 , (X-B)is pgrα- closed and B is pgrα- open.

**Remark: 4.11** For any A⊂X, pint(pcl(A)-A) =∅.

**Theorem: 4.12** If A⊂X is pgrα- closed then pcl(A)- A is pgrα- open.

**Proof:** Let A be pgrα- closed. Let F be a regular α- closed set such that F⊂(pcl(A)-A). By theorem 3.15, F=∅. So F⊂pint(pcl(A)-A).This shows (pcl(A)-A) is pgrα- open.

The converse implication does not hold.

**Example: 4.13** Let X= {a, b, c} and τ= {X, ∅{a},{b},{a,b}}. Then PGRαO(X) = {X,∅, {b,c},{a,c},{c},{a},{b}}.

Consider A= {c} then pcl(A)= {c}, (pcl(A)-A)= ∅, which is pgrα-open in (X,τ). But A={c}is not pgrα-closed in (X, τ).

**Definition: 4.14** A space (X, τ) is called pgrα- $T_{1/2}$ space if every pgrα-closed set is pre closed.

**Remark: 4.15** Any set with indiscrete topology is an example for a pgrα- $T_{1/2}$ space. The notion pgrα- $T_{1/2}$ and $T_{1/2}$ are independent of each other.

**Example: 4.16** Let X= {a, b, c},τ= {∅,X,{a},{b},{a,b}}. Then (X, τ) is a $T_{1/2}$ space but it is not a pgrα-$T_{1/2}$ space.

**Example: 4.17** Let X= {a, b, c}, τ= {∅, X, {c}, {a, b}}. Then (X, τ) is a pgrα- $T_{1/2}$ space but it is not $T_{1/2}$ space.

**Theorem: 4.18** For a space (X, τ) the following are equivalent.
(1) X is a pgrα- $T_{1/2}$ space.
(2) Every singleton is either regular closed or pre- open.

**Proof:** Suppose {x} is not a regular α-closed subset for some x∈X. Then X- {x} is not regular α- open and hence X is the only regular α- open set containing X- {x}. Therefore X- {x} is pgrα- closed. Since (X, τ) is pgrα- $T_{1/2}$ , X-{x} is pre closed and then {x}is pre- open. Hence (1) implies (2).

 

Let A be a pgrα- closed subset of (X, τ) and x∈pcl(A). We will show that x∈A. If {x} is regular α-closed and x∉A, then {x}∈(pcl(A)-A). Thus (pcl(A)-A) contains a nonempty regular α- closed set {x}, a contradiction to Theorem 3.15. So x∈A.

If{x} is pre- open, since x∈pcl(A) then for every pre -open set U containing x. We have U∩A≠ ∅.But {x} is pre open then {x}∩A≠ ∅. Hence x∈A, so in both cases we have x∈A. Therefore A is pre- closed set. Hence (2) implies (1).

**5. pgrα- continous and pgrα- irresolute functions**

**Definition: 5.1** A map f :(X, τ)→ (Y, σ) is called pgrα- continous if $f^{-1}$ (V) is pgrα- closed in (X, τ) for every closed set V of (Y, σ).

**Example: 5.2** Let X= {a, b}, τ= {X,∅, {a},{b}} and Y= {a, b, c},σ= {Y, ∅,{a}}. Define f:(X,τ) →(Y,σ), by f(a) = a, and f(b)=c. Since every subset of (X, τ) is pgrα- closed. Hence f is pgrα- continous.

**Remark: 5.3** The composition of two pgrα- continous functions need not be pgrα- continous.

**Example: 5.4** Let X= {a, b, c},τ= {∅,X,{a},{b},{a,b}},σ= {∅,X,{c},{b,c}} and η={∅,X,{b,c}}. Define f:(X,τ)→(X,σ) by f(a)=b, f(b)=c, f(c)=a ,and define g: (X, σ)→(X,η) by g(a)=b,g(b)=a,g(c)=c then f and g are pgrα- continous. {a} is closed in (X, η), then $(g∘f)^{-1}$({a})= $f^{-1}$($g^{-1}$({a}))= $f^{-1}$({b})= {a}, which is not pgrα- closed. Hence g∘f is not pgrα- continous.

