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On (1, 2)*- πwg-Closed Sets in Bitopological Spaces

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ABSTRACT

The aim of this paper is to introduce a new class of sets called (1, 2)-πwg-closed sets in bitopological spaces and to study their properties. Further, we define and study (1, 2)*- πwg-continuity, (1, 2)*- πwg-irresolute maps and (1, 2)*-πwg-space.*

Mathematics Subject Classification: *54E55.*

Key Words: *(1, 2)*- πwg-closed sets, (1, 2)*- πwg-continuous and (1, 2)*- πwg-irresolute maps, πwg-space, (1, 2)*-πwg-$T_{1/2}$- Space.*

1. INTRODUCTION

The study of bitopological spaces was first initiated by J.C. Kelly [6]in the year 1963. Levine [7] introduced generalized closed sets and studied their properties. In 1985, Fukutake [4], introduced the concepts of g-closed sets in bitopological spaces. Dontchev. J, Noiri. T [3] introduced and studies the concepts of πg- closed set in topological spaces. Recently Ravi, Lellis Thivagar, Ekici and many others [8,9,12,13-17] have defined different weak forms of the topological notions namely , semi open, pre open, regular open and α-open sets in bitopological spaces.

In this paper, we introduce the notion of (1, 2)*- πwg-closed sets and investigate their properties. By using the class of (1, 2)*- πwg -closed sets in bitopological spaces, we study (1, 2)*- πwg -continuous, (1, 2)*- πwg-irresolute maps, πwg-space, (1, 2)*- πwg-$T_{1/2}$- space. In most of the properties and conditions, our ideas are discussed with suitable examples.

2. PRELIMINARIES

Throughout this paper, X and Y denote the bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) respectively, on which no separation axioms are assumed.

Definition: 2.1 Let A be a subset of X. Then A is called $\tau_{1,2}$ -open [1,14] if $A = A_1 \cup B_1$, where $A_1 \in \tau_1$, $B_1 \in \tau_2$. The complement of $\tau_{1,2}$ -open set[14] is $\tau_{1,2}$ -closed set. The family of all $\tau_{1,2}$ -open (resp. $\tau_{1,2}$-closed) sets of X is denoted by (1,2)* -O(X) and (resp. (1,2)* -C(X)).

Example: 2.2 Let X= {a, b, c}, τ_1 = {φ, X, {a}, {a, c}}, τ_2 = {φ, X, {c}}.

Then $\tau_{1,2}$ -open sets ={φ, X,{a},{c},{a, c}} and $\tau_{1,2}$ -closed sets ={φ, X,{b, c},{a, b},{b}}

Definition: 2.3 Let A be a subset of a bitopological space X. Then

(i) $\tau_{1,2}$ -closure of A [1,14] denoted by $\tau_{1,2}$ -cl(A) is defined by the intersection of all $\tau_{1,2}$ -closed sets containing A.
(ii) $\tau_{1,2}$-interior of A [1,14]denoted by $\tau_{1,2}$ -int (A) is defined by the union of all open sets contained in A.

Remark: 2.4 Notice that $\tau_{1,2}$ -open subsets of X need not necessarily form a topology.

Now, we recall some definitions and results which are used in this paper.

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Definition: 2.5 A subset A of a bitopological space X is said to be

(i) (1, 2)* -pre –open [18] if A⊂ $\tau_{1,2}$ -int ($\tau_{1,2}$ -cl(A)).
(ii) (1, 2)* -semi open [18] if A⊂ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)).
(iii) regular (1,2)* -open [10] if A = $\tau_{1,2}$ -int ($\tau_{1,2}$ -cl(A)).
(iv) (1, 2)* - α-open [18] if A⊂ $\tau_{1,2}$ -int ($\tau_{1,2}$ -cl ($\tau_{1,2}$ -int (A))).
(v) (1, 2)* -π- open [19] if A is the finite union of regular (1, 2)* -open sets.

The complements of all the above mentioned open sets are called their respective closed sets. The family of all (1, 2)* - open sets [(1, 2)* -regular open, (1, 2)* - π-open, (1, 2)* -semi open, (1, 2)* - regular semi open set) sets of X will be denoted by (1, 2)* O(X)(resp. (1, 2)* RO(X), (1, 2)* - πO(X), (1, 2)*-SO(X),(1,2)*-RSO(X)].

Definition: 2.6 A subset A of bitopological space X is said to be

(i) a $\tau_{1,2}$ - ω - closed [5] if $\tau_{1,2}$ -cl (A) ⊂ U whenever A⊂ U and U ∈ (1, 2)* -SO(X).

(ii) a (1, 2)* - generalized closed set [12] ((1, 2)* -g closed set) if $\tau_{1,2}$ -cl (A) ⊂ U whenever A⊂ U and U ∈ (1,2)*-O(X).

(iii) a regular (1, 2)* - generalized closed [16] (briefly (1, 2)* - rg closed set) if $\tau_{1,2}$-cl(A)⊂U whenever A ⊂ U and U∈ (1, 2)* - RO(X).

(iv) a (1, 2)* - generalized pre regular closed set [13] (briefly (1, 2)* -gpr –closed set) if (1, 2)* - pcl (A) ⊂U whenever A ⊂ U and U ∈ (1, 2)* - RO(X) .

(v) a weakly (1, 2)* - generalized closed [20] (briefly (1, 2)*-wg closed) if $\tau_{1,2}$-cl($\tau_{1,2}$-int (A)) ⊂ U whenever A⊂ U and U ∈ (1, 2)* -O(X).

(vi) a (1, 2)*-π-generalized closed [19] (briefly (1, 2)* - πg closed set) if $\tau_{1,2}$ - cl (A) ⊂U whenever A⊂U and U∈ (1,2)* -πO(X).

(viii) a (1,2) * - πgα - closed set [2] if $\tau_{1,2}$ - αcl (A) ⊂ U whenever A ⊂ U and U ∈ (1,2)* -πO(X).

(ix) a (1,2)* - regular semi open set[11] if there is a (1,2)* - RO(X) , U such that U⊂A ⊂ $\tau_{1,2}$ Cl(U).

(x) a (1,2)* - rw- closed set [11] if $\tau_{1,2}$ -cl(A) ⊂ U, whenever A⊂ U and U is (1,2)* - regular semi open set in X.

(xi) a (1, 2)*- regular α-open [11] in X if there is a (1, 2)* -regular open set U such that U ⊂ A⊂ $\tau_{1,2}$ - αcl(U).

