International Journal of Mathematical Archive-3(5), 2012, 2029-2034
Available online through www.ijma.info ISSN 2229 – 5046

$g^{*\alpha}$- CLOSED SETS IN BITOPOLOGICAL SPACES

MRS.VERONICA VIJAYAN*
Department of Mathematics, Nirmala College for Women (Autonomous), Coimbatore-6410018, Tamil Nadu, India

A. DHANIS ARUL MARY
Department of Mathematics with Computer Application, Nirmala College for Women (Autonomous), Coimbatore-6410018, Tamil Nadu, India

(Received on: 19-05-12; Accepted on: 31-05-12)

ABSTRACT

In this paper we introduce a new class of sets (i, j) - $g^{*\alpha}$ - closed sets in bitopological spaces. Properties of these sets are investigated and we introduce new bitopological spaces (i, j)-$T^{*\alpha}_{1/2}$ and (i, j)-$*^{\alpha}T_{1/2}$ as applications.

***Key words:** (i, j)-g-closed sets, (i, j)-g*-closed sets, (i, j) - $g^{*\alpha}$ - closed sets; (i, j) -$T^{*\alpha}_{1/2}$ spaces, and (i, j) -$*^{\alpha}T_{1/2}$ space.*

1. INTRODUCTION

A triple (X, τ_1, τ_2) where X is non-empty set and τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly [10] initiated the study of such spaces. In 1985, Fukutake introduced the concepts of g-closed sets [8] in bitopological spaces. Recently Veera Kumar introduced and studied the concepts of g* -closed set [19] and g* -continuity in topological spaces.

The purpose of this paper is to introduce the new class of sets, namely $g^{*\alpha}$ - closed sets. Applying these sets, the author introduced the new class of spaces, namely (i, j) - $T^{*\alpha}_{1/2}$ spaces, (i, j)-$*^{\alpha}T_{1/2}$ spaces for bitopological spaces and investigate some of their properties. In this paper we study the relationship of $g^{*\alpha}$ - closed sets with the class of closed sets namely (i, j)-g*-closed[17],(i, j)-gs-closed[5],(i, j)-αg-closed [14], (i, j)-gsp-closed[5] , (i, j)-rg-closed set[16], (i, j) -sg-closed[6], and (i, j)-gpr-closed sets. Also we study the relationship of (i, j) - $T^{*\alpha}_{1/2}$, (i, j)-$*^{\alpha}T_{1/2}$ spaces with (i, j)-$T_{1/2}$ [7], (i, j) - $T^{*}_{1/2}$ [17], (i, j)-T_b spaces.

2. PRELIMINARIES

If A is a subset of X with a topology τ, then the closure of A is denoted by τ-cl (A) or cl(A), the interior of A is denoted by τ-int(A), or int(A) and the complement of A in X is denoted by A^c.

Definitions 2.1: A subset A of a bitopological space (X, τ_1, τ_2) is called
(i) a generalized open (g-open) set[17] if F ⊆ int(A) whenever F ⊆ A and F is closed in (X,τ).
(ii) a α-open set[15] if A ⊆ int(cl(int(A))).
(iii) a regular closed set[16] if A = cl(int(A)).
(iv) a semi-open set[12] if A ⊆ cl (int(A)).
(v) a semi-pre-open set[1] if cl(int(cl(A))) ⊆ A.
(vi) a preclosed set[17] if A = int(cl(A)).

Definition 2.2: A subset A of a bitopological space (X, τ_1, τ_2) is called
(i) (i, j)-g*-closed[17] if τ_j-cl(A) ⊆ U whenever A ⊆ U and U∈GO(X, τ)
(ii) (i, j)-gs-closed[5] if τ_j-scl(A) ⊆ U whenever A ⊆ U and U is open in τ_i.
(iii) (i, j)-αg-closed [14] if τ_j-αcl(A) ⊆ U whenever A ⊆ U and U∈τ_i.
(iv) (i, j)-gsp-closed[5] if τ_j-spcl(A) ⊆ U whenever A ⊆ U and U is open in τ_i.
(v) (i, j)-rg-closed set[16] if τ_j -cl(A) ⊆ U whenever A ⊆ U is and U is regular open in τ_i.
(vi) (i, j)-sg-closed[7] if τ_j-scl(A) ⊆ U whenever A ⊆ U and U is semi open in τ_i.
(vii) (i, j)-gpr-closed[9] if τ_j-pcl(A) ⊆ U whenever A ⊆ U and U is regular open in τ_i.

