International Journal of Mathematical Archive-3(5), 2012, 2083-2088
Available online through www.ijma.info ISSN 2229 – 5046

(1, 2)*-$M_{\delta\pi}$-Closed Sets in Bitopological Spaces

K. Mohana* & I. Arockiarani**

****Lecturer, Department of Quantitative Techniques, T. John College, Bangalore-560 083, India***
*****Associate Professor Department of Mathematics, Nirmala College for Women, Coimbatore (TN), India***

(Received on: 07-02-12; Revised & Accepted on: 11-06-12)

ABSTRACT

In this paper, a new class of sets, namely (1, 2)-$M_{\delta\pi}$-closed sets is introduced in bitopological spaces. We prove that this class lies between the class of (1, 2)*- δ-closed sets and the class of (1, 2)*- δg-closed sets. Also we discuss some basic properties and applications of (1, 2)*-$M_{\delta\pi}$-closed sets, which defines a new class of space namely (1, 2)*-$T_{\delta\pi g}$-space.*

Keywords: *(1, 2)*- πg-closed set, (1, 2)*- δ-closed set, (1, 2)*-$M_{\delta\pi}$-closed set, (1, 2)*- $T_{\delta\pi g}$-space.*

2000 mathematics subject classification: *54E55, 54C55.*

1. INTRODUCTION

Njastad[9], Velicko[10] introduced the concept of α-open sets and δ-closed sets respectively. Dontchev and Ganster [3] studied δ-generalized closed set in topological spaces. Levine[5] introduced generalization of closed sets and discussed their properties. Also M. E. Abd El-Monsef [1] et al investigated α-closed sets in topological spaces. Thivagar et al [5] have developed the concepts of (1, 2)*-semi-open sets, (1, 2)*-α-open sets, (1, 2)*-generalised closed sets, (1, 2)*-semi generalised closed sets and (1, 2)*-α-generalized closed sets in bitopological spaces. Recently Arockiarani and Mohana [2] discussed (1, 2)*-πgα-closed sets in bitopological spaces. The purpose of the present paper is to define a new class of closed sets called (1, 2)*- $M_{\delta\pi}$-closed sets and we discuss some basic properties of (1, 2)*-$M_{\delta\pi}$-closed sets in bitopological spaces. Applying these sets, we obtain a new space called (1, 2)*- $T_{\delta\pi g}$-space.

2. PRELIMINARIES

Throughout this paper by a space X and Y represent non-empty bitopological spaces on which no separation axioms are assumed, unless otherwise mentioned. We recall the following definitions and results which are useful in the sequel.

Definition: 2.1 [5] A subset S of a bitopological space X is said to be $\tau_{1,2}$-open if S=A∪B where $A \in \tau_1$ and $B \in \tau_2$. A subset S of X is said to be (i) $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open. (ii) $\tau_{1,2}$-clopen if S is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed.

Definition: 2.2 [5] Let S be a subset of the bitopological space X. Then the $\tau_{1,2}$-interior of S denoted by $\tau_{1,2}$-int(S) is defined by ∪{G: G⊆S and G is $\tau_{1,2}$-open} and the $\tau_{1,2}$-closure of S denoted by $\tau_{1,2}$-cl(S) is defined by ∩{F: S⊆F and F is $\tau_{1,2}$- closed}.

Definition: 2.3 A subset A of a bitopological space X is called

(i) (1, 2)*-regular open [5] if A= $\tau_{1,2}$-int ($\tau_{1,2}$-cl (A)).
(ii) (1, 2)*-α -open [5] if A ⊆ $\tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A))).
(iii) (1, 2)*-semi-open [5] if A ⊆ $\tau_{1,2}$-cl ($\tau_{1,2}$-int(A))

The complement of the sets mentioned from (i) to (iii) are called their respective closed sets.

Definition: 2. 4 [2] Let S be a subset of the bitopological space X. Then

(i) The (1, 2)*-α-interior of S denoted by (1, 2)*-α-int(S) is defined by ∪{G: G⊆S and G is (1, 2)*-α-open}.
(ii) The (1, 2)*-α-closure of S denoted by (1, 2)*-α-cl (S) is defined by ∩{F: S⊆F and F is (1, 2)*-α-closed}.