(xii) a regular (1,2)* - generalized α- closed set [11](briefly (1,2)* - rgα - closed set) if $\tau_{1,2}$ -αcl (A) ⊂ U whenever A ⊂ U and U∈ (1,2)* - Rα O(X).[Rα O(X)- Collection of all regular (1,2)*- α -open set in X]

(xiii) a regular (1, 2)*-weakly generalized closed [11] (briefly (12) *- rwg closed) if $\tau_{1,2}$- cl($\tau_{1,2}$-int (A)) ⊂U whenever A⊂ U and U ∈ (1,2)*-RO(X).

(xiv) a (1,2)*- $T_{1/2}$-space[8] if every (1,2)*- g-closed set in X is $\tau_{1,2}$ -closed in X.

Definition: 2.7 A Bitopological space X is called

(i) a (1,2)*- T_{wg}-Space[20] if every (1,2)*- wg-closed subset of X is closed in X.
(ii) a (1,2)*-T_{α}- Space [18] if every (1,2)*- α -closed sunset of X is closed in X.
(iii) a (1,2)*- T_{ω} -Space [5] if every (1,2)*- ω -closed subset of X is closed in X.

Definition: 2.8 A map f: X→Y is said to be

(i) (1, 2)*- continuous [12] if f^{-1}(V) is $\tau_{1,2}$ -closed in X for every $\sigma_{1,2}$ - closed set V in Y.
(ii) (1, 2)*- semi continuous [18] if f^{-1} (V) is (1,2)*- semi closed in X for every $\sigma_{1,2}$ -closed set V in Y.
(iii) (1, 2)*- ω - continuous [5] if f^{-1} (V) is (1,2)*- ω- closed in X for every $\sigma_{1,2}$-closed set V in Y.
(iv) (1, 2)*- rg -continuous [16] if f^{-1}(V) is (1,2)*- rg closed in X for every $\sigma_{1,2}$ - closed set V in Y.
(v) (1, 2)*- π-continuous [19] if f^{-1} (V) is (1,2)*- π closed in X for every $\sigma_{1,2}$ - closed set V in Y.
(vi) (1, 2)*- πg-continuous [19] if f^{-1}(V) is (1, 2)*-πg closed in X for every $\sigma_{1,2}$-closed set V in Y.
(vii) (1, 2)*- g-continuous [12] if f^{-1} (V) is (1,2)*- g closed in X for every $\sigma_{1,2}$ - closed set V in Y.

(viii) (1, 2)*- gpr-continuous [13] if f^{-1}(V) is (1, 2)*- gpr closed in X for every $\sigma_{1,2}$ -closed set V in Y.
(ix) (1, 2)*- wg-continuous [20] if f^{-1} (V) is (1, 2)*- wg- closed in X for every $\sigma_{1,2}$ - closed set V in Y.

3. (1, 2) * - πwg – Closed Sets in Bitopological Spaces

Definition: 3.1A subset A of X is called (1, 2) *-πwg- closed set in X if $\tau_{1,2}$-cl ($\tau_{1,2}$ -int(A)) ⊂ U whenever A⊂ U and U ∈(1,2)* -πO(X).

The complement of (1, 2)* - πwg -closed set is (1, 2)*-πwg-open set.

We denote the family of all (1,2)*- πwg-closed (resp. πwg-open)sets in X by (1,2)*- πwGC(X)(resp. (1,2)*-πwGO(X)).

Theorem: 3.2

1. Every $\tau_{1,2}$ -closed set is (1, 2) *-πwg- closed set .
2. Every (1, 2)* - πg -closed set is (1, 2)* - πwg -closed set.
3. Every (1, 2)* - g - closed set is (1, 2)*- πwg -closed set.
4. Every (1, 2)* - πwg - closed set is (1, 2)* - gpr-closed set.
5. Every (1, 2)* - α - closed set is (1, 2)* - πwg -closed set.
6. Every (1, 2)* - wg - closed set is (1, 2)* - πwg -closed set.

Proof: Straight forward.

Remark: 3.3 The converse of the above results need not be true as seen in the following examples.

Example: 3.4 Let X = {a, b, c, d}, τ_1 = {φ, X, {b},{b, c}}, τ_2 = {φ, X, {c},{b, c, d}}.Then $\tau_{1,2}$ -open = {φ, X, {b},{c},{b, c},{b, c, d}} and $\tau_{1,2}$ -closed = {φ, X, {a, c, d},{a, b, d},{a, d},{a}}
Here A = {d} is (1, 2)* - πwg- closed set but not $\tau_{1,2}$ -closed set.

Example : 3.5 Let X = {a, b, c, d}, τ_1 = {φ, X, {a},{c, d},{a, c, d}}, τ_2 = {φ, X, {a},{c},{a, c}}.

Then $\tau_{1,2}$-open = {φ, X,{a},{c},{a, c},{c, d},{a, c, d}} and $\tau_{1,2}$-closed = {φ, X,{b, c, d},{a, b, d}, {b, d} {a, b},{b}} .

Here A = { d} is (1,2)* - πwg- closed set but not (1,2)* - πg- closed set.

Example: 3.6 In the above example A={d} is (1,2)* - πwg- closed set but not (1,2)* - g-closed set.

Example: 3.7 Let X={a, b, c, d}, τ_1={φ, X, {b},{b, c, d}}, τ_2 = {φ, X, {b},{d},{b, d},{a, b, d},{b, c, d}}.

Then $\tau_{1,2}$ -open = {φ, X, {b},{d},{b, d},{a, b, d},{b, c, d}} and $\tau_{1,2}$ -closed = {φ, X,{a, c, d},{a, b, c}, {a, c },{c},{a}}.

Here A = {a, b} is (1,2)* - πwg- closed set but not (1,2)* - wg closed set.

Example: 3.8 Let X= {a, b, c}, τ_1 = {φ, X, {a},{c},{a, c},{b, c}}, τ_2 = {φ, X, {a},{a, b}}. Then $\tau_{1,2}$ -open ={φ, X, {a},{c},{a, b},{a, c},{b, c}} and $\tau_{1,2}$ -closed = {φ, X, {b, c},{a, b},{c},{b},{a}}

Here A = {a, c} is (1, 2)* - gpr closed but not (1, 2)* -πwg- closed set.

Example: 3.9 Let X= {a, b, c, d}, τ_1 = {φ, X, {b},{b, c}}, τ_2 = {φ, X, {c},{b, c, d}}.Then $\tau_{1,2}$ –open = {φ, X, b},{c} , {b, c},{b, c, d}} and $\tau_{1,2}$ -closed = {φ, X, {({a, c, d},{a, b, d},{a, d},{a}}

Here A = {c, d} is (1, 2)* - πwg- closed set but not (1, 2)* -α-closed set.