Corresponding author: MRS.VERONICA VIJAYAN*
Department of Mathematics, Nirmala College for Women (Autonomous),Coimbatore-6410018,Tamil Nadu, India

The family of all (i, j)-g-closed (resp (i, j)-gsp-closed, (i, j)-gp-closed, (i, j)-gpr-closed, (i, j)-αgr-closed,(i, j)-αg-closed, (i, j)-gs-closed) subsets of a bitopological space (X,τ_1,τ_2) is denoted by D(i, j).

Definition 2.3: (i) A bitopological space (X, τ_1, τ_2) is said to be an (i, j) - $T^*_{1/2}$ space if every (i, j)-g*-closed set is τ_j-closed.
(ii) A bitopological space (X, τ_1, τ_2) is said to be an (i, j)-T_b space if every (i, j)-gs-closed set is τ_j-closed.

3. (i, j) - $g^{*\alpha}$- CLOSED SETS

In this section we introduce the concept of (i, j) -$g^{*\alpha}$- closed sets in bitopological spaces.

Definition 3.1: A subset A of a bitopological space (X, τ_1, τ_2) is said to be an (i, j) - $g^{*\alpha}$- closed set if τ_j - $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in GO(X, \tau_i)$.

We denote the family of all (i, j) - $g^{*\alpha}$- closed sets in (X, τ_1, τ_2) by $D^{*\alpha}$ (i, j).

Remark 3.2: By setting $\tau_1 = \tau_2$, in Definition 3.1, a (i, j) –$g^{*\alpha}$- closed set is a g^*- closed set.

If A is (i) τ_j-closed, (ii) τ_j-α-closed, (iii) (i, j) - g^*- closed subset of (X, τ_1, τ_2), then A is (i,j)- $g^{*\alpha}$- closed.

The following examples show that the reverse implications of the above proposition are not true.

Example 3.4: Let X= {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ, {a}, {a, b}, X}. Then the subset A = {b} is (1, 2) - $g^{*\alpha}$-closed but not τ_2- closed in (X, τ_1, τ_2).

Example 3.5: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = {ϕ, {a}, X}. Then the subset {a, c} is $g^{*\alpha}$-closed but not τ_2 -α-closed.

Example 3.6: let X= {a, b, c}, τ_1= {ϕ, {a}, X}; τ_2 = {ϕ, {a}, {a, b}, X}. The subset {b} is (1, 2) - $g^{*\alpha}$ -closed but not (1, 2)-g*-closed.

Proposition 3.7: In a bitopological space (X, τ_1, τ_2) every (i, j) - $g^{*\alpha}$- closed set is (i) (i, j)-gs-closed, (ii) (i, j)-αg-closed, (iii) (i, j)-αgr-closed, (iv) (i, j)-gsp-closed.

The following examples support that the converse of the above theorem is not true.

Example 3.8: Let X = {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ, {a}, {a, b}, X}. In this example, the subsets {a, b}, {a, c} are (1, 2) - gs-closed but not (1, 2) - $g^{*\alpha}$-closed.

Example 3.9: Let X = {a, b, c}, τ_1= {ϕ, {c}, {a, b}, X}, τ_2= {ϕ, {a}, X,}. The subset{a, c} is (1, 2)- αg-closed but not (1, 2) - $g^{*\alpha}$-closed.

Example 3.10: let X = {a, b, c}, τ_1= {ϕ, {a}, X}; τ_2= {ϕ, {a}, {b, c}, X}. The subsets {b}, {c}, {a, b}, {a, c} are (1, 2) - αgr-closed but not (1, 2) - $g^{*\alpha}$-closed.

Example 3.11: let X= {a, b, c}, τ_1= {ϕ, {a}, X}; τ_2= {ϕ, {a}, {a, b}, X}. The subsets {a, b}, {a, c} are (1, 2)- gsp-closed sets but not (1, 2) - $g^{*\alpha}$-closed.

