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THERMAL RADIATION AND ROTATION EFFECT ON AN UNSTEADY MHD MIXED CONVECTION FLOW THROUGH A POROUS MEDIUM WITH HALL CURRENT

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ABSTRACT

The effect of thermal radiation and rotation on an unsteady magnetohydrodynamic mixed convection flow through a porous medium with hall current has been studied. The similarity solutions were obtained using suitable transformations and the resulting similarity partial differential equations were solved by using Galerkin finite element method.. A uniform magnetic field is applied in the direction normal to the planes of the plates. The entire system rotates about an axis normal to the planes of the plates with uniform angular velocity Ω. The temperature of one of the plates varies periodically and the temperature difference of the plates is high enough to induce radiative heat transfer. The effects of various parameters on the velocity profiles, the skin friction, temperature field, rate of heat transfer in terms of their amplitude and phase angles are shown graphically.

Keywords: *Thermal radiation, Rotation, MHD, Mixed Convection flow, Hall current, Finite Element Method.*

1. INTRODUCTION

The study of flow in rotating porous media is motivated by its practical applications in geophysics and engineering. Among the applications of rotating flow in porous media to engineering disciplines, one can find the food processing industry, chemical process industry, centrifugation filtration processes and rotating machinery. Also the hydrodynamic rotating flow of electrically conducting viscous incompressible fluids has gained considerable attention because of its numerous applications in physics and engineering. In geophysics, it is applied to measure and study the positions and velocities with respect to a fixed frame of reference on the surface of earth, which rotate with respect to an inertial frame in the presence of its magnetic field. The subject of geophysical dynamics now – a – days has become an important branch of fluid dynamics due to the increasing interest to study environment. In astrophysics, it is applied to study the stellar and solar structure, inter planetary and inter stellar matter, solar storms etc. In engineering, it finds its application in MHD generators, ion propulsion, MHD bearings, MHD pumps MHD boundary layer control of reentry vehicles etc. Several scholars viz. Crammer and Pai [5], Ferraro and plumpton [6], Shercliff [17] have studied such flows because of their varied importance. MHD channel or duct flows are important from its practical point of view. Chang and Lundgren [3] have studied a hydromagnetic flow in a duct. Yen and Chang [22] analyzed the effect of wall electrical conductance on the magnetohydrodynamic Couette flow. From the technological point of view and due to practical applications, free convective flow and heat transfer problems are always important. This process of heat transfer is encountered in cooling of nuclear reactors, providing heat sinks in turbine blades and aeronautics. Ostrach [13] studied the combined effects of natural and forced convection laminar flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperature. Jain and Gupta [8] studied three dimensional free convection Couette flow with transpiration cooling.

There are numerous important engineering and geophysical applications of the channel flows through porous medium, for example in the fields of agricultural engineering for channel irrigation and to study the underground water resources, in petroleum technology to study the movement of natural gas, oil and water through the oil channels/reservoirs. Transient natural convection between two vertical walls with a porous material having variable porosity has been studied by Paul *et al.* [14]. Sahin [16] investigated the three – dimensional free convective channel flow through porous medium.

In recent years, the effects of transversely applied magnetic field on the flows of electrically conducting viscous fluids have been discussed widely owing to their astrophysics, geophysical and engineering applications. Attia and Kotb [2] studied MHD flow between two parallel plates with heat transfer. When the strength of the magnetic field is strong, one cannot neglect the effects of Hall current. The rotating flow of an electrically conducting fluid in the presence of a

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magnetic field is encountered in geophysical and cosmical fluid dynamics. It is also important in the solar physics involved in the sunspot development. Soundalgekar [21] studied the Hall effects in MHD Couette flow with heat transfer. Mazumder *et al* [9, 10] have studied the effects of Hall current on MHD Ekman layer flow and heat transfer over porous plate and on free and forced convective hydromagnetic flow through a channel. Hall effects on unsteady MHD free and forced convection flow in a porous rotating channel has been investigated by Sivaprasad *et al.* [20] Singh and Kumar [18] studied the combined effects of Hall current and rotation on free convection MHD flow in a porous channel. Ghosh *et al.* [7] studied the Hall effects on MHD flow in a rotating system with heat transfer characteristics.

Radiative convective flows have gained attention of many researchers in recent years. Radiation plays a vital role in many engineering, environment and industrial processes e.g. heating and cooling chambers, fossil fuel combustion energy processes astrophysical flows and space vehicle re – entry. Raptis [15] studied the radiation and free convection flow through a porous medium. Alagoa *et al* [1] analyzed the effects of radiation on free convective MHD flow through a porous medium between infinite parallel plates in the presence of time – dependent suction. Mebine [11] studied the radiation effects on MHD Couette flow with heat transfer between two parallel plates. Singh and Kumar [19] have studied radiation effects on the exact solution of free convective oscillatory flow through porous medium in a rotating vertical porous channel.

The MHD free convective flow in a rotating channel filled with porous medium has been studied in the present paper. The transverse magnetic field applied is strong enough so that the Hall currents are induced. The temperature difference between the walls of channel is sufficiently high to radiate the heat.