Theorem: 3.10 Every (1, 2)* - πwg- closed set in X is (1, 2)* - rwg closed.

Proof: Let A be a (1,2)* - πwg- closed set in X and A⊂ U and U is (1,2)* - RO(X).

Since every (1, 2)* - RO(X) is (1, 2)* -πO(X) and A is (1, 2)* - πwg- closed set, then $\tau_{1,2}$- cl ($\tau_{1,2}$ -int (A))⊂U whenever A⊂ U and U ∈ (1,2)* - RO(X).

The above implies A is (1, 2)* - rwg -closed.

Remark: 3.11 The converse of the above need not be true as seen in the following example.

Example: 3.12 Let X = {a, b, c, d}, τ_1 = {φ, X, {d}, {b, c, d}}, τ_2 = {φ, X, {b}, {d}, {b, d},{a, b, d},{b, c, d}}. Then $\tau_{1,2}$-open = {φ, X, {b},{d},{b, d},{a, b, d},{b, c, d}} and $\tau_{1,2}$-closed = {φ, X,{a, c, d},{a, b, c}, {a, c },{c},{a}}.

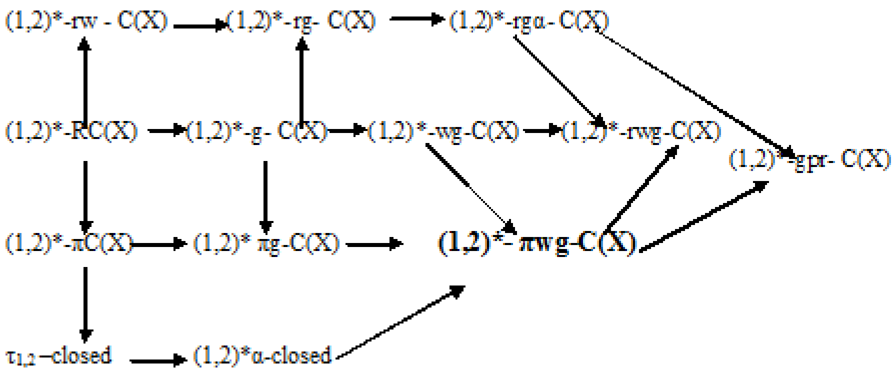
Here A = {b, d} is (1, 2)* - rwg- closed set but not (1, 2)* - πwg closed set.

Remark: 3.13 The concepts of (1, 2)*- πwg -closed set, (1, 2)* -rgα closed set are independent.

Example: 3.14 Let X = {a, b, c, d}, τ_1 = {φ, X, {b},{d}{b, d}}, τ_2 = {φ, X, {b},{d},{b, d},{a, b, d},{b, c, d}}. Then $\tau_{1,2}$ -open = {φ, X, {b},{d},{b, d},{a, b, d},{b, c, d}} and $\tau_{1,2}$ -closed = {φ, X,{a, c, d},{a, b, c}, {a, c },{c},{a}}. Here the (1,2)*- rgα -closed sets are { φ, X,{a},{c},{a, c},{b, d},{a, b, c},{a, b, d} {a, c, d},{b, c, d}} and the (1, 2)*- πwg -closed sets are {φ, X,{a},{c},{a, b},{a, c},{a, d},{b, c}, {c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}} .

The set A = {b, d} is (1, 2)* - rgα -closed but not (1,2)*- πwg -closed and B= {a, b} is πwg -closed but not (1, 2)*- rgα -closed .

Remark: 3.15 The above discussions are summarized in the following diagram.



Remark: 3.16 Finite union of (1, 2)* - πwg closed sets need not be (1, 2)* - πwg closed set.

Example: 3.17 Let X= {a, b, c}, τ_1 = {φ, X, {a},{c},{a, c},{b, c}}, τ_2 = {φ, X, {a},{a, b}}.Then $\tau_{1,2}$ -open = {φ, X, {a},{c},{a, b},{a, c},{b, c}} and $\tau_{1,2}$ -closed = {φ, X, {b, c},{a, b},{c},{b},{a}}

Here A = {a} and B= {c} are two (1, 2)* -πwg- closed sets, but A ∪ B = {a, c} is not (1, 2)*- πwg closed.

Remark: 3.18 Finite intersection of two (1, 2)* - πwg closed sets need not be (1, 2)* - πwg closed set.

Example: 3.19 Let X = {a, b, c, d}, τ_1 = {φ, X, {d},{b, c, d}}, τ_2 = {φ, X, {b},{b, d},{a, b, d}}. Then $\tau_{1,2}$ -open ={φ, X, {b},{d},{b, d},{a, b, d},{b, c, d}} and $\tau_{1,2}$ -closed ={φ, X,{a, c, d},{a, b, c},{a, c}, {c},{a}}. Here A = {a, b, d} and B= {b, c, d} are (1,2)* - πwg closed sets ,but A∩B= {b, d} is not (1,2)*- πwg closed.

Theorem: 3.20 If A is (1, 2)* - πwg closed set and $A \subset B \subset \tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) . Then B is also (1, 2)* - πwg -closed set in X.

Proof: Let $B \subset U$, where U is (1, 2)* - π- open. Then $A \subset B \Rightarrow A \subset U$, U is (1, 2)* - π- open. Since A is (1, 2)* - πwg closed, $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) ⊂ U. By hypothesis, $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(B)) ⊂U. Hence B is also (1, 2)* - πwg - closed.

Theorem: 3.21 If A is both (1, 2)*- Regular open and (1, 2)* -πwg closed, then it is (1, 2)*- clopen.

Proof: Since A is (1, 2)* -Regular open, A is $\tau_{1,2}$ -open.

Then A= $\tau_{1,2}$-int(A).Also, A⊂A and A is (1,2)* - πwg closed.
⇒ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) ⊂A. Now, $\tau_{1,2}$ -cl (A) = $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) ⊂A.
⇒ $\tau_{1,2}$ -cl (A)= A. Hence A is $\tau_{1,2}$ -clopen.

Theorem: 3.22 The following properties are equivalent for a subset A of X.
1. A is $\tau_{1,2}$ -clopen.
2. A is (1, 2)* - regular open and (1, 2)* - πwg closed.
3. Ais (1, 2)* - π-open and (1, 2)*- πwg closed.

Proof:
(1)⇒(2): let A is $\tau_{1,2}$ -clopen. Then A = $\tau_{1,2}$-int (A)= $\tau_{1,2}$-cl (A).

⇒ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A))=A.