Corresponding author: K. Mohana*
****Department of Quantitative Techniques, T. John College, Bangalore-560 083, India***

Definition: 2. 5 A subset A of a bitopological space X is called

(i) (1, 2)*- $\pi g\alpha$ - closed [2] if (1, 2)*-αcl (A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$- π - open.
(ii) (1, 2)*-πg- closed [7] if $\tau_{1,2}$-cl (A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$- π-open.
(iii) (1, 2)*- strongly- $\pi g\alpha$ - closed [8] if (1, 2)*-αcl (A) $\subseteq$ U whenever $A \subseteq U$ and U is (1, 2)*-πg-open.
(iv) (1, 2)*-g- closed [5] if $\tau_{1,2}$-cl (A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$-open.
(v) (1, 2)*- gα- closed [5] if (1, 2)*-αcl (A) $\subseteq$ A whenever $A \subseteq U$ and U is (1, 2)*-α-open.
(vi) (1, 2)*- αg- closed [5] if (1, 2)*-αcl (A) $\subseteq$ A whenever $A \subseteq U$ and U is $\tau_{1,2}$-open.
(vii) (1, 2)*- gs- closed [5] if (1, 2)*-scl (A) $\subseteq$ A whenever $A \subseteq U$ and U is $\tau_{1,2}$-open.
(viii) (1, 2)*- $\hat{g}$ - closed [4] if $\tau_{1,2}$-cl (A) $\subseteq$ A whenever $A \subseteq U$ and U is (1, 2)*-semi-open. and the complement of the sets mentioned from (i) to (viii) are called their respective open sets.

Definition: 2.6 [5] A space X is called (1, 2)*-$T_{1/2}$-space if every (1, 2)*-g-closed set in it is an $\tau_{1,2}$-closed.

3. (1, 2)*-$M_{\delta\pi}$-Closed sets

Definition: 3.1 The (1, 2)*-δ-interior of a subset A of X is the union of all (1, 2)*-regular open set of X contained in A and is denoted by (1, 2)*-δ-int (A). The subset A is called (1, 2)*-δ-open if A = (1, 2)*-δ-int (A), (i. e), a set is (1, 2)*-δ-open if it is the union of (1, 2)*-regular open sets. The complement of a (1, 2)*-δ-open set is called (1, 2)*-δ-closed. Alternatively, a sub set A in X is called (1, 2)*-δ-closed if A = (1, 2)*-δ-cl (A), where (1, 2)*-δ-cl (A) = {$x \in X$: $\tau_{1,2}$-int ($\tau_{1,2}$-cl (U)) ∩ A $\neq \phi$, U $\in \tau_{1,2}$ and $x \in U$}.

Definition: 3. 2 A subset A of a space X is called (1, 2)*-$M_{\delta\pi}$-closed set if (1, 2)*-δcl (A)$\subseteq$U whenever A$\subseteq$U and U is (1, 2)*-πg-open set in X.

Proposition: 3. 3 Every (1, 2)*-δ-closed set is (1, 2)*-$M_{\delta\pi}$-closed set.

Proof: Let A be an (1, 2)*-δ-closed set and U be any (1, 2)*-πg-open set containing A. Since A is (1, 2)*-δ-closed, (1, 2)*-δcl (A) =A, for every subset A of X. Therefore, (1, 2)*-δcl (A) $\subseteq$ U and hence A is (1, 2)*-$M_{\delta\pi}$-closed set.

Remark: 3. 4 The converse of the above theorem is not true as shown in the following example.

Example: 3. 5 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {d}, {a, d}}, τ_2 = {Φ, X, {a, c}, {c, d}, {a, c, d}}. Here {a, b}, {a, b, d} are (1, 2)*-$M_{\delta\pi}$-closed set, but not (1, 2)*-δ-closed set in X.

Proposition: 3. 6 Every (1, 2)*-$M_{\delta\pi}$-closed set is (1, 2)*-πg-closed set.