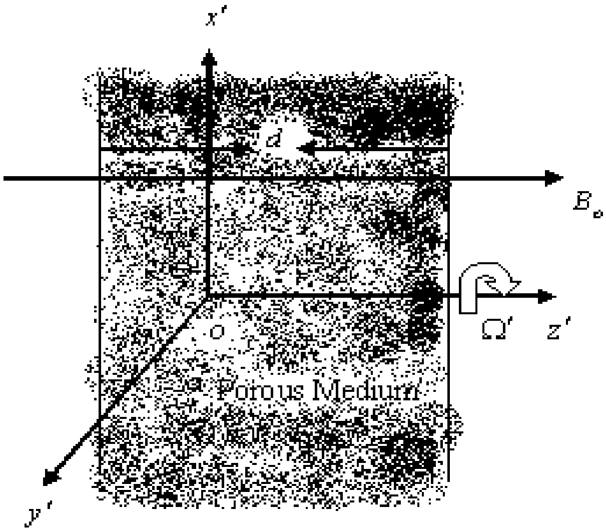

Figure 1. Schematic diagram of the physical problem

2. BASIC EQUATIONS:

The equations governing the unsteady free convective flow of an incompressible, viscous and electrically conducting fluid in a rotating vertical channel filled with porous medium in the presence of magnetic field are:

Equation of Continuity:

$$div\,\overline{V} = 0 \tag{1}$$

Momentum Equation:

$$\rho\left[\frac{\partial \overline{V}}{\partial t'} + \overline{\Omega}\times\overline{V} + \left(\overline{V}.\nabla\right)\overline{V}\right] = -\nabla p + \overline{J}\times\overline{B} + \mu\nabla^2\overline{V} - \frac{\mu}{K'}\overline{V} + g\beta T' \tag{2}$$

Energy Equation:

$$\rho\, C_P\left[\frac{\partial T'}{\partial t'} + \left(\overline{V}.\nabla\right)T'\right] = k\nabla^2 T' - \nabla q \tag{3}$$

Kirchhoff's First Law:

$$div\bar{J} = 0 \tag{4}$$

General Ohm's Law:

$$\bar{J} + \frac{\omega_e \tau_e}{B_0}\left(\bar{J} \times \bar{B}\right) = \sigma\left[\bar{E} + \bar{V} \times \bar{B} + \frac{1}{e\eta_e}\nabla p_e\right] \tag{5}$$

Gauss's Law of Magnetism:

$$div\bar{B} = 0$$
(6)

Where $\bar{V}$ is the velocity vector, $\bar{\Omega}$ the angular velocity of the fluid, p the pressure, ρ the density, $\bar{B}$ the magnetic induction vector, $\bar{J}$ the current density, μ the coefficient of viscosity, t' the time, g the acceleration due to gravity, β the coefficient of volume expansion, K' is the permeability of the porous medium, C_p the specific heat at constant pressure, T' the temperature, T_o the reference temperature that of the left plate, k the thermal conductivity, q the radiative heat, σ the electrical conductivity, B_o the strength of the applied magnetic field, e the electron charge, ω_c the electron frequency, τ_c the electron collision time, p_c the electron pressure, $\bar{E}$ the electric field and η_c is the number density of electron.

3. MATHEMATICAL FORMULATION:

Consider an unsteady magnetohydrodynamic free convective flow of an electrically conducting, viscous, incompressible fluid through a porous medium bounded between two insulated infinite vertical plates in the presence of Hall current and thermal radiation. The plates are at a distance d apart. A Cartesian coordinate system with x' – axis oriented vertically upward along the centerline of the channel introduced. The z' – axis is taken perpendicular to the planes of the plates and is the axis of rotation and the entire system rotates about this axis with uniform angular velocity Ω'. The schematic diagram of the physical problem is shown in Figure 1. Since the plates of the channel are of infinite extent, all the physical quantities depend only on z' and t' only. The temperature $T_w' \cos\omega' t'$ of the right plate at $z' = d/2$ is considered to be varying periodically with time and the temperature $T' = T_o = 0$ of the left plate at $z' = d/2$ is taken to be zero. Let (u', v', w') be the components of velocity in the directions (x', y', z') respectively. Since the plates are non – porous, therefore equation of continuity (1) on integration gives $w' = 0$. A strong transverse magnetic field of uniform strength B_0 is applied along the z' – axis. So, the equation (6) for the magnetic field $\bar{B} = \left(B_x', B_y', B_z'\right)$ gives $B_z' = B_0$ (constant).

If $\left(J_x', J_y', J_z'\right)$ are the components of electric current density $\bar{J}$ then equation of conservation of electric charge in equation (4) gives $J_z' =$ constant. For non – conducting plates

$$J_z' = 0 \tag{7}$$

At the plates and hence zero everywhere in the fluid. Under the usual assumptions that the electron pressure (for a weakly ionized gas), the thermoelectric pressure, ion slip and the external electric field arising due to polarization of charges are negligible. It is assumed that no applied and polarization voltage exists. This corresponds to the case where no energy is being added or extracted from the fluid by electrical means (Meyer [12]) i.e., electrical field $\bar{E} = 0$

Therefore, the equation (5) takes the form:

$$\bar{J}+\frac{\omega_e\tau_e}{B_0}\left(\vec{J}\times\vec{B}\right)=\sigma\left(\vec{V}\times\vec{B}\right) \tag{8}$$

After using equation (7), equation (8) in component form becomes:

$$J'_y+\omega_e\tau_e J'_x=\sigma B_0 v' \tag{9}$$

$$J'_y-\omega_e\tau_e J'_x=-\sigma\, B_0 u' \tag{10}$$

Solving equations (9) and (10) for J'_x and J'_y, we get

$$J'_x=\frac{\sigma B_0}{\left(1+m^2\right)}\left(mu'+v'\right) \text{ and } J'_y=\frac{\sigma B_0}{\left(1+m^2\right)}\left(mv'-u'\right)$$

Where $m=\omega_e\tau_e$ is the Hall parameter. Under the foregoing assumptions and reference temperature $T_o=0$, Equation (2) in Cartesian components reduces to:

$$\frac{\partial u'}{\partial t'}=-\frac{1}{\rho}\frac{\partial p'}{\partial x'}+\upsilon\frac{\partial^2 u'}{\partial z'^2}+2\Omega' v'+\frac{\sigma\, B_0^2}{\rho\left(1+m^2\right)}\left(mv'-u'\right)-\frac{\upsilon}{K'}u'+g\beta T' \tag{11}$$

$$\frac{\partial v'}{\partial t'}=-\frac{1}{\rho}\frac{\partial p'}{\partial y'}+\upsilon\frac{\partial^2 v'}{\partial z'^2}-2\Omega' u'-\frac{\sigma\, B_0^2}{\rho\left(1+m^2\right)}\left(mu'+v'\right)-\frac{\upsilon}{K'}u' \tag{12}$$