**Definition: 5.5** A function f: (X, τ)→(Y,σ) is called pgrα- irresolute if $f^{-1}$(V) is pgrα- closed in (X, τ) for every pgrα- closed set V of (Y, σ).

**Example: 5.6** Let X= {a, b, c}, τ= {∅, X, {a, b}}and σ= {∅,X,{a,b},{c}}. Define f(X, τ)→(X,σ) by f(a)=c, f(c)=a, f(b)= b.
The inverse image of every pgrα- closed set is pgrα- closed under f. Hence f is pgrα- irresolute.

**Remark: 5.7** Every pgrα- irresolute function is pgrα- continous.

**Example: 5.8** Converse of the above need not be true.

Consider X={a, b, c},τ= {∅,X,{a},{b},{a,b}},σ= {∅,X,{c},{b,c}}. Define f:(X, τ)→(X,σ) by f(a)= b, f(b)= c,f(c)=a. Here inverse image of all pgrα- closed sets are not pgrα-closed set.

**Definition: 5.9** A map f:X→Y is called pre- regular α- open if f(U) is regular α- open in Y for every regular α- open set U in X.

**Definition: 5.10** A map f: X→Y is called pre- irresolute if $f^{-1}$(V) is pre- closed in (X, τ) for every pre closed set V of (Y, σ)

**Theorem: 5.11** If f: X→Y is a pre- irresolute, pre- regular α- open and bijective then f is pgrα- irresolute.

**Proof:** Let F be pgrα- closed in Y. Let $f^{-1}$(F) ⊂U where U is regular α- open in X.

Since f is pre-regular α- open and F⊂f(U) ,we have pcl(F) ·⊂f(U). This implies $f^{-1}$(pcl(F))⊂U.

Since f is pre- irresolute, $f^{-1}$(pcl(F)) is pre- closed in X, then pcl($f^{-1}$(pcl(F))= $f^{-1}$ (pcl(F))⊂ U.

This implies $f^{-1}$(F) is pgrα- closed in X. Therefore f is pgrα- irresolute.

**Theorem: 5.12** Let f:(X, τ)→(Y,σ) and g:(Y,σ)→(Z,η) be any two functions , then
(i) g∘f is pgrα- continous if g is continous and f is pgrα- continous.
(ii) g∘f is pgrα-irresolute, if g is pgrα-irresolute and f is pgrα- irresolute.
(iii) g∘f is pgrα- continous , if g is pgrα- continous and f is pgrα- irresolute.

**Proof:** (i) Let V be closed in (Z, η). Then $g^{-1}$(V) is closed in (Y, σ), since g is continuous.

Since f is pgrα- continous, $f^{-1}$($g^{-1}$(V)) is pgrα- closed in (X,τ). That is $(g∘f)^{-1}$ (V) is pgrα- closed in (X, τ). Hence g∘f is pgrα- continous.

(ii) Let V be pgrα- closed in (Z, η). Since g is pgrα- irresolute, $g^{-1}(V)$ is pgrα-closed in (Y, σ).Since f is pgrα- irresolute, then $f^{-1}(g^{-1}(V))$ is pgrα- closed in (X, τ). That is $(g∘f)^{-1}(V)$ is pgrα- closed in (X, τ). Hence g∘f is pgrα- irresolute.

(iii)Let V be closed in (Z, η). Since g is pgrα- continous then $g^{-1}(V)$ is pgrα- closed in (Y, σ).Since f is pgrα- irresolute then $f^{-1}(g^{-1}(V))$ is pgrα- closed in (X, τ). That is $(g∘f)^{-1}$ (V) is pgrα- closed in (X, τ). Hence g∘f is pgrα- continous.

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