Proof: Let A be an (1, 2)*-$M_{\delta\pi}$-closed set and U be an any $\tau_{1,2}$-π-open set containing A in X. Since every $\tau_{1,2}$-π-open set is $\tau_{1,2}$-open set and therefore (1, 2)*-πg-open set. Also A is (1, 2)*-$M_{\delta\pi}$ -closed, (1, 2)*-δcl (A) $\subseteq$U, for every subset A of X. Since $\tau_{1,2}$-cl (A) $\subseteq$ (1, 2)*-δcl (A) $\subseteq$U implies $\tau_{1,2}$- cl (A) $\subseteq$ U and hence A is (1, 2)*-πg-closed set.

Remark: 3. 7 An (1, 2)*-πg-closed set need not be (1, 2)*- $M_{\delta\pi}$ -closed set as shown in the following example.

Example: 3. 8 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {b}, {a, b}}, τ_2 = {Φ, X, {a, b, d}}. Then the sets {c}, {d}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d} are (1, 2)*-πg-closed set, but not (1, 2)*- $M_{\delta\pi}$ -closed set in X.

Proposition: 3. 9 Every (1, 2)*- $M_{\delta\pi}$ -closed set is (1, 2)*-πgα-closed set.

Proof: It is true that (1, 2)*-αcl (A) $\subseteq$(1, 2)*-δcl (A) for every subset A of X.

Remark: 3. 10 A (1, 2)*-πgα-closed set need not be (1, 2)*- $M_{\delta\pi}$ -closed set as shown in the following example.

Example: 3. 11 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {a, b, d}}, τ_2 = {Φ, X, {b}, {a, b}}. Then the set {a, b, c} is (1, 2)*-πgα-closed set, but not (1, 2)*- $M_{\delta\pi}$ -closed set in X.

Proposition: 3. 12 Every (1, 2)*- $M_{\delta\pi}$ -closed set is (1, 2)*-strongly-πgα-closed set.

Proof: It is true that (1, 2)*-αcl (A) $\subseteq$(1, 2)*-δcl (A) for every subset A of X.

Remark: 3. 13 A (1, 2)*-strongly-πgα-closed set need not be (1, 2)*- $M_{\delta\pi}$ -closed set as shown in the following example.

Example: 3. 14 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {d}, {c, d}, {a, d}, {a, c, d}}, τ_2 = {Φ, X, {a, c}}. Then the set {b, d} is (1, 2)*-strongly-πgα-closed set, but not (1, 2)*- $M_{\delta\pi}$ -closed set in X.

Definition: 3. 15 A subset A of a bitopological space X is called (1, 2)*-δg- closed if (1, 2)*-δcl (A) $\subseteq$A whenever $A \subseteq U$ and U is $\tau_{1,2}$-open.

Proposition: 3. 16 Every (1, 2)*-$M_{\delta\pi}$ -closed set is (1, 2)*-δg-closed set.

Proof: Let A be an (1, 2)*-$M_{\delta\pi}$ -closed set and U be an any $\tau_{1,2}$-open set containing A in X. Since every $\tau_{1,2}$-open set is (1, 2)*-πg-open set, (1, 2)*-δcl (A)$\subseteq$U, whenever A$\subseteq$U and U is (1, 2)*-πg-open set. Therefore, (1, 2)*-δcl (A)$\subseteq$U and U is $\tau_{1,2}$-open. Hence A is (1, 2)*-δg-closed set.

Remark: 3. 17 A (1, 2)*-δg-closed set need not be (1, 2)*-$M_{\delta\pi}$ -closed set as shown in the following example.

Example: 3. 19 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {c}, {a, c}, {a, b, d}}, τ_2 = {Φ, X, {b, c, d}}. Then the sets {b}, {d}, {c, d}, {a, d}, {b, c}, {a, c, d}, {a, b, c} are (1, 2)*-δg-closed set, but not (1, 2)*-$M_{\delta\pi}$-closed set in X.

Remark: 3. 20 The class of (1, 2)*-$M_{\delta\pi}$ -closed sets is properly placed between the classes of (1, 2)*-δ-closed and (1, 2)*-δg-closed sets.

Proposition: 3. 21 Every (1, 2)*-$M_{\delta\pi}$ -closed set is (1, 2)*-gs-closed set.

Proof: It is true that (1, 2)*-scl(A) $\subseteq$ (1, 2)*-δcl(A) $\subseteq$ U, (1, 2)*-scl(A) $\subseteq$ U and hence A is (1, 2)*-gs-closed set.