And equation (3) becomes:

$$\rho C_P\frac{\partial T'}{\partial t'}=k\frac{\partial^2 T'}{\partial z'^2}-\frac{\partial q}{\partial z'} \tag{13}$$

The boundary conditions for the flow problem are:

$$\left.\begin{aligned} &u'=v'=T'=0 \quad at \quad z'=-\frac{d}{2} \\ &u'=v'=0,\, T'=T'_w\cos\omega' t'\, at \quad z'=\frac{d}{2} \end{aligned}\right\} \tag{14}$$

Where T'_w is the mean temperature of the plate at $z'=d/2$ and ω' is the frequency of oscillation.

Following Cogley *el al* [4]. the last term in the energy equation (13), $\frac{\partial q}{\partial z'}=4\alpha^2(T'-T_0)$ stands for radiative heat flux modifies to:

$$\frac{\partial q}{\partial z'}=4\alpha^2 T' \tag{15}$$

In view of the reference temperature $T_0=0$, where α is the mean radiation absorption coefficient.

Introducing the following non dimensional quantities

$$\left.\begin{aligned}&\eta=\frac{z'}{d},x=\frac{x'}{d},y=\frac{y'}{d},u=\frac{u'}{U},V=\frac{v'}{U},T=\frac{T'}{T'_w},\ t=\frac{t'U}{d},\omega=\frac{\omega' d}{U},p=\frac{p'}{\rho U^2},\ \mathrm{Re}=\frac{Ud}{\nu},\\&\Omega=\frac{\Omega' d^2}{\nu},\ K=\frac{K'}{d^2},\ Gr=\frac{g\beta d^2 T'_w}{\nu U},\ Pe=\frac{\rho C_P dU}{k},\ M=\frac{\sigma B_o^2\upsilon}{\rho U^2},\quad N=\frac{2\alpha\ d}{\sqrt{k}}\end{aligned}\right\}\tag{16}$$

Using non – dimensional quantities from (16), the equations (11), (12) and (13) reduces to:

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial\eta^2}+\frac{2\Omega}{\mathrm{Re}}v+\frac{M\left(mv-u\right)}{\mathrm{Re}\left(1+m^2\right)}-\frac{1}{K\,\mathrm{Re}}u+\frac{Gr}{\mathrm{Re}}T\tag{17}$$

$$\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\frac{\partial^2 v}{\partial\eta^2}-\frac{2\Omega}{\mathrm{Re}}u-\frac{M\left(mu+v\right)}{\mathrm{Re}\left(1+m^2\right)}-\frac{1}{K\,\mathrm{Re}}v\tag{18}$$

$$\frac{\partial T}{\partial t}=\frac{1}{Pe}\frac{\partial^2 T}{\partial\eta^2}-\frac{N^2 T}{Pe}\tag{19}$$

Where U is the mean axial velocity.

The corresponding transformed boundary conditions are:

$$\left.\begin{aligned}&u=v=T=0\ at\ \eta=-\frac{1}{2}\\&u=v=0,\ T=\cos\omega t\ \ at\ \eta=\frac{1}{2}\end{aligned}\right\}\tag{20}$$

For the oscillatory internal flow, we shall assume that the fluid flows only under the influence of a non dimensional pressure gradient oscillating in the direction of x' – axis only which is of the form $-\frac{\partial p}{\partial x}=P\cos\omega t$.

4. METHOD OF SOLUTION:

By applying Galerkin finite element method for equation (17) over the element (e), $\left(\eta_j\le\eta\le\eta_k\right)$ is

$$\int_{\eta_j}^{\eta_k}\left\{N^{(e)T}\left[\frac{\partial^2 u^{(e)}}{\partial\eta^2}-\mathrm{Re}\frac{\partial u^{(e)}}{\partial t}-Bu+P\right]\right\}d\eta=0\tag{21}$$

Where $P=Amv+2\Omega v-\mathrm{Re}\frac{\partial p}{\partial x}+GrT,\ B=A+\frac{1}{K},\ A=\frac{M}{1+m^2}$

Integrating the first term in equation (21) by parts one obtains

$$N^{(e)^T}\left\{\frac{\partial u^{(e)}}{\partial\eta}\right\}_{\eta_j}^{\eta_k}-\int_{\eta_j}^{\eta_k}\left\{\frac{\partial N^{(e)^T}}{\partial\eta}\frac{\partial u^{(e)}}{\partial\eta}+N^{(e)^T}\left(\mathrm{Re}\frac{\partial u^{(e)}}{\partial t}+Bu^{(e)}-P\right)\right\}d\eta=0\tag{22}$$

Neglecting the first term in equation (22), one gets:

$$\int_{\eta_j}^{\eta_k}\left\{\frac{\partial N^{(e)^T}}{\partial\eta}\frac{\partial u^{(e)}}{\partial\eta}+N^{(e)^T}\left(\mathrm{Re}\frac{\partial u^{(e)}}{\partial t}+Bu^{(e)}-P\right)\right\}d\eta=0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ be the linear piecewise approximation solution over the element $(e), (\eta_j \le \eta \le \eta_k)$, where $N^{(e)} = \begin{bmatrix} N_j & N_k \end{bmatrix}$, $\phi^{(e)} = \begin{bmatrix} u_j & u_k \end{bmatrix}^T$ and $N_j = \dfrac{\eta_k - \eta}{\eta_k - \eta_j}$, $N_k = \dfrac{\eta - \eta_j}{\eta_k - \eta_j}$ are the basis functions. One obtains:

$$\int_{\eta_j}^{\eta_k} \left\{ \begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N_k' N_k' \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} d\eta + \mathrm{Re} \int_{\eta_j}^{\eta_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} \right\} d\eta + B \int_{\eta_j}^{\eta_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} d\eta = P \int_{\eta_j}^{\eta_k} \begin{bmatrix} N_j \\ N_k \end{bmatrix} d\eta$$