⇒ A is (1, 2)* -regular open and hence A is (1, 2)* - π-open .Then $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A))=A⊂A.

⇒A is (1, 2)* - πwg -closed. Hence (2) holds.

(2)⇒(3): Obvious.

(3)⇒(4): let A is (1,2)*- π-open and (1,2)*- πwg-closed. Since A⊂A, a(1, 2)* - π-open set and $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) ⊂ A.

⇒ A is both $\tau_{1,2}$ -closed and $\tau_{1,2}$- open.

⇒A is $\tau_{1,2}$-clopen .

Theorem: 3.23 If A is (1, 2)* - πwg -closed, then $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) – A contains no non-empty (1,2)*- π -closed set.

Proof: Suppose that F is a non-empty (1, 2)*- π-closed subset of $\tau_{1,2}$-cl($\tau_{1,2}$ -int(A)) – A.

Now, F ⊂ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) – A

⇒ F ⊂ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) ∩ A^C .So,F ⊂ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) and F ⊂A^C . F⊂ A^C implies A⊂ F^C.

Since F^C is π-open and A is (1,2)*- πwg -closed. We have, $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) ⊂ F^C.

⇒F ⊂ $[\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A))$]^C$.

Hence F ⊂ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) ∩ $[\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A))$]^C$.

⇒F ⊂ φ, which is a contradiction.

⇒ [$\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A))] – A contains no non-empty (1,2)*- πwg- closed set.

Theorem: 3.24 Suppose that B ⊂A⊂X, B is (1,2)* - πwg -closed set relative to A and that A is both (1,2)*-regular open and (1,2)*- πwg- closed subset of X, then B is (1,2)*- πwg -closed set relative to X.

Proof: Let B ⊂ G and G be (1, 2) * -π -open set in X. Given B⊂A⊂X.

⇒ B ⊂ A∩G. Since B is (1,2)* - πwg- closed set relative to A, then $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int_A(B)) ⊂ A∩G.

Also, A∩ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(B)) ⊂ A∩G.Then A∩ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(B)) ⊂ G. Since A is (1,2)* - regular open and (1,2)* - πwg -closed set, then A is $\tau_{1,2}$ –clopen. i,e., A = $\tau_{1,2}$ -cl(A) and $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(B))⊂ $\tau_{1,2}$ -cl(B)⊂ $\tau_{1,2}$ -cl(A)=A. Hence $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(B))∩A = $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(B)) and $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int B) ⊂ G whenever B ⊂G and G is (1,2)*- π-open in X. Hence B is (1,2)*- πwg -closed.

Theorem: 3.25 Let A⊂Y⊂X. Suppose that A is (1, 2)*- πwg-closed in X and Y is π-open in X, then A is (1, 2)*- πwg-closed set relative to Y.

Proof: Given A⊂Y⊂X and A is (1, 2)*- πwg-closed in X. Let A ⊂Y∩G, where G is π-open in X. Since A is (1, 2)* - πwg closed in X, A⊂G ⇒ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A))⊂ G.

Y ∩ $\tau_{1,2}$ -cl ($\tau_{1,2}$ -int(A)) ⊂ Y∩G. Therefore, A is (1, 2)*- πwg-closed set relative to Y.

Theorem: 3.26
1. Every $\tau_{1,2}$- open set is (1,2)*- πwg-open.
2. Every $\tau_{1,2}$- g -open set is (1,2)*- πwg-open.
3. Every $\tau_{1,2}$- wg-open set is (1,2)*- πwg-open.

Proof: Straight forward.

Remark: 3.27 The converse of the above need not be true as seen in the following examples.

Example: 3.28 Let X={a, b, c, d}, τ_1 = {φ, X,{a},{b},{a, b}}, τ_2 = {φ, X, {a},{b},{a, b},{a, b, c},{a, b, d}}. Then $\tau_{1,2}$-open ={φ, X, {a},{b},{a, b},{a, b, c},{a, b, d}},$\tau_{1,2}$-closed ={φ, X,{b, c, d},{a, c, d},{c, d} ,{d},{c}}. Then the (1,2)*-πwg-open sets are{ φ, X,{a, b, d},{a, b, c},{b, d},{b, c},{a, d},{a, c},{a, b},{d},{c},{b},{a}}. Here A= {b, d}, B= {b, c}, {a, c}, {c}, {d} are not $\tau_{1,2}$-open but they are (1, 2)*- πwg-open.

Example: 3.29 Let X= {a, b, c, d}, τ_1 = {φ, X, {b}, {b, c}}, τ_2 = {φ, X, {c}, {b, c, d}}. Then $\tau_{1,2}$–open = {φ, X, {b}, {c},{b, c},{b, c, d}} and $\tau_{1,2}$-closed = {φ, X, {({a, c, d},{a, b, d},{a, d},{a}}

Here A = {a, b}, B= {a, c}, C= {a}, D= {b} are (1, 2)* - πwg-open but not (1, 2)* -g-open.

Example: 3.30 In example 3.29, the sets {a}, {a, b}, {a, c} are (1, 2)* - πwg-open but not (1, 2)* -wg-open.

Theorem: 3.31 If A is (1, 2)*- πwg-open and $\tau_{1,2}$-int $\tau_{1,2}$-cl (A) ⊂B ⊂A. Then B is (1, 2)*- πwg-open.

Proof: Let A be (1, 2)*- πwg-open set, A^c is (1,2)*- πwg-closed set. Since $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) ⊂B⊂A, $A^C \subset B^C \subset$ [$\tau_{1,2}$-int ($\tau_{1,2}$-cl (A))]^C. By theorem (3.20), B^C is (1,2)*- πwg-closed.

⇒ B is (1, 2)*- πwg-open.

4. (1, 2)* - πwg – Continuous and (1,2)*- πwg -Irresolute function.

Definition: 4.1 A function f: (X,τ_1, τ_2) →(Y, σ_1, σ_2) is called (1,2)* - πwg- continuous if every f^{-1}(V) is (1,2)* - πwg-closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2).

Definition: 4.2 A function f: (X, τ_1, τ_2) →(Y, σ_1, σ_2) is called (1, 2)* - πwg- irresolute if every f^{-1}(V) is (1, 2)* - πwg-closed in (X, τ_1, τ_2) for every (1, 2)* - πwg -closed set V of (Y, σ_1, σ_2).

Theorem: 4.3: Every (1, 2)*- continuous map is (1, 2)*- πwg-continuous.