Proposition 3.12: In a bitopological space (X, τ_1, τ_2) every (i, j) - $g^{*\alpha}$- closed set is (i) (i, j)-gpr-closed, (ii) (i, j)-gp-closed, (iii) (i, j)-pre-semi-closed.

The following examples support that the reverse implications of the above propositions are not true.

Example 3.13: Let X= {a, b, c}, τ_1 = {ϕ, {a, b}, X}, τ_2 = {ϕ, {c}, X}. Then the subsets {a, c}, {b, c} are (1,2)-gpr-closed but not (1, 2) - $g^{*\alpha}$-closed.

Example 3.14: Let X = {a, b, c}, τ_1 = {ϕ, {a, b}, X}, τ_2 = {ϕ, {b}, {a, c}, X}. Then the subsets {a}, {c}, {a, b}, {b, c} are (1, 2)-gp-closed but not (1, 2) - $g^{*\alpha}$-closed.

Example 3.15: Let X = {a, b, c}, τ_1 = {ϕ, {a, b}, X}, τ_2 = {ϕ, {b}, {a, c}, X}. Then the subsets {a}, {c}, {a, b}, {b, c} are (1, 2)-pre semi-closed but not (1, 2) - $g^{*\alpha}$-closed.

Remark 3.16: The following examples show that (i, j) - g- closed sets and (i, j)- $g^{*\alpha}$- closed sets are independent.

Example 3.17: Let X = {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ, {a}, {a, b}, X}. Then the subsets A = {a, b}, {a, c} are (1, 2)-g-closed but not (1, 2) - $g^{*\alpha}$-closed.

Example 3.18: Let X= {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = {ϕ, {a}, X}. Then the subset B = {b} is (1, 2) - $g^{*\alpha}$-closed set but not (1, 2)-g-closed set.

Remark 3.19: The following examples show that (i, j) - $g^{*\alpha}$- closed sets and (i, j) - rg- closed sets are independent.

Example 3.20: Let X = {a, b, c}, τ_1 = {ϕ, {c}, {a, b}, X}, τ_2 = {ϕ, {a}, X}. Then the subsets A = {b}, {c} are (1, 2) - $g^{*\alpha}$-closed sets but not (1, 2) - rg-closed sets.

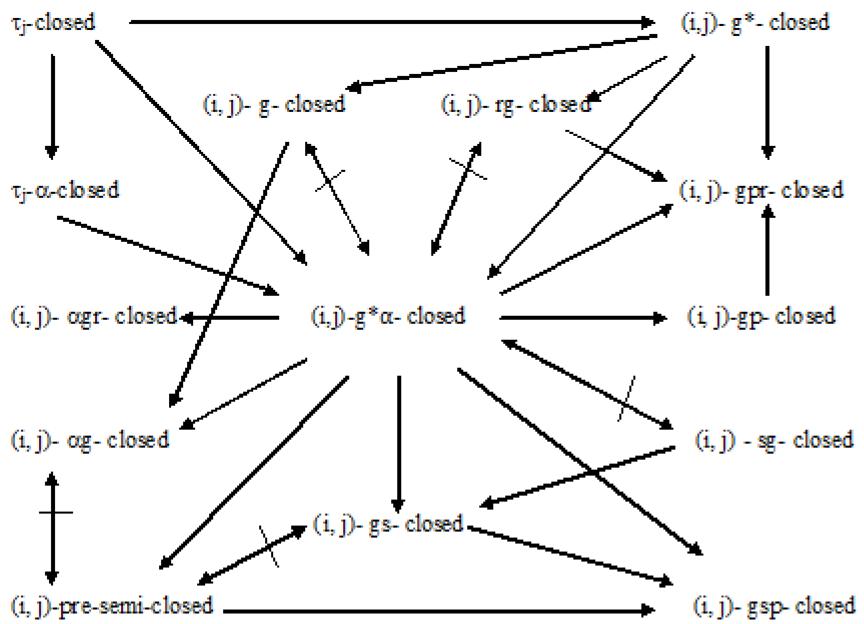
In the same example 3.20, the subset B = {a, c} is a (1, 2)-rg-closed set but not a (1, 2) - $g^{*\alpha}$-closed set.