Simplifying we get

$$\frac{1}{l^{(e)^2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{\mathrm{Re}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} + \frac{B}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

where prime and dot denotes differentiation w.r.t η and time t respectively. Assembling the element equations for two consecutive elements $(\eta_{i-1} \le \eta \le \eta_i)$ and $(\eta_i \le \eta \le \eta_{i+1})$ following is obtained:

$$\frac{1}{l^{(e)^2}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{\mathrm{Re}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} + \frac{B}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{23}$$

Now put row corresponding to the node i to zero, from equation (23) the difference schemes with $l^{(e)} = h$ is:

$$\frac{1}{h^2}\left[-u_{i-1} + 2u_i - u_{i+1}\right] + \frac{\mathrm{Re}}{6}\left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}\right] + \frac{B}{6}\left[u_{i-1} + 4u_i + u_{i+1}\right] = P \tag{24}$$

Applying the trapezoidal rule, following system of equations in Crank – Nicholson method are obtained:

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^{n} + A_5 u_i^{n} + A_6 u_{i+1}^{n} + 12Pk \tag{25}$$

Applying similar procedure to equations (18) and (19) the following equations are obtained:

$$B_1 v_{i-1}^{n+1} + B_2 v_i^{n+1} + B_3 v_{i+1}^{n+1} = B_4 v_{i-1}^{n} + B_5 v_i^{n} + B_6 v_{i+1}^{n} + 12Qk \tag{26}$$

$$C_1 T_{i-1}^{n+1} + C_2 T_i^{n+1} + C_3 T_{i+1}^{n+1} = C_4 T_{i-1}^{n} + C_5 T_i^{n} + C_6 T_{i+1}^{n} \tag{27}$$

Where $A_1 = 2\,\mathrm{Re} - 6r + Bk,\ A_2 = 8\,\mathrm{Re} + 12r + 4Bk,\ A_3 = 2\,\mathrm{Re} - 6r + Bk,\ A_4 = 2\,\mathrm{Re} + 6r - Bk,$

$$A_5 = 8\,\mathrm{Re} - 12r - 4Bk,\ A_6 = 2\,\mathrm{Re} + 6r - Bk,\ B_1 = 2\,\mathrm{Re} - 6r + Bk,\ B_2 = 8\,\mathrm{Re} + 12r + 4Bk,$$
$$B_3 = 2\,\mathrm{Re} - 6r + Bk,\ B_4 = 2\,\mathrm{Re} + 6r - Bk,\ B_5 = 8\,\mathrm{Re} - 12r - 4Bk,\ B_6 = 2\,\mathrm{Re} + 6r - Bk,$$
$$C_1 = 2Pe - 6r + N^2k,\ C_2 = 8Pe + 12r + 4N^2k,\ C_3 = 2Pe - 6r + N^2k,$$
$$C_4 = 2Pe + 6r - N^2k,\ C_5 = 8Pe - 12r - 4N^2k,\ C_6 = 2Pe + 6r - N^2k,$$
$$P = Amv_i^j + 2\Omega v_i^j - \mathrm{Re}\frac{\partial p}{\partial x} + GrT_i^j,\ Q = Amu_i^j + 2\Omega u_i^j + \mathrm{Re}\frac{\partial p}{\partial y};$$

Here $r = \dfrac{k}{h^2}$ and $h,\ k$ are mesh sizes along η – direction and time – direction respectively. Index i refers to space and j refers to the time. In equations (25), (26) and (27) taking $i = 1(1)n$ and using boundary conditions (20), then the following system of equations are obtained:

$$A_i X_i = B_i \quad i = 1(1)3 \tag{28}$$

Where $A_i^{'}$s are matrices of order η and X_i, $B_i^{'}$s are column matrices having η – components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C – programme. In order to prove the convergence and stability of Galerkin finite element method, the same C – programme was run with smaller values of h and k and no significant change was observed in the values of u, v and T. Hence the Galerkin finite element method is stable and convergent.

5. SKIN FRICTION AND RATE OF HEAT TRANSFER:

We now calculate from the velocity field the skin friction. It is given in non – dimensional form as

$$\tau_1 = \left[\frac{du}{d\eta}\right]_{\eta=0} \text{ and } \tau_2 = \left[\frac{dv}{d\eta}\right]_{\eta=0} \tag{29}$$

Heat transfer coefficient (Nu) at the plate is

$$Nu = -\left[\frac{dT}{d\eta}\right]_{\eta=0} \tag{30}$$

6. RESULTS AND DISCUSSIONS:

In order to study the effect of different parameters appearing in the flow problem, we have carried out numerical calculations for the velocity field, temperature field, skin friction, and rate of heat transfer. To assess the effects of each parameter for small and large rotations, two values of the rotation parameter Ω (= 5 and 10) are considered.

Figure 2 shows the variation of velocity profiles under the influence of the rotation parameter Ω. The velocity decreases when Ω is increased. Figure 3 shows the variation of with Reynolds number Re. It is evident from figure 3 that the increasing value of Re leads to increase of velocity. It is interesting to note that for large rotation the maximum of velocity no longer occurs at the centre of the channel but shifted towards the walls.