Proof: Let f: (X, τ_1, τ_2) →(Y, σ_1, σ_2) be a (1, 2)*- continuous map and V be any $\sigma_{1,2}$-closed set in Y. Then f^{-1}(V) is $\tau_{1,2}$-closed in X. Every $\tau_{1,2}$-closed set is (1,2)*- πwg-closed. Then f^{-1}(V) is (1, 2)*- πwg-closed in X. Therefore, f is (1, 2)*-πwg-continuous.

Remark: 4.4 The converse of the above need not be true as shown in the following example.

Example: 4.5 Let X= {a, b, c}, τ_1 = {φ, X, {b}}, τ_2 = {φ, X, {a, c}}. Then $\tau_{1,2}$-open = {φ, X, {b},{a, c} and $\tau_{1,2}$-closed = {φ, X, {a, c},{b}} .Then (1,2)* - πwg-closed sets are ={φ, X,{a},{b},{c},{a, b} ,{a, c},{b, c}}. Let Y= {a, b, c}, σ_1 = {φ, X, {b, c}}, σ_2 = {φ, X, {c}}, $\sigma_{1,2}$-open = {φ, Y,{c},{b, c}} $\sigma_{1,2}$-closed = { φ, Y, {a, b},{a}}. Define f: (X, τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=a, f(b)=b, f(c)=c. The inverse image of the closed set in $\sigma_{1,2}$ are (1,2)*- πwg-closed in X.

Hence f is (1, 2)*- πwg-continuous. But f is not (1, 2)*- continuous, because f^{-1} ({c}) = {c} and f^{-1}({b, c}) = {b, c} are not $\tau_{1,2}$-closed in X.

Theorem: 4.6 If f is (1, 2)*- g- continuous, then f is (1, 2)*- πwg- continuous.

Proof: Similar to that of the proof in theorem 4.3.

Remark: 4.7 The converse of the above need not be true as seen in the following example.

Example: 4.8 Let X = Y = {a, b, c, d}, τ_1 = {φ, X, {d}, {b, c, d}}, τ_2 = {φ, X, {b}, {d}, {b, d}, {a, b, d}, {b, c, d}}. Then $\tau_{1,2}$-open = {φ, X, {b},{d},{b, d},{a, b, d},{b, c, d}} and $\tau_{1,2}$-closed = {φ, X, {a, c, d},{a, b, c},{a, c },{c},{a}}. Let σ_1= {φ, Y, {a}}, σ_2= {φ, Y, {d}, {a, d}}. Then $\sigma_{1,2}$- open ={φ, Y, {a},{d},{a, d}}. $\sigma_{1,2}$- closed ={ φ, Y, {b, c, d},{a, b, c}, {b, c}}. Define f: (X,τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=a, f(b)=b, f(c)=c, f(d)=d. Then the inverse images of the above are the same. Here the inverse image of the elements in $\sigma_{1,2}$- closed set are (1, 2)*- πwg- closed in X and f^{-1}({b, c, d}) = {b, c, d} , f^{-1}(b, c)={b, c} are not (1,2)*- g -closed in X.

Theorem: 4.9 If f is wg- continuous, then f is (1, 2)*- πwg- continuous.

Proof: Similar to the proof as in theorem 4.3

Remark: 4.10 The converse of the above need not be true is shown in the following example.

Example: 4.11 Let X = Y={a, b, c, d}, τ_1 = {φ, X, {b}, τ_2 = {φ, X, {c},{b, c},{b, c, d}}. Then $\tau_{1,2}$ -open = {φ, X, {b},{c},{b, c},{b, c, d}} and $\tau_{1,2}$ -closed = {φ, X, {a, c, d},{a, b, d },{a, d },{a}}. Let σ_1={ φ, Y, {a, b, c}}, σ_2={ φ, Y,{a, c}}, $\sigma_{1,2}$-open={φ, Y,{a, c},{a, b, c}}, $\sigma_{1,2}$-closed ={ φ, Y,{b, d},{d}} Define f: (X,τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=a, f(b) = b, f(c) = c, f(d) = d. Here the inverse image of the elements in $\sigma_{1,2}$- closed set are (1, 2)*- πwg- closed in X and f^{-1}({b, d}) = {b, d} is not (1, 2)*- wg -closed in X.

Theorem: 4.12 Every (1, 2)*- πg- Continuous map is (1,2)*- πwg-continuous.

Proof: Similar to that of the proof in theorem 4.3

Remark: 4.13 The converse of the above need not be true as seen in the following example.

Example: 4.14 Let X ={a, b, c, d}=Y, τ_1 ={φ, X,{a},{c, d},{a, c, d}},τ_2 = {φ, X, {a},{c},{a, c}}. Then $\tau_{1,2}$ -open ={φ, X,{a},{c},{a, c},{c, d},{a, c, d}} ,$\tau_{1,2}$ -closed={φ, X,{b, c, d},{a, b, d},{b, d},{a, b},{b}} , σ_1={φ, Y,{a}},σ_2={φ, Y,{a, b, c}}, $\sigma_{1,2}$-open ={φ, Y,{a},{a, b, c}}, $\sigma_{1,2}$-closed={ φ, Y,{b, c, d},{d}} Define f:(X,τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=a, f(b)=b, f(c)=c, f(d)=d. The map is (1, 2)*- πwg- continuous, but f^{-1}{d} = {d} is not (1, 2)*- πg-closed .Hence the map is not (1, 2)*- πg- continuous.

Theorem: 4.15 Every (1, 2)*- πwg - continuous map is (1, 2)* - rwg continuous.

Proof: Straight forward.

Remark: 4.16 The converse of the above need not be true as shown in the following example.

Example: 4.17 Let X = {a, b, c}, τ_1 = {φ, X,{b},{b, c}}, τ_2 = {φ, X,{c}}. Then $\tau_{1,2}$ -open = {φ, X,{b},{c}, {b, c}} and $\tau_{1,2}$ -closed ={φ, X,{a, c},{a, b},{a}}. Let Y= {a, b, c},σ_1 ={φ, X,{a}}, σ_2 = {φ, X,{b, c}}, $\sigma_{1,2}$ -open = {φ, Y, {a},{b, c}}, $\sigma_{1,2}$ -closed = {φ, Y,{b, c},{a}}. Define f: (X, τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=a, f(b)=b, f(c)=c. Then the inverse images are also the same. The inverse image of the closed set in $\sigma_{1,2}$ are (1,2)*- rwg-closed in X. Hence f is (1, 2)*- rwg-continuous. But f is not (1, 2)*- πwg-continuous, because f^{-1}({b, c}) = {b, c} is not (1, 2)*- πwg- closed in X.

Theorem: 4.18 Every (1, 2)* - πwg - continuous map is (1, 2)* - gpr- continuous.