Remark 3.21: The following examples show that (i, j) - sg- closed sets and (i, j) - $g^{*\alpha}$- closed sets are independent.

Example 3.22: Let X = {a, b, c}, τ_1 = {ϕ, {a, b}, X}, τ_2 = {ϕ, {b}, {a, c}, X}. Then the subsets A = {c}, {b, c} are (1, 2)-sg-closed but not (1, 2) - $g^{*\alpha}$-closed.

Example 3.23: Let X= {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ, {a}, {b, c}, X}. Then the subset B = {a} is (1, 2) - $g^{*\alpha}$-closed set but not a (1, 2)-sg-closed set.

All the above results can be represented by the following diagram.



Where A → B (resp. A↮ B) represents A implies B but not conversely (resp. A and B are independent)

Proposition 3. 24: If A, B ∈ $D^{*\alpha}$(i, j), then A∪B ∈ $D^{*\alpha}$(i, j).

Remark 3.25: $D^{*\alpha}$(1, 2) is generally not equal to $D^{*\alpha}$(2, 1). This is proved by an example below.

Example 3.26: Let X= {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = {ϕ, {a}, X}. Then the subset {b} is in $D^{*\alpha}$(1, 2) but not in $D^{*\alpha}$(2, 1).

Proposition 3.27: If $\tau_1 \subseteq \tau_2$, then $D^{*\alpha}$(2, 1) ⊆ $D^{*\alpha}$(1, 2).

The converse of the above proposition is not true as seen from the following example.

Example 3.28: Let X= {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = {ϕ, {a}, X}. Then $D^{*\alpha}$(2, 1) ⊆ $D^{*\alpha}$(1, 2) but the subsets {b}, {a, b} of τ_1 not contained in τ_2.

Proposition 3.29: If A is (i, j) - $g^{*\alpha}$- closed set, then τ_j - αcl(A) – A contains no non-empty τ_i – g-closed set.

Proof: Let A be (i, j)-$g^{*\alpha}$ -closed set and F be a τ_j-g-closed set such that F$\subseteq$ τ_j-αcl(A)-A then $F^c \supset$ A, And F^c is g-open. We have τ_j-αcl(A) $\subseteq$ F^c. Thus F$\subseteq$ τ_j-αcl (A) $\cap$ (τ_j-αcl(A)^c = ϕ

Corollary 3.30: If A is (i, j) - $g^{*\alpha}$ -closed in (X,τ_1,τ_2), then A is τ_j-α-closed if and only if τ_j-αcl(A)-A is τ_i-g-closed.

Proof: Necessity: If A is τ_j-αclosed then, τ_j-αcl(A) = A. i.e., τ_j-αcl(A)-A = ϕ and hence τ_j-αcl(A)-A is τ_i - g-closed.

Sufficiency: Suppose τ_j-αcl(A)-A = F $\neq$ ϕ, Then by the proposition 3.29, F contains a g- closed set B. τ_j-αcl(A)-A = ϕ, since A is $g^{*\alpha}$-closed. Therefore A is τ_j-α-closed.

Proposition 3.31: If A is an (i, j) - $g^{*\alpha}$ -closed in (X,τ_1,τ_2), then τ_i - cl(x)$\cap$A $\neq$ ϕ holds for each x$\in\tau_j$-αcl(A).

Proof: If A is (i, j) - $g^{*\alpha}$ -closed set. And let x$\in\tau_j$-αcl(A). Suppose τ_i -cl(x)$\cap$A = ϕ . Then, A $\subseteq$ (τ_i -cl(x))^c = U where U is open in $\tau_{i.}$. Thus A$\subseteq$U where U is g-open in τ_i. Then τ_j-αcl(A) $\subseteq$ U , since A is (i, j)- $g^{*\alpha}$ -closed. Hence τ_i- cl(x) $\cap$ A $\neq$ ϕ.

Proposition 3.32: If A is (i, j) - $g^{*\alpha}$ -closed set of (X,τ_1,τ_2) such that A $\subseteq$ B$\subseteq$ τ_j-αcl(A), then B is also a (i, j)- $g^{*\alpha}$ -closed set of (X, τ_i, τ_j).