The variations of the velocity profiles with the Grashof number Gr are shown in Figure 4. For small rotations $(\Omega = 5)$, the velocity increases with the increasing Grashof number. The maximum of the velocity profiles shifts towards right half of the channel due to the greater buoyancy force in this part of the channel due to the presence of hotter plate. For large rotation $(\Omega = 10)$, the Grashof number has opposite effect on the velocity profiles in the right half and the left half of the channel. In the right half there lies hot plate at $\eta = 1/2$ and heat is transferred from the hot plate to the fluid and consequently buoyancy force enhances the flow velocity further. In the left half of the channel, the transfer of heat takes place from the fluid to the cooler plate at $\eta = 1/2$. Thus, the effect of Grashof number on the velocity is reversed i.e. velocity decreases with increasing Gr. It is evident from figure 5 that the velocity decreases with the increase of Hartmann number M. This is because of the reason that effects of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive type force (called Lorentz force) similar to drag force and upon increasing the values of M increases the drag force which has tendency to slow down the motion of the fluid. Figure 6 shows the variation of the velocity with Hall parameter m. The velocity increases with the increase of m in the middle of the channel for small rotation $(\Omega = 5)$ of the channel while for the large rotation $(\Omega = 10)$ of the channel there is no significant effect of m on velocity (Figure 6).

The variation of the velocity profile with permeability of the porous medium K is shown in Figure 7. It is observed from Figure 7 that in the rotating channel the velocity decreases with increasing K. It is expected physically also because the resistance posed by the porous medium to the decelerated flow due to rotation reduces with increasing permeability K which leads to decrease in the velocity. For $(\Omega = 10)$ velocity is much less than $(\Omega = 5)$. Figure 8 shows that with increasing Peclet number Pe the velocity decreases. The variation of velocity profile with radiation parameter N is shown in Figure 9.

In the left half of the channel, the effect of N on the velocity is insignificant while in the right half of the channel velocity decrease with increase of N . It is evident from the Figure10 that the increasing pressure gradient P leads to the increase of velocity. Figure11 shows the variation of the velocity with frequency of oscillations ω . The velocity decreases with the increase of frequency of oscillations ω (Figure 11).

The temperature profile is shown in Figure 12. The temperature decreases with the increase of radiation parameter N , the Peclet number Pe and the frequency of oscillations ω (Figure 12). It is interesting to note that the flow of heat is reversed with the increase of Peclet number Pe. From table 1 we observed that the skin friction τ_1 decreases with increase of rotation parameter Ω, Peclet number Pe and Hartmann number M. The skin friction τ_1 increases with increase of Reynolds number Re, Grashof number Gr, Hall parameter m, Permeability of the porous medium K, Pressure gradient P. From table 1 we observed that the skin friction τ_2 decreases with increase of rotation parameter Ω, Peclet number Pe and Hartmann number M. The skin friction τ_2 increases with increase of Reynolds number Re, Grashof number Gr, Hall parameter m, Permeability of the porous medium K, Pressure gradient P. From table 2 we observed that the rate of heat transfer decreases with increase of Peclet number Pe and Radiation parameter N.

Table 1. Skin – friction coefficients (τ_1 & τ_2)

Ω	Re	Gr	M	m	K	Pe	N	P	τ_1	τ_2
5	1	1	1	1	1	0.71	1	5	1.769	-0.591
10	1	1	1	1	1	0.71	1	5	2.310	-0.183
5	0.5	1	1	1	1	0.71	1	5	0.930	-0.606
5	1	3	1	1	1	0.71	1	5	1.941	-0.520
5	1	1	2	1	1	0.71	1	5	1.602	-0.681
5	1	1	1	3	1	0.71	1	5	1.907	-0.401
5	1	1	1	1	0.5	0.71	1	5	1.712	-0.653
5	1	1	1	1	1	7.0	1	5	1.755	-0.495
5	1	1	1	1	1	0.71	5	5	1.701	-0.684
5	1	1	1	1	1	0.71	1	10	3.501	-0.410

Table 2. Rate of heat transfer

Pe	N	Nu
0.71	1.0	1.032
7.0	1.0	0.987
0.71	5.0	0.964

7. CONCLUSIONS:

This work investigated the effect of thermal radiation and rotation on an unsteady magnetohydrodynamic mixed convection flow through a porous medium with hall current has been studied. The governing equations are approximated to a system of linear partial differential equations by using Galerkin finite element method. The results are presented graphically and we can conclude that the flow field and the quantities of physical interest are significantly influenced by these parameters.

1. The velocity increases as rotation parameter Ω, Reynolds number Re, Grashof number Gr, Hall parameter m and Pressure gradient P increases. However, the velocity was found to decreases as the Hartmann number M, Permeability of the porous medium K, Peclet number Pe, Thermal radiation parameter N and Frequency of oscillation ω are increases.
2. The fluid temperature was found to decreases as the thermal radiation parameter N and Peclet number Pe are increases.
3. The skin friction τ_1 decreases with increase of rotation parameter Ω, Peclet number Pe Thermal radiation parameter N and Hartmann number M. However, it increases with increase of Reynolds number Re, Grashof number Gr, Hall parameter m, Permeability of the porous medium K, Pressure gradient P.

4. The skin friction τ_2 decreases with increase of rotation parameter Ω, Peclet number Pe Thermal radiation parameter N and Hartmann number M. However it increases with increase of Reynolds number Re, Grashof number Gr, Hall parameter m, Permeability of the porous medium K, Pressure gradient P.
5. The rate of heat transfer decreases with increase of Peclet number Pe and Thermal radiation parameter N.