Proof: Straight forward.

Remark: 4.19 The converse of the above need not be true as shown in the following example.

Example: 4.20 In Example 4.17, the map f is (1, 2)* - gpr continuous but f^{-1}({b, c}) = {b, c} is not (1, 2)*- πwg- closed in X. Hence f is not (1, 2)*- πwg-continuous.

Remark: 4.21 The concepts of (1, 2)*- πwg-continuous and (1, 2)*- rg- continuous are independent.

Example: 4.22 Let X=Y= {a, b, c, d}, τ_1 = {φ, X, {a},{c, d},{a, c, d}}, τ_2 = {φ, X, {a},{c},{a, c}}. Then $\tau_{1,2}$ -open ={φ, X,{a},{c},{a, c},{c, d},{a, c, d}} ,$\tau_{1,2}$ -closed={φ, X,{b, c, d},{a, b, d},{b, d},{a, b},{b}} σ_1= { φ, Y,{a},{a, b}}, σ_2= {φ, Y,{b}}, $\sigma_{1,2}$-open= {φ, Y,{a},{b},{a, b}}, $\sigma_{1,2}$- closed= { φ, Y,{b, c, d},{a, c, d},{c, d}}, Define f: (X,τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=c, f(b)=b, f(c)=a, f(d)=d. Here The inverse image of all $\sigma_{1,2}$- closed sets are (1,2)*- rg-closed in X, but not (1,2)*- πwg-closed in X .Hence the function f is (1,2)*- rg-continuous and not (1,2)*- πwg-continuous. (i.e, f^{-1}{a, c, d}={a, c, d} is not (1, 2)*- πwg-closed in X)

Let X, Y, τ_1, τ_2, $\tau_{1,2}$ -open ,$\tau_{1,2}$ –closed be as above in the same example.

Let σ_1= { φ, Y,{a}},σ_1={ φ, Y,{a, b, c}}.Then $\sigma_{1,2}$-open ={ φ, Y,{a},{a, b, c}} and $\sigma_{1,2}$-closed ={φ, Y,{b, c, d},{d}}.

Define f: (X, τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=c, f(b)=b, f(c)=a, f(d)=d. Here the inverse image of all $\sigma_{1,2}$- closed sets are (1,2)*- πwg-closed in X, but not (1,2)*-rg-closed in X. Hence f is (1, 2)*- πwg-continuous in X and not (1, 2)*- rg-continuous in X. (i.e . f^{-1}{d}= {d} is not (1,2)*-rg-closed in X)

Remark: 4.23 The concepts of (1, 2)*- πwg continuous, (1, 2)*- rgα- continuous are independent.

Example: 4.24 Let X = Y= {a, b, c}, τ_1 = {φ, X,{b}, τ_2 = {φ, X,{c},{b, c}}. Then $\tau_{1,2}$ -open ={φ, X, {b} , {c},{b, c}} and $\tau_{1,2}$ -closed = {φ, X,{a, c},{a, b},{a}}. Let σ_1={ φ, Y, {a}}, σ_2={φ, Y}}, $\sigma_{1,2}$-open= {φ, Y,{a}},$\sigma_{1,2}$ -closed ={φ, Y, {b, c}}. Define f: (X,τ_1, τ_2)→(Y, σ_1, σ_2) by f(a)=a, f(b)=b , f(c)=c . Here f^{-1}({b, c}) = {b, c}, is not (1, 2)*- πwg-closed in X. But the inverse image of $\sigma_{1,2}$ -closed sets are (1, 2)*- rgα-closed in X. Hence f is (1, 2)* - rgα -continuous and not (1, 2)*- πwg -continuous.

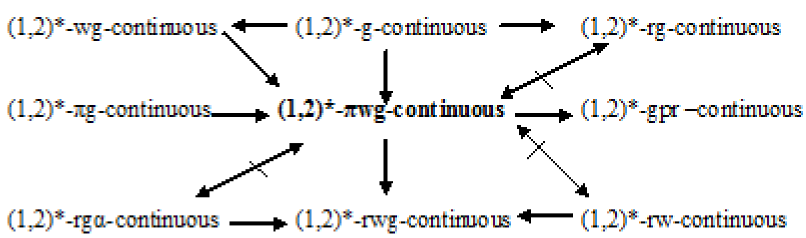
Let X = Y= {a, b, c, d}, τ_1 = {φ, X, {d}, {b, c, d}}, τ_2 = {φ, X, {b}, {d}, {b, d},{a, b, d}, {b, c, d}}. Then $\tau_{1,2}$ -open = {φ, X, {b},{d},{b, d},{a, b, d},{b, c, d}} and $\tau_{1,2}$ -closed = {φ, X, {a, c, d},{a, b, c},{a, c},{c},{a}}. Let σ_1= {φ, Y, {d}}, σ_2= { φ, Y, {c, d}}. Then $\sigma_{1,2}$- open ={φ, Y, {d},{c, d}}.$\sigma_{1,2}$- closed ={φ, Y, {a, b, c},{a, b}}. Define f: (X,τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=a, f(b)=b, f(c)=c, f(d)=d. Here the (1,2)*- πwg- closed sets are {φ, X, {a}, {c} ,{a, b} ,{a, c} ,{a, d}, {b, c},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}} and (1,2)*- rgα-closed sets are {φ, X, {a}, {c},{a, c} , {b, d}, {a, b, c},{a, b, d},{a, c, d},{b, c, d}}. Here the inverse image of all $\sigma_{1,2}$- closed sets are (1,2)*- πwg- closed in X, but f^{-1}{a, b}={a, b} is not (1,2)*- rgα-closed in X. Hence f is (1, 2)*- πwg -continuous in X and not (1, 2)*- rgα-continuous in X.

Remark: 4.25 The concepts of (1, 2)*- πwg-continuous, (1, 2)* - rw-continuous are independent.