Remark: 3. 22 A (1, 2)*-gs-closed set need not be (1, 2)*-$M_{\delta\pi}$-closed set as shown in the following example.

Example: 3. 23 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {a, b, d}}, τ_2 = {Φ, X, {c}, {a, c}, {b, c, d}}. Then the sets {a}, {b}, {d}, {a, d}, {c, d}, {b, c}, {a, b, c}, {a, c, d} are (1, 2)*-gs-closed set, but not (1, 2)*- $M_{\delta\pi}$ -closed set in X.

Proposition: 3. 24 Every (1, 2)*- $M_{\delta\pi}$ -closed set is (1, 2)*-αg-closed set.

Proof: It is true that, (1, 2)*-αcl (A) $\subseteq$ (1, 2)*-δcl (A) for every subset A of X.

Remark: 3. 25 A (1, 2)*-αg-closed set need not be (1, 2)*- $M_{\delta\pi}$ -closed set as shown in the following example.

Example: 3. 26 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {a, b}}, τ_2 = {Φ, X, {b}}. Then the sets {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d} are (1, 2)*-αg-closed set, but not (1, 2)*- $M_{\delta\pi}$ -closed set in X.

Proposition: 3. 27 Every (1, 2)*- $M_{\delta\pi}$ -closed set is (1, 2)*-g-closed set.

Proof: The proof is obvious.

Remark: 3. 28 A (1, 2)*- g-closed set need not be (1, 2)*- $M_{\delta\pi}$ -closed set as shown in the following example.

Example: 3. 29 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}}, τ_2 = {Φ, X, {b}}. Then the sets {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d} are (1, 2)*- g-closed set, but not (1, 2)*- $M_{\delta\pi}$ -closed set in X.

Definition: 3. 30 A subset A of a bitopological space X is called (1, 2)*-δg- closed if (1, 2)*-δcl (A) $\subseteq$A whenever $A \subseteq U$ and U is (1, 2)*- $\hat{g}$ -open.

Proposition: 3. 31 Every (1, 2)*- $M_{\delta\pi}$ -closed set is (1, 2)* - δ$\hat{g}$ -closed set.

Proof: The proof is straightforward, since every (1, 2)*- $\hat{g}$ -closed set is (1, 2)*-πg-closed.

Remark: 3. 32 A (1, 2)*-δ$\hat{g}$ -closed set need not be (1, 2)*- $M_{\delta\pi}$ -closed set as shown in the following example.

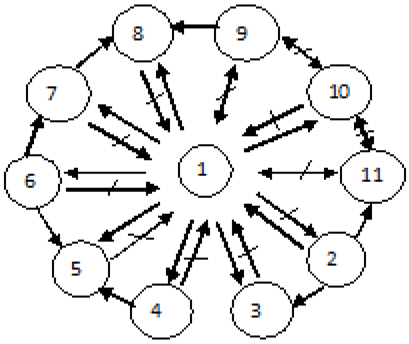
Example: 3. 33 Let X = {a, b, c}, τ_1 = {Φ, X, {a}, {a, b}}, τ_2 = {Φ, X, {b}, {a, c}}. Then the sets {c}, {b, c} are (1, 2)*- $\delta\hat{g}$ -closed set, but not (1, 2)*- $M_{\delta\pi}$ -closed set in X.

Remark: 3. 34 The following examples show that (1, 2)*-$M_{\delta\pi}$–closedness is independent from (1, 2)*- gα-closed set, (1, 2)*-α-closed.

Example: 3. 35 Let X = {a, b, c}, τ_1 = {Φ, X, {a}, {a, b}}, τ_2 = {Φ, X, {b}, {a, c}}. Then the set {b} & {a, c} are (1, 2)*- $M_{\delta\pi}$ -closed set, but neither (1, 2)*-gα-closed nor (1, 2)*-α-closed.

Example: 3. 36 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}}, τ_2 = {Φ, X, {b}, {a, b}}. Then the sets {c} and {d} are (1, 2)*-α-closed set and (1, 2)*-gα-closed, but not (1, 2)*- $M_{\delta\pi}$ -closed set in X.

Remark: 3. 37 The following diagram shows the relationships of (1, 2)*- $M_{\delta\pi}$ -closed sets with other known existing sets. A⟶B represents A implies B, but not conversely.