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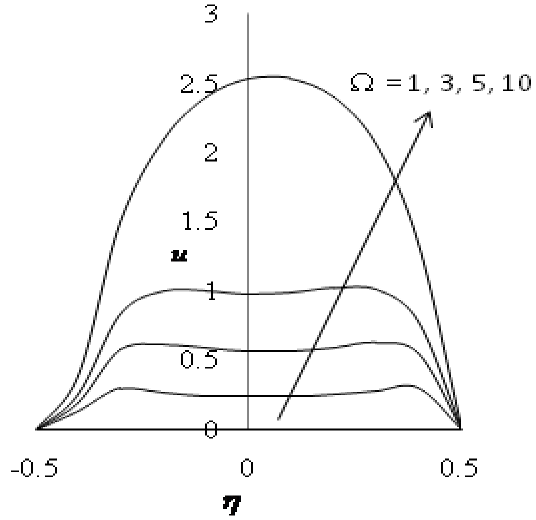

Figure 2. Variation of velocity profiles with Ω for Re = 1.0, Gr =1.0, M =1.0, m = 1.0, K = 1.0, Pe =0.71, N =1.0, P =5.0, ω =5.0 and t =1.0.

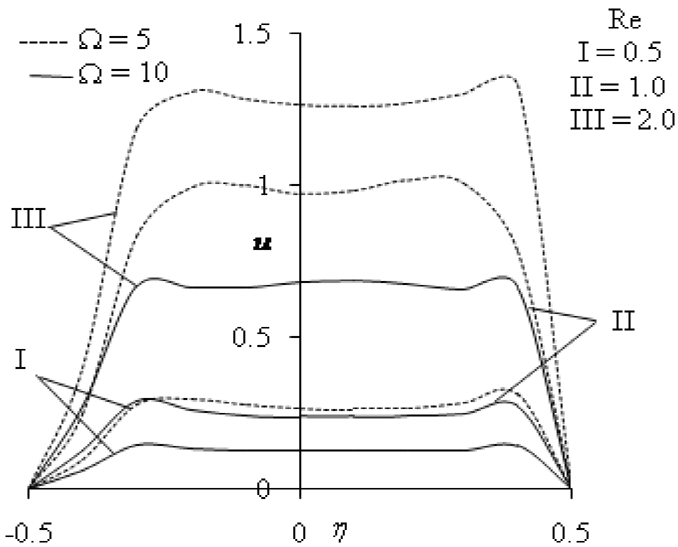

Figure 3. Variation of velocity profiles with Re for Gr =1.0, M =1.0, m = 1.0, K = 1.0, Pe =0.71, N =1.0, P =5.0, ω =5.0 and t =1.0.

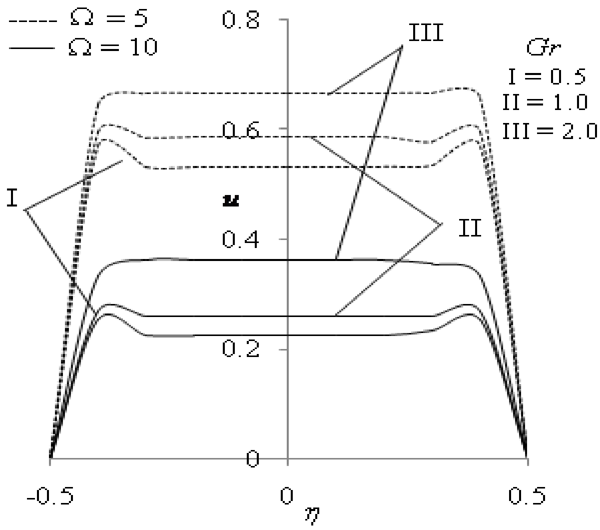

Figure 4. Variation of velocity profiles with Gr for Re =1.0, M =1.0, m = 1.0, K = 1.0, Pe =0.71, N =1.0, P =5.0, ω = 5.0 and t =1.0.

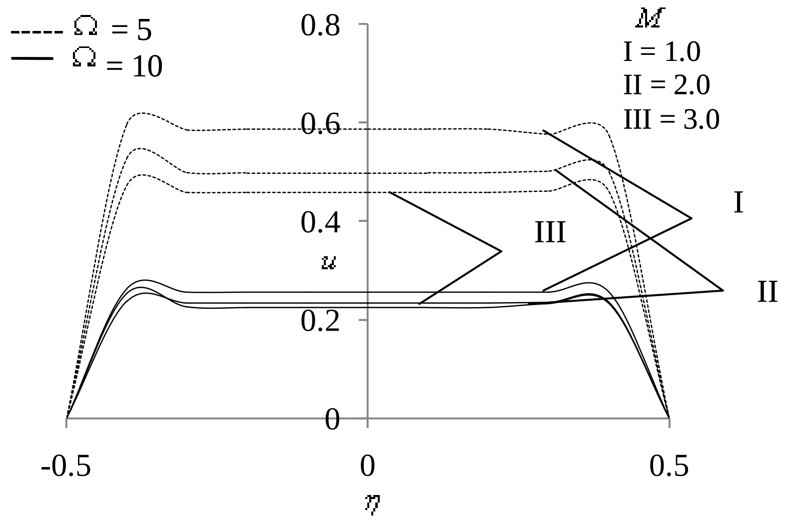

Figure 5. Variation of velocity profiles with M for $\text{Re}=1.0$, $Gr=1.0$, $m=1.0$, $K=1.0$, $Pe=0.71$, $N=1.0$, $P=5.0$, $\omega=5.0$ and $t=1.0$.