Example: 4.26 Let X = Y= {a, b, c, d}, τ_1 = {φ, X, {d}, {b, c, d}}, τ_2 = {φ, X,{b},{d},{b, d},{a, b, d}, {b, c, d}}. Then $\tau_{1,2}$ -open = {φ, X, {b},{d},{b, d},{a, b, d},{b, c, d}} and $\tau_{1,2}$ -closed = {φ, X, {a, c, d},{a, b, c},{a, c},{c},{a}}. Let σ_1= {φ, Y, {a}, {a, c}}, σ_2= {φ, Y, {c}}. Then $\sigma_{1,2}$- open ={φ, Y,{a},{c},{a, c}}. $\sigma_{1,2}$- closed ={ φ, Y, {b, c, d},{a, b, d},{b, d}}. Define f: (X,τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=a, f(b)=b, f(c)=c, f(d)=d. Here the (1,2)*- πwg- closed sets are { φ, X, {a}, {c} ,{a, b} ,{a, c} ,{a, d}, {b, c},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}} and (1, 2)*- rw-closed sets are {φ, X, {a}, {c},{a, c} ,{b, d}, {a, b, c},{a, b, d},{a, c, d},{b, c, d}}. Here the inverse image of all $\sigma_{1,2}$- closed sets are (1,2)*- rw- closed in X, but not (1,2)*- πwg -closed in X (i.e,f^{-1}{b, d}={b, d} is not (1,2)*- πwg -closed in X). Hence f is (1, 2)*-rw-continuous and not (1, 2)*- πwg -continuous in X.

Suppose, let f: (X,τ_1, τ_2) →(Y, σ_1, σ_2) be defined as f{a}={b}, f{b}={a}, f{c}= {c}, f(d)={d}. Then the inverse image of all $\sigma_{1,2}$- closed sets are (1, 2)*- πwg-closed in X, but not (1, 2)*- rw-closed in X (i.e, f^{-1}{b, d}={a, d} is not (1,2)*-rw-closed in X). Hence f is (1, 2)*- πwg -continuous but not (1, 2)*- rw-continuous.

Remark: 4.27 From the above discussions and known results we have the following implications.



Remark: 4.26 The composition of two (1, 2)*-πwg-continuous functions need not be (1, 2)*-πwg continuous.

The fact given above is shown in the following example.

Example: 4.27 Let X=Y=Z={a, b, c}, τ_1={φ, X,{a},{a, b}}, τ_2={φ, X,{b}},$\tau_{1,2}$ -open ={φ, X,{a},{b},{a, b}}, $\tau_{1,2}$ -closed ={φ, X, {b, c},{a, c},{c}},σ_1= {φ, Y, {a}}, σ_2= {φ, Y, {a, b}}, $\sigma_{1,2}$-open = {φ, Y, {a} , {a, b}}, $\sigma_{1,2}$- closed = {φ, Y, {b, c},{c}}, η_1= { φ, Z, {a}}, η_2= {φ, Z, {a, b}}, $\eta_{1,2}$ -open ={φ, Z, {a},{a, b}}, $\eta_{1,2}$ -closed= {φ, Z, {b, c},{c}}. Define f: X→Y by f(a)= b, f(b)=a, f(c)=c. Here f is (1, 2)*- πwg continuous. Define g: Y→Z by g(a)=a, g(b)=b, g(c)=c. Also the map g is (1, 2)*- πwg- continuous. But $(gof)^{-1}$({b, c}) = {a, c} is not (1,2)*- πwg continuous.

Theorem: 4.28 Every (1, 2)*- πwg -irresolute function is (1, 2)*- πwg - continuous, but not conversely.

Proof: Let f: (X, τ_1, τ_2) →(Y, σ_1, σ_2) is (1, 2)*- πwg - irresolute and V is $\sigma_{1,2}$-closed set in Y. Then V is (1, 2) *- πwg - closed in Y. Also, f is (1, 2)* πwg -irresolute, f^{-1}(V) is (1, 2)*- πwg-closed in X. Hence f is (1,2)*- πwg -continuous. The converse of the above need not be true. We show the converse by the following example.

Example: 4.29 Let X = Y={a, b, c}, τ_1= {φ, x, {a}, {a, c}}, τ_2= { φ, x, {c}}, $\tau_{1,2}$ -open = { φ, x,{a}, {c},{a, c}}, $\tau_{1,2}$ -closed= { φ, x,{b},{a, b},{b, c}}, σ_1= {φ, Y, {a}, {a, b}}, σ_2= { φ, Y}, $\sigma_{1,2}$-open= { φ, Y, {a}, {a, b}}, $\sigma_{1,2}$- closed= {φ, Y, {b, c }, {c}}, Define f: (X,τ_1, τ_2) →(Y, σ_1, σ_2) by f(a)=a, f(b)=c, f(c)=b. Here the map f is (1, 2)*- πwg-continuous. But f^{-1}{b} ={c} and f^{-1} ({a, b}) = {a, c} are not (1, 2)*- πwg- closed in X. Hence f is not (1, 2)*- πwg - irresolute.

Theorem: 4.30 Let f: X→Y and g: Y→Z be any two functions. Then (gof) is (1, 2)*- πwg -continuous if g is (1, 2)* - continuous and f is (1, 2)*- πwg-continuous.

Proof: Let V be any $\eta_{1,2}$ -closed set in Z. Then g^{-1}(V) is $\sigma_{1,2}$-closed in Y. Since g is (1, 2)*- continuous.

Thus $f^{-1}[g^{-1}(V)]$ is (1, 2)*- πwg - closed in X and f is (1, 2)*- πwg -continuous. Then (gof) is (1, 2)*- πwg- continuous.

Theorem: 4.31 Let f: X→Y and g: Y→Z be any two functions. Then (gof) is (1, 2)*- πwg -irresolute if g is (1, 2)* -irresolute and f is (1, 2)*- πwg- irresolute.

Proof: Let U be any (1, 2)*- πwg- closed set in Z. Since g is (1, 2)*- πwg irresolute, $g^{-1}(U)$ is (1, 2)*- πwg-closed in Y. Then $f^{-1}[g^{-1}(U)]= (gof)^{-1}(U)$ is (1, 2)*- πwg -closed in X. Therefore, (gof) is (1, 2)*- πwg -irresolute.

Theorem: 4.31 Let f: X→Y and g: Y→Z be any two functions. Then (gof) is (1, 2)*- πwg -continuous if g is (1, 2)* -πwg -continuous and f is (1, 2)*- πwg- irresolute.

Proof: Let V be any $\eta_{1,2}$ - closed set in Z. Since g is (1, 2)*- πwg -continuous, $g^{-1}(V)$ is (1, 2)*- πwg -closed in Y. Then $f^{-1}[g^{-1}(V)]= (gof)^{-1}(V)$ is (1, 2)*- πwg- closed in X and f is (1, 2)* - πwg -irresolute. Therefore (gof) is (1, 2)*-πwg -continuous.

5. APPLICATIONS

Here, we introduce and study (1, 2)*- $T_{\pi wg}$ -Space and study its relationship with other existing spaces.

Definition: 5.1 A Bitopological space X is called (X, τ_1, τ_2) is

1) (1,2)* - πwg -$T_{1/2}$- space if every (1,2)*- πwg -closed set in X is (1,2)*-g-closed in X.
2) (1,2)*-$T_{\pi wg}$-space if every (1,2)*- πwg -closed subset of X is closed in X.