(1, 2)*-δg-closed set.

1. (1, 2)*- $M_{\delta\pi}$ -closed set
2. (1, 2)*-δ-closed set 3. (1, 2)*-δg-closed set.
4. (1, 2)*- πg-closed
5. (1, 2)*- πgα-closed set
6. (1, 2)*- strongly-πgα-closed set
7. (1, 2)*- g-closed set
8. (1, 2)*- αg-closed set
9. (1, 2)*-α-closed
10. (1, 2)*- $\delta\hat{g}$ -closed set
11. (1, 2)*- gα-closed set.

4. CHARACTERISATION

Theorem: 4. 1 The union of (1, 2)*-$M_{\delta\pi}$ -closed sets is (1, 2)*-$M_{\delta\pi}$ -closed.

Proof: Let {A_i / i=1, 2,…n} be a finite class of (1, 2)*-$M_{\delta\pi}$ -closed subsets of a space X. Then for each (1, 2)*-πg-open set, $\mathbf{U}_i A_i \subseteq \mathbf{U}_i U_i = V$. Since arbitrary union of (1, 2)*- πg-open sets in X is also (1, 2)*- πg-open set in X, V is (1, 2)*- πg-open set in X. Also $\mathbf{U}_i$ (1, 2)*-δcl (A_i) = (1, 2)*-δcl ($\mathbf{U}_i A_i$) ⊆V. Therefore, $\mathbf{U}_i A_i$ is (1, 2)*- $M_{\delta\pi}$ -closed set in X.

Remark: 4. 2 Intersection of any (1, 2)*-$M_{\delta\pi}$-closed sets in X need not be (1, 2)*-$M_{\delta\pi}$-closed. Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {a, b, d}}, τ_2 = {Φ, X, {c}, {b, c, d}}. Then the sets {b, d} and {a, b, c} are (1, 2)*-$M_{\delta\pi}$-closed sets, but the intersection {b} is not (1, 2)*-$M_{\delta\pi}$-closed.

Proposition: 4. 3 Let A be a (1, 2)*-$M_{\delta\pi}$-closed set of X. Then (1, 2)*-δcl (A)-A does not contain non-empty (1, 2)*-πg-closed set.

Proof: Suppose that A is (1, 2)*-$M_{\delta\pi}$-closed, let F be a (1, 2)*-πg-closed set contained in (1, 2)*-δcl (A)-A. Now F^C is (1, 2)*- πg-open set of X such that $A \subseteq F^C$. Since A is (1, 2)*-$M_{\delta\pi}$-closed set of X, then (1, 2)*-δcl (A) ⊆ F^C.

Thus F ⊆ $[(1, 2)^*\text{-}\delta cl\,(A)]^C$. Also F⊆(1, 2)*-δcl (A)-A. Therefore, F ⊆ $[(1, 2)^*\text{-}\delta cl\,(A)]^C \cap [(1, 2)^*\text{-}\delta cl\,(A)]$ = Φ. Hence F=Φ.

Proposition: 4. 4 If A is (1, 2)*- πg-open and (1, 2)*-$M_{\delta\pi}$-closed subset of X, then A is an (1, 2)*- δ-closed subset of X.

Proof: Since A is (1, 2)*-πg-open and (1, 2)*-$M_{\delta\pi}$-closed, (1, 2)*-δcl (A) $\subseteq$ A. Hence A is (1, 2)*-δ-closed.

Theorem: 4. 5 The intersection of a (1, 2)*-$M_{\delta\pi}$-closed set and (1, 2)*-δ-closed set is always (1, 2)*- $M_{\delta\pi}$-closed.

Proof: Let A be (1, 2)*- $M_{\delta\pi}$-closed and F be (1, 2)*-δ-closed. If U is an (1, 2)*-πg-open set with A∩ F $\subseteq$ U, then A $\subseteq$ U U F^C and so (1, 2)*-δcl (A) $\subseteq$ U U F^C. Now (1, 2)*-δcl (A∩ F) $\subseteq$ (1, 2)*-δcl (A) ∩F $\subseteq$ U.

Hence A∩ F is (1, 2)*- $M_{\delta\pi}$-closed.