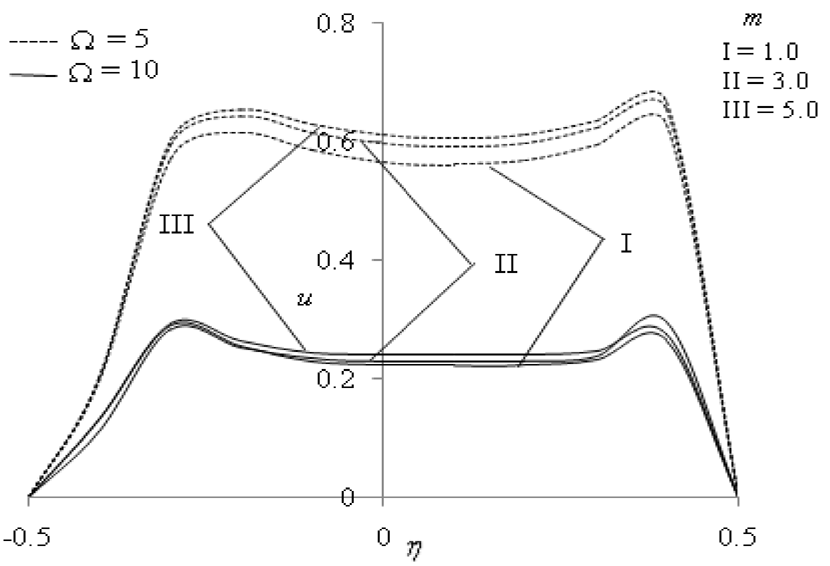

Figure 6. Variation of velocity profiles with m for $\text{Re}=1.0$, $Gr=1.0$, $M=1.0$, $K=1.0$, $Pe=0.71$, $N=1.0$, $P=5.0$, $\omega=5.0$ and $t=1.0$.

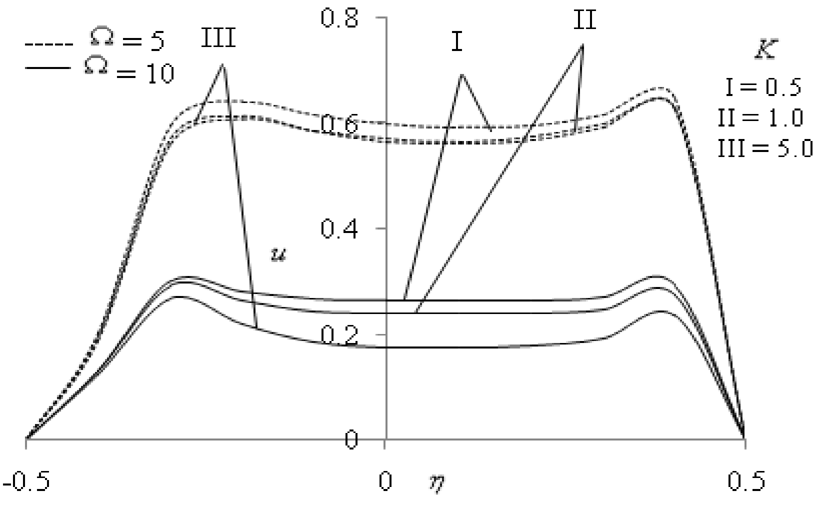

Figure 7. Variation of velocity profiles with K for $\text{Re}=1.0$, $Gr=1.0$, $M=1.0$, $m=1.0$, $Pe=0.71$, $N=1.0$, $P=5.0$, $\omega=5.0$ and $t=1.0$.

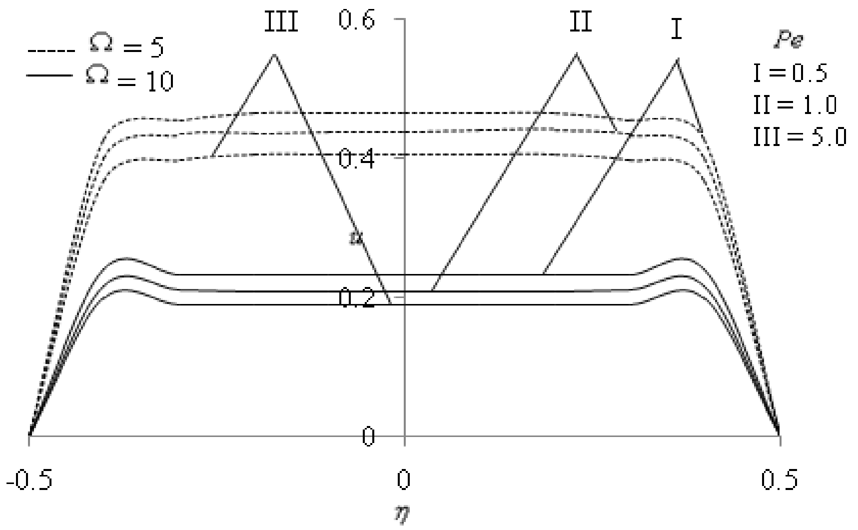

Figure 8. Variation of velocity profiles with Pe for $\mathrm{Re} = 1.0$, $Gr = 1.0$, $M = 1.0$, $m = 1.0$, $N = 1.0$, $K = 1.0$, $P = 5.0$, $\omega = 5.0$ and $t = 1.0$.

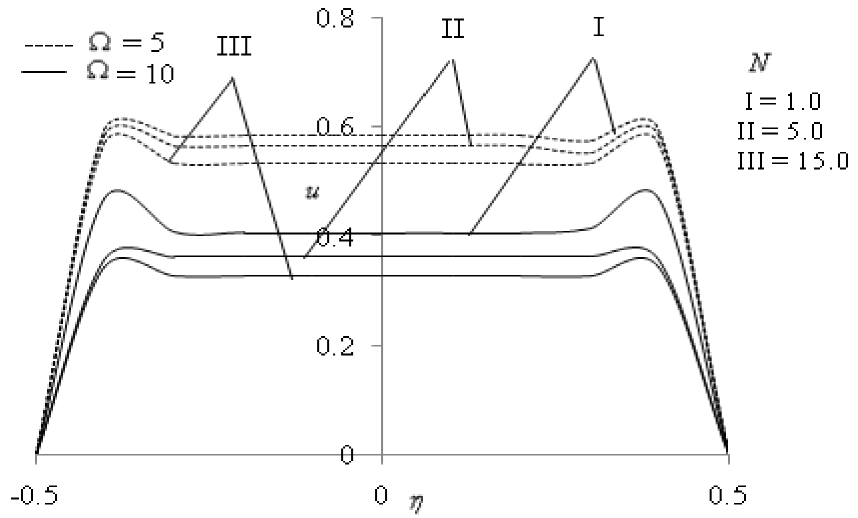

Figure 9. Variation of velocity profiles with N for $\mathrm{Re} = 1.0$, $Gr = 1.0$, $M = 1.0$, $m = 1.0$, $Pe = 0.71$, $K = 1.0$, $P = 5.0$, $\omega = 5.0$ and $t = 1.0$.