Proof: If A is (i, j) - $g^{*\alpha}$ -closed of (X, τ_1, τ_2) such that A $\subseteq$ B$\subseteq$ τ_j-αcl(A). Let B$\subseteq$U where U is g-open in τ_i. Since A$\subseteq$B, A$\subseteq$U where U is g-open in τ_i. Then, τ_j-αcl(A) $\subseteq$ U since A is (i, j)- $g^{*\alpha}$ -closed. i.e., B $\subseteq$ τ_j-αcl(A) $\subseteq$ U. Hence τ_j-αcl(B) $\subseteq$ U. Therefore B is (i, j) - $g^{*\alpha}$ -closed set of (X, τ_i, τ_j).

4. (i, j)- $T^{*\alpha}_{1/2}$ and (i, j)-$*^{\alpha}T_{1/2}$ spaces

In this section, we introduce (i, j) - $T^{*\alpha}_{1/2}$ and (i, j)-$*^{\alpha}T_{1/2}$ bitopological spaces.

Definition 4.1: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)- $T^{*\alpha}_{1/2}$ space if every (i, j) - $g^{*\alpha}$ -closed set is τ_j-closed.

Proposition 4.2: If (X, τ_1, τ_2) is said to be an (i, j) - $T^{*\alpha}_{1/2}$ space. Then it is an (i, j) - $T^{*}_{1/2}$ space but not conversely.

Example 4.3: Let X= {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ, {a}, {a, b}, X}. Then (X, τ_1, τ_2) is (1, 2) - $T^{*}_{1/2}$ space, since the subsets {b}, {c}, {b, c} are (1, 2)-g*-closed and τ_2-closed, but (X, τ_1, τ_2) is not (1, 2)- $T^{*\alpha}_{1/2}$, since the subset {b} is (1, 2) - $g^{*\alpha}$ -closed but not τ_2-closed.

Proposition 4.4: Every (i, j)-T_b- space is (i, j) - $T^{*\alpha}_{1/2}$ space but not conversely.

Example 4.5: Let X= {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ, {a}, {b, c}, X}. Then the space (X, τ_1, τ_2) is (1, 2)- $T^{*\alpha}_{1/2}$ space since the subsets {a}, {b, c} are (1, 2)- $g^{*\alpha}$ -closed and τ_2-closed but (X, τ_1, τ_2) is not (1, 2)-T_b, since the subsets {b}, {c}, {a, b}, {a, c} are (1, 2)-gs-closed but not τ_2-closed.

Definition 4.12: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)-$*^{\alpha}T_{1/2}$ space if every (i, j) - $g^{*\alpha}$ -closed set is (i, j) - g*-closed.

Proposition 4.13: Every (i, j) - $T^{*\alpha}_{1/2}$ space is (i, j)-$*^{\alpha}T_{1/2}$ but not conversely.

Example 4.14: Let X= {a, b, c}, τ_1 = {ϕ, {c}, {a, c}, X}, τ_2 = {ϕ, {a}, X}. Then (X, τ_1, τ_2) is an (1, 2)-$*^{\alpha}T_{1/2}$ space, since the subsets {b}, {c}, {a, b}, {b, c} are (1, 2) - $g^{*\alpha}$ -closed and g*-closed but not an (1, 2)- $T^{*\alpha}_{1/2}$–space, since the subset {b} is (1, 2)-$g^{*\alpha}$-closed but not τ_2-closed.

Proposition 4.15: Every (i, j)-T_b- space is (i, j)-$*^{\alpha}T_{1/2}$ space but not conversely.

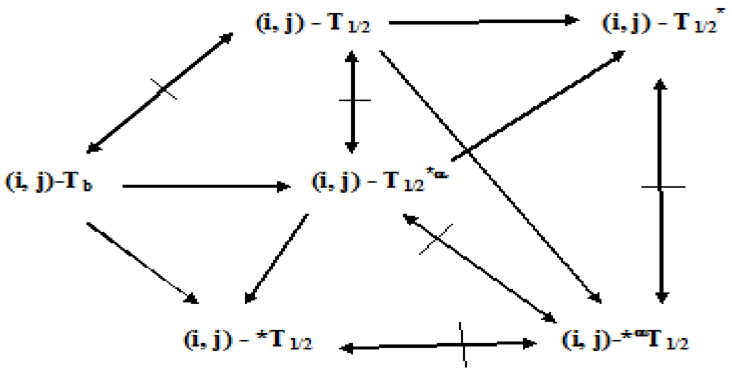
Example 4.16: Let X= {a, b, c}, τ_1 = {ϕ,{c}, {a, c}, X}, τ_2 = {ϕ, {a}, X}. Then (X, τ_1, τ_2) is an (1, 2)-$*^{\alpha}T_{1/2}$ space,since the subsets {b}, {c}, {a, b}, {b, c} are (1, 2) - $g^{*\alpha}$-closed and g*-closed but not an (1, 2)-T_b–space, since the subsets {b}, {c}, {a, b} are (1, 2)-gs-closed and not τ_2-closed.