Proposition: 5.2 Every (1, 2)*-$T_{\pi wg}$ -Space is

(i) (1, 2)*-T_{wg}–space,
(ii) (1, 2)*-α-space,
(iii) (1, 2)*-$T_{1/2}$-space and
(iv) (1, 2)*-T_{ω}-space.

Proof: Let (X,τ_1, τ_2) is (1,2)*- $T_{\pi wg}$ -Space and let A be (1,2)*-wg closed set in X. Then it is (1, 2)*- πwg -closed. Since X is (1, 2)*- $T_{\pi wg}$ -space, A is closed, hence X is (1, 2)*- T_{wg}-space.

Remark 5.3: Similar arguments for (ii), (iii) and (iv).

Remark 5.4: The converse of the above need not be true as seen in the following examples.

Example: 5.5 Let X = {a, b, c, d}, τ_1= {φ, X, {a},{b}, {a, b}}, τ_2= { φ, X ,{a},{b},{a, b},{a, b, d},{a ,b, c}}, $\tau_{1,2}$ -open ={ φ, X,{a},{b},{a, b},{a, b, d},{a, b, c}}, $\tau_{1,2}$ -closed={φ, x,{b, c, d},{a, c, d},{c, d},{d},{c}}. Here the (1, 2)*-wg -closed sets are in $\tau_{1,2}$ - closed in X and not (1,2)*- πwg -closed in X. Hence the space is T_{wg}-space but not $T_{\pi wg}$- space.

Example: 5.6 In Example 5.4, (1, 2)*- α closed sets are $\tau_{1,2}$- closed in X. Hence the space is (1, 2)*- α space. But the (1, 2)*- πwg -closed sets are not $\tau_{1,2}$ - closed in X. Hence the (1, 2)*- α- space need not be a (1, 2)*- πwg - space.

Example: 5.7 Let X= {a, b, c}, τ_1= {φ, X, {b}}, τ_2= {φ, X, {b}, {a, b}, {b, c}}, $\tau_{1,2}$ -open = {φ, X,{b},{a, b},{b, c}}, $\tau_{1,2}$ - closed= {φ, X,{a, c},{c},{a}}. Here the (1, 2)*-g-closed sets are closed in X. Hence the space X is (1, 2)*- $T_{1/2}$-Space. But the (1, 2)*- πwg -closed sets are not $\tau_{1,2}$ -closed in X. Hence every (1, 2)*- πwg-space is a (1, 2)*-$T_{1/2}$-space but not conversely.

Example: 5.8 In Example 3.12, the (1, 2)* - w-closed sets are $\tau_{1,2}$ -closed in X. Hence the space X is a (1, 2)*-T_{ω}-space, but the (1, 2)*- πwg -closed sets are not $\tau_{1,2}$ - closed in X. So, (1, 2)*-T_{ω} -space need not be a (1, 2)*-$T_{\pi wg}$ -Space.

Proposition: 5.8 If a space X is (1, 2)*- πwg-$T_{1/2}$ -Space, then every singleton set of X is either (1, 2)*- π-closed or (1, 2)*- g -open.

Proof: Let x∈X and assume that {x} is not (1, 2)*- π-closed. Then clearly X-{x} is trivially a (1, 2)*- πwg- closed set. By our assumption, {x} is (1, 2)*-g -open.

Proposition: 5.9 For a space (X, τ_1, τ_2),

(i) (1, 2)*-GO(X, τ_1, τ_2) ⊂ (1, 2)*- πWGO(X, τ_1, τ_2).
(ii) A space is (1, 2)*- πwg-$T_{1/2}$-space iff (1, 2)*-GO(X, τ_1, τ_2) = (1, 2)*- πWGO(X, τ_1, τ_2).

Proof (i): Let A be (1, 2)*- g -open set, then X-A is (1,2)*- g-closed set. Since every (1, 2)*-g-closed set is (1, 2)*-

Hence X-A is (1, 2)*- πWGC(X) and hence A is (1, 2)*- πWGO(X).

$\Rightarrow$ (1, 2)*-GO(X,τ_1, τ_2) $\subset$ (1,2)*- πWGO(X)(X, τ_1, τ_2).

Proof (ii): Let X be (1, 2)*- πwg-$T_{1/2}$-space .Then A$\in$(1,2)*- πwg-open (X, τ_1, τ_2).

Then X-A is (1, 2)*- πwg-closed in X. By hypothesis, X-A is (1, 2)*-g -closed and then A$\in$(1,2)*- GO(X, τ_1, τ_2).

Therefore, (1, 2)*- GO (X, τ_1, τ_2) = (1, 2)* - πwg-open (X, τ_1, τ_2).

Conversely, let (1, 2)*- GO (X, τ_1, τ_2) = (1, 2)* - πwg-open (X, τ_1, τ_2).

Let A be (1, 2)*- πwg-closed set. Then X-A is (1, 2)*- πwg-open set. By assumption, X-A is (1, 2)*-GO(X). And then A is (1, 2)*-g-closed in X. Hence X is (1, 2)*- πwg-$T_{1/2}$ -Space.

Theorem: 5.10 Every (1, 2)*-$T_{\pi wg}$- Space is (1, 2)*- πwg-$T_{1/2}$-Space.

Proof: Straight forward.

Remark: 5.11 The converse of the above need not be true as shown in the following example.

Example: 5.12 In Example 4.8,the (1,2)*- πwg-closed sets are (1,2)*-g-closed in X but the (1,2)*- πwg-closed sets are not $\tau_{1,2}$ -closed in X .Hence the space is (1,2)*- πwg-$T_{1/2}$-Space but not (1,2)*- πwg-Space.

Theorem: 5.13 Let f: X$\rightarrow$Y and g: Y$\rightarrow$Z be any two functions. Then (gof) is (1, 2)*- g- continuous if f is (1, 2)*- πwg-irresolute, g is (1,2)*- πwg-continuous and Y is a (1,2)*- πwg-$T_{1/2}$-space.

Proof: Let V be a $\eta_{1,2}$ -closed set in Z. Then g^{-1}(V) is (1, 2)*- πwg closed in Y, since g is (1, 2)*- πwg-continuous. As Y is a (1, 2)*- πwg-$T_{1/2}$-space,g^{-1}(V) is (1,2)*-g-closed in Y. Irresoluteness of f implies that f^{-1} [g^{-1}(V)] is (1,2)*-g-closed in X. Hence (gof) is (1, 2)*-g-continuous.

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