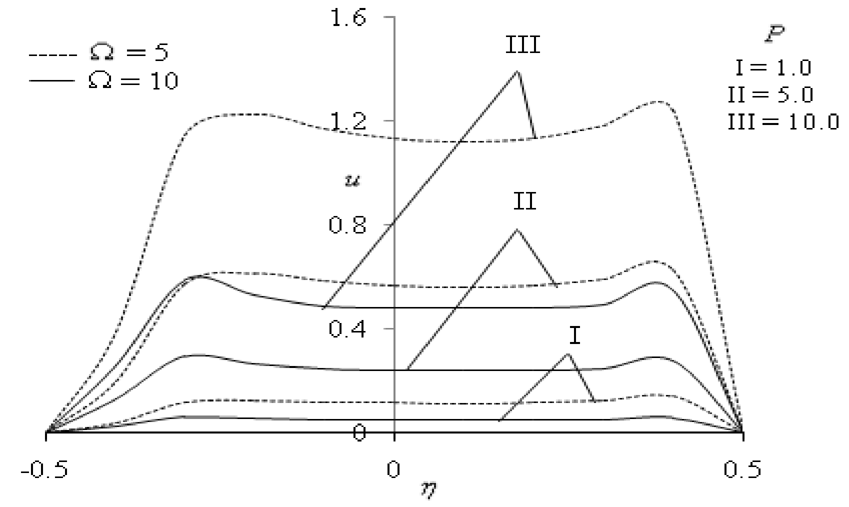

Figure 10. Variation of velocity profiles with P for $\mathrm{Re} = 1.0$, $Gr = 1.0$, $M = 1.0$, $m = 1.0$, $Pe = 0.71$, $K = 1.0$, $N = 1.0$, $\omega = 5.0$ and $t = 1.0$.

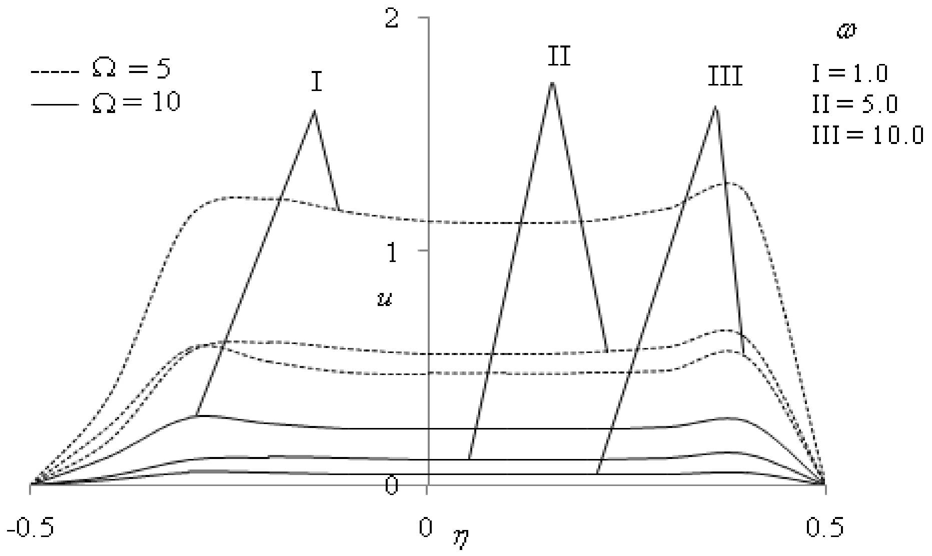

Figure 11. Variation of velocity profiles with ω for $\mathrm{Re} = 1.0$, $Gr = 1.0$, $M = 1.0$, $m = 1.0$, $Pe = 0.71$, $K = 1.0$, $N = 1.0$, $P = 5.0$ and $t = 1.0$.

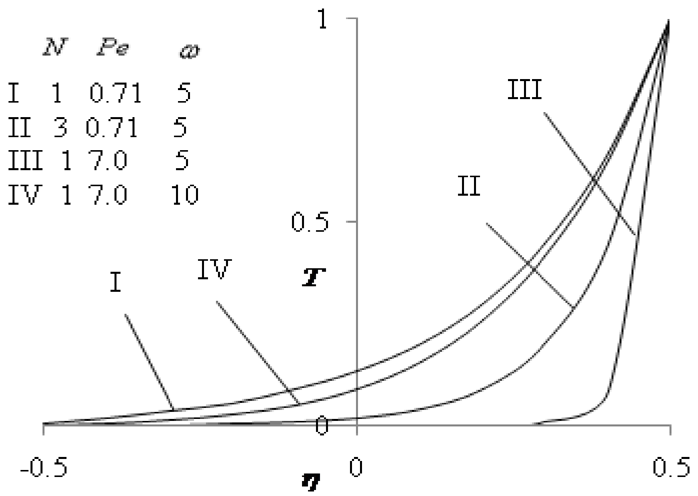

Figure 12. Variation of temperature profiles for $t = 1.0$.

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