Proposition: 4. 6 If A is a (1, 2)*-$M_{\delta\pi}$-closed set in a space X and A$\subseteq$ B $\subseteq$(1, 2)*-δcl (A), then B is also (1, 2)*-$M_{\delta\pi}$-closed set.

Proof: Let U be (1, 2)*-πg-open set of X such that B $\subseteq$ U. Then A$\subseteq$U. Since A is (1, 2)*-$M_{\delta\pi}$-closed set, (1, 2)*-δcl (A) $\subseteq$ U. Also, since, B$\subseteq$(1, 2)*-δcl (A), (1, 2)*-δcl(B)$\subseteq$(1, 2)*-δcl((1, 2)*-δcl(A)) = (1, 2)*-δcl(A). Hence (1, 2)*-δcl (B) $\subseteq$ U. Therefore, B is also a (1, 2)*-$M_{\delta\pi}$-closed set.

Proposition: 4. 7 Let A be a (1, 2)*-$M_{\delta\pi}$-closed set of X. Then A is (1, 2)*-δ-closed iff (1, 2)*-δcl (A)-A is (1, 2)*- πg-closed set.

Proof: Necessity. Let A be (1, 2)*-δ-closed subset of X. Then, (1, 2)*-δcl (A) =A and so (1, 2)*-δcl (A) –A= Φ, which is (1, 2)*-πg-closed set.

Sufficiency. Since A is (1, 2)*-$M_{\delta\pi}$-closed, by proposition 4. 3, (1, 2)*-δcl (A) –A does not contain a non-empty (1, 2)* -πg-closed set. But (1, 2)*-δcl (A) –A= Φ. That is, (1, 2)*-δcl (A) =A. Hence A is (1, 2)*-δ-closed.

5. APPLICATIONS

Definition: 5. 1 A space X is called (1, 2)*-$T_{\delta\pi g}$-space if every (1, 2)*-$M_{\delta\pi}$-closed set in it is an (1, 2)*-δ-closed.

Definition: 5. 2 A space X is called a (1, 2)*-$T_{\delta g}$-space if every (1, 2)*-δg-closed set in it is (1, 2)*-δ-closed.

Theorem: 5. 3 For a bitopological space X, the following conditions are equivalent.
(1) X is a (1, 2)*-$T_{\delta\pi g}$-space.
(2) Every singleton {x} is either (1, 2)*-πg-closed or (1, 2)*-δ-open.

Proof:
(1) => (2): Let x∈X. Suppose {x} is not a (1, 2)*-πg-closed set of X. Then X-{x} is not a (1, 2)*-πg-open set. Thus X-{x} is an (1, 2)*-$M_{\delta\pi}$-closed set of X. Since X is (1, 2)*-$T_{\delta\pi g}$-space, X-{x} is an (1, 2)*-δ-closed set of X, i. e., {x} is (1, 2)*-δ-open set of X.

(3) => (1): Let A be an (1, 2)*-$M_{\delta\pi}$-closed set of X. Let x ∈(1, 2)*-δcl (A). By (2), {x} is either (1, 2)*-πg-closed or (1, 2)*-δ-open.

Case (i): Let {x} be (1, 2)*- πg-closed set. If we assume that x∉A then we would have x∈(1, 2)*-δcl (A)-A, which cannot happen according to proposition 4. 3. Hence x∈X.

Case (ii): Let {x} be (1, 2)*-δ-open set. Since x∈(1, 2)*-δcl (A), then {x} ∩ A ≠ Φ. This shows that x∈X. So in both cases we have (1, 2)*-δcl (A)$\subseteq$A. Trivially A $\subseteq$(1, 2)*-δcl (A). Therefore, A= (1, 2)*-δcl (A) or equivalently A is (1, 2)*-δ-closed. Hence X is a (1, 2)*-$T_{\delta\pi g}$-space.

Theorem: 5. 4 Every (1, 2)*-$T_{\delta g}$-space is a (1, 2)*-$T_{\delta\pi g}$-space.

Proof: The proof is straight forward, since every (1, 2)*-$M_{\delta\pi}$-closed set is (1, 2)*-δg-closed set.

Remark: 5. 5 The converse of the above theorem is not true as it can be seen from the following example.