Proposition 4.17: (i, j) - $T^{*\alpha}_{1/2}$ and (i, j)-$*^{\alpha}T_{1/2}$ are independent as seen from the following two examples.

Example 4.18: Let X= {a, b, c}, τ_1 = {ϕ, {c}, {a, c}, X}, τ_2 = {ϕ, {a}, X}. Then (X, τ_1, τ_2) is an (1, 2)-$*^{\alpha}T_{1/2}$ space since the subsets {b}, {c}, {a, b}, {b, c} are (1, 2)- $g^{*\alpha}$-closed and g*-closed but not an (1, 2)-T*–space, since the subsets {b}, {c}, {a, b} are (1, 2)-g*-closed and not τ_2-closed.

Example 4.19: Let X= {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ, {a}, {a, b}, X}. Then (X, τ_1, τ_2) is (1, 2) - $T^*_{1/2}$ – space since the subsets {c}, {b, c} are (1, 2) - g*-closed and τ_2 -closed but not an (1, 2) - $*^{\alpha}T_{1/2}$ space, since the subset {b} is (1, 2) - $g^{*\alpha}$-closed and not τ_2-closed.

All the above results can be represented by the following diagram.



Where A → B (resp. A↮B) represents A implies B but not conversely (resp. A and B are independent)

REFERENCES

[1] D. Andrijevic, Semi-preopen sets, Mat. Vesnik, 38(1) (1986), 24-32.

[2] I. Arockiarani, Studies on generalized closed sets and maps in topological spaces, Ph. D., Thesis, Bharathiar University., Coimbatore, 1997.

[3] P. Bhattacharya and B.K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29(3)1987), 375-382.

[4] K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Kochi Univ. Ser. A, Math., 12(1991), 5-13.

[5] J. Dontchev, On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 16(1995), 35-48.

[6] R. Devi H. Maki and K. Balachandran, Semi-generalized closed maps and generalized closed maps, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 14(1993), 41-54.

[7] R. Devi and K. Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser.A, Math., **15**(1994), 51-63.

[8] T. Fukutake, On generalized closed sets in bitopological spaces, Bull. Fukuoka Univ. Ed. Part III, 35 (1985), 19-28.

[9] Y. Gnanambal, Studies on generalized pre-regular closed sets and generalizations of locally closed sets.

[10] J.C. Kelley, Bitopological spaces, Proc. London Math. Sci., 13 (1963), 71 – 89.

[11] J. L. Kelly, General topology, D. Van Nostrand company Inc., Princeton, New Jersey, 1955.

[12] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[13] N. Levine, Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19(2) (1970), 89-96.

[14] H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., **15**(1994), 51-63.

[15] O. Njåstad, On some classes of nearly open sets, Pacific J. Math., 15(1965), 961- 970.

[16] N. Palaniappan and K.C. Rao, Regular generalized closed sets, Kyungpook Math. J., 33(2) (1993), 211-219.

[17] M.Sheik john and P.Sundaram, g^*-Closed sets in Bitopological spaces.

[18] P. Sundaram, H. Maki and K. Balachandran, Semi-generalized continuous maps and semi-$T_{1/2}$ spaces, Bull. Fukuoka Univ. Ed. Part III, 40(1991), 33-40.

[19] M.K.R.S. VeeraKumar, g*- Closed Sets in topological space, Mem. Fac, Sci. Kohi Univ. (Math), 21(2000), 1-19.

Source of support: Nil, Conflict of interest: None Declared