Example: 5. 6 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {a, b, d}}, τ_2 = {Φ, X, {c}, {b, c, d}}. Then X is a (1, 2)*-$T_{\delta\pi g}$-space, but not (1, 2)*-$T_{\delta g}$-space.

Definition: 5. 7 A space X is called a (1, 2)*-$T_\delta \hat{g}$ -space if every (1, 2)*-$\delta \hat{g}$ -closed set in it is (1, 2)*-δ-closed.

Theorem: 5. 8 Every (1, 2)*-$T_\delta \hat{g}$ -space is a (1, 2)*-$T_{\delta\pi g}$-space.

Proof: The proof is straight forward, since every (1, 2)*-$M_{\delta\pi}$-closed set is (1, 2)*-$\delta \hat{g}$ -closed set.

Remark: 5. 9 The converse of the above theorem is not true as it can be seen from the example.

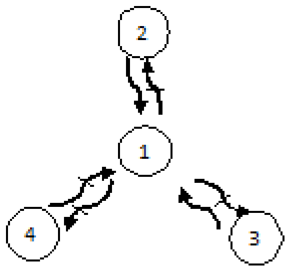
Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}}, τ_2 = {Φ, X, {a, b, d}}, X is (1, 2)*-$T_{\delta\pi g}$-space, but not (1, 2)*-$T_\delta \hat{g}$ -space.

Remark: 5. 10 (1, 2)*-$T_{\delta\pi g}$-space and (1, 2)*-$T_{1/2}$-space are independent of one another as the Following examples show.

Example: 5. 11 Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}}, τ_2 = {Φ, X, {b}}, X is (1, 2)*-$T_{\delta\pi g}$-space, but not (1, 2)*-$T_{1/2}$-space.

Example: 5. 12 Let X = {a, b, c}, τ_1 = {Φ, X, {a}}, τ_2 = {Φ, X, {a, b, d}}. Then X is a (1, 2)*-$T_{1/2}$-space, but not (1, 2)*-$T_{\delta\pi g}$-space.

Remark: 5. 13 The following diagram shows the relationships (1, 2)*-$T_{\delta\pi g}$-space with other known existing spaces.
A⟶B represents A implies B, but not conversely.



1. (1, 2)*-$T_{\delta\pi g}$-space **2.** (1, 2)*-$T_{\delta g}$-space **3.** (1, 2)*-$T_\delta \hat{g}$ -space **4.** (1, 2)*-$T_{1/2}$-space

REFERENCES

[1] M. E. Abd El-Monsef, S. Rose Mary and M. Lellis Thivagar, On $\alpha \hat{g}$ -closed sets in topological spaces, Assiut University Journal of Mathematics and Computer science, Vol. 36(1), P-P. 43-51(2007).

[2] I. Arockiarani and K. Mohana, (1, 2)*-πgα-closed sets and (1, 2)*-Quasi-α-normal Spaces in Bitopological settings, Antarctica J. Math., 7(3) (2010), 345-355.

[3] J. Dontchev and M. Ganster, On δ-generalised closed set and $T_{3/4}$-spaces, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 17(1996), 15-31.

[4] S. Jafari, M. Lellis Thivagar and Nirmala Mariappan, On (1, 2)*-$\alpha \hat{g}$ -closed sets, J. Adv. Math. Studies, 2(2)(2009), 25-34.

[5] M. Lellis Thivagar, O. Ravi, On stronger forms of (1, 2)*-quotient mappings in bitopological spaces. Internat. J. Math. Game theory and Algebra. Vol. 14. No. 6 (2004), 481-492.

[6] N. Levine, Generalised closed sets in topology, Rend. Circ. Mat. Palermo, 19(1970), 89-96.

[7] K. Mohan, I. Arockiarani, (1, 2)*-πg-homeomorphims in Bitopological spaces, CiiT Inter. J. Automation and Autonomous System, April 2011.

[8] K. Mohana, I. Arockiarani, (1, 2)*-strongly -πgα-closed sets, Journal of Advanced studies in Topology, Vol.2, No. 2, 2011, 31-36.

[9] O. Njastad, On some classes of nearly open sets, Pacific J Math., 15(1965), 961-970.

[10] N. V. Velicko, H-closed topological spaces, Amer. Math. Soc. Transl., 78(1968), 103-118.

