

g^{} - closed sets in topological spaces**

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ABSTRACT

*In this paper, we introduce a new class of sets namely, g^{**} -closed sets, which is settled in between the class of closed sets and the class of g -closed sets. Applying these sets, we introduce the new class of spaces, namely $T_{1/2}^{**}$ spaces, $^{**}T_{1/2}$ spaces, $^*T_{1/2}^*$ spaces, T_c^* spaces and ${}_aT_c^*$ spaces. Further we introduce g^{**} continuous maps and g^{**} irresolute maps.*

Key words: g^{**} -closed sets; g^{**} continuous maps; g^{**} irresolute maps; $T_{1/2}^{**}$ spaces; ${}_aT_c^*$ spaces, $^*T_{1/2}^*$ spaces, T_c^* spaces and $^{**}T_{1/2}$ spaces.

1. Introduction

Levine [13] introduced the class of g -closed sets in 1970. Maki et.al [15] defined ag -closed sets and $\alpha^{**}g$ -closed sets in 1994. S.P. Arya and N. Tour [3] defined gs -closed sets in 1990. Dontchev [10], Gnanambal [12] and Palaniappan and Rao [22] introduced gsp -closed sets, gpr -closed sets and rg -closed sets respectively. M.K.R.S. Veerakumar [24] introduced g^* -closed sets in 1991. We introduce a new class of sets called g^{**} -closed sets, which is properly placed in between the class of closed sets and the class of g -closed sets.

Levine [13] Devi et.al [7] and Devi et.al [6] introduced $T_{1/2}$ spaces, T_b spaces and ${}_aT_b$ spaces respectively. M.K.R.S. Veerakumar [24] introduced T_c , $T_{1/2}^*$ and ${}_aT_c$ spaces. Applying g^{**} -closed sets, five new spaces namely, $T_{1/2}^{**}$ spaces, $^{**}T_{1/2}$ spaces, $^*T_c^*$, $T_{1/2}^*$ and ${}_aT_c^*$ spaces are introduced.

2. Preliminaries

Throughout this paper (X, τ) , (Y, σ) and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $cl(A)$ and $int(A)$ denote the closure and the interior of A respectively.

The class of all closed subsets of a space (X, τ) is denoted by $C(X, \tau)$. The smallest semi-closed (resp. pre-closed and α -closed) set containing a subset A of (X, τ) is called the semi-closure (resp. pre-closure and α -closure) of A and is denoted by $scl(A)$ (resp. $pcl(A)$ and $\alpha cl(A)$).

Definition 2-1: A subset A of a topological space (X, τ) is called

- 1) a *pre-open* set [18] if $A \subseteq int(cl(A))$ and a *pre-closed* set if $cl(int(A)) \subseteq A$.
- 2) a *semi-open* set [14] if $A \subseteq cl(int(A))$ and a *semi-closed* set if $int(cl(A)) \subseteq A$.
- 3) a *semi-pre-open* set [1] if $A \subseteq cl(int(cl(A)))$ and a *semi pre-closed* set [1] if $int(cl(int(A))) \subseteq A$.
- 4) an α -open set [20] if $A \subseteq int(cl(int(A)))$ and an α -closed set [19] if $cl(int(cl(A))) \subseteq A$.

Definition 2-2: A subset A of a topological space (X, τ) is called

- 1) a *generalized closed* set (briefly *g-closed*) [13] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
 - 2) a *semi-generalized closed* set (briefly *sg-closed*) [5] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
 - 3) a *generalized semi-closed* set (briefly *gs-closed*) [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
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- 4) an α -generalized closed set (briefly αg -closed) [15] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 5) a generalized α -closed set (briefly ga -closed) [16] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ) .
- 7) an α^{**} -generalized closed set (briefly $\alpha^{**}g$ -closed) [15] if $\alpha cl(A) \subseteq \text{int}(\text{cl}(U))$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 8) a generalized semi-preclosed set (briefly gsp -closed [10] if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 9) a regular generalized closed set (briefly rg -closed) [22] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
- 10) a generalized preclosed set (briefly gp -closed) [17] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 11) a generalized preregular closed set (briefly gpr -closed) [12] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
- 12) a $g\alpha^*$ -closed set [16] if $\alpha cl(A) \subseteq \text{int}(U)$ whenever $A \subseteq U$ and U is α -open in (X, τ) .
- 13) a g^* -closed set [24] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in (X, τ) .
- 14) a wg – closed set [21] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 14) a wg – closed set [21] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .

Definition 2.3: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

- 1) a g -continuous [4] if $f^{-1}(V)$ is a g -closed set of (X, τ) for every closed set V of (Y, σ) .
- 2) an αg -continuous [12] if $f^{-1}(V)$ is an αg -closed set of (X, τ) for every closed set V of (Y, σ) .
- 3) a gs -continuous [8] if $f^{-1}(V)$ is a gs -closed set of (X, τ) for every closed set V of (Y, σ) .
- 4) a gsp -continuous [10] if $f^{-1}(V)$ is a gsp -closed set of (X, τ) for every closed set V of (Y, σ) .
- 5) a rg -continuous [22] if $f^{-1}(V)$ is a rg -closed set of (X, τ) for every closed set V of (Y, σ) .
- 6) a gp -continuous [2] if $f^{-1}(V)$ is a gp -closed set of (X, τ) for every closed set V of (Y, σ) .
- 7) a gpr -continuous [12] if $f^{-1}(V)$ is a gpr -closed set of (X, τ) for every closed set V of (Y, σ) .
- 8) a g^* -continuous [24] if $f^{-1}(V)$ is a g^* -closed set of (X, τ) for every closed set V of (Y, σ) .
- 9) a g^* -irresolute [24] if $f^{-1}(V)$ is a g^* -closed set of (X, τ) for every g^* -closed set V of (Y, σ) .

Further we call a function $f : (Y, \sigma) \rightarrow (X, \tau) \rightarrow$ as $\alpha^{**}g$ -continuous [15] if $f^{-1}(V)$ is an $\alpha^{**}g$ -closed set of (X, τ) whenever V is a closed set of (Y, σ) and wg -continuous [21] if $f^{-1}(V)$ is an wg -closed set of (X, τ) whenever V is a closed set of (Y, σ) .

Definition 2.4: A topological space (X, τ) is said to be

- 1) a $T_{1/2}$ space [13] if every g -closed set in it is closed.
- 2) a T_b space [7] if every gs -closed set in it is closed.
- 3) a T_d space [7] if every gs -closed set in it is g -closed.
- 4) an αT_d space [4] if every αg -closed set in it is g -closed.
- 5) an αT_b space [4] if every αg -closed set in it is closed.
- 6) a $T_{1/2}^*$ [24] space if every g^* -closed set in it is closed.
- 7) $*T_{1/2}$ [24] space if every g -closed set in it is g^* -closed set.

3. Basic properties of g^{**} – closed sets

We now introduce the following definitions.

Definition 3.1: A subset A of (X, τ) is said to be a g^{**} – closed set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* – open in X .

The class of g^{**} – closed subset of (X, τ) is denoted by $G^{**}C(X, \tau)$.

Proposition 3.2: Every closed set is g^{**} – closed .

Proof follows from the definition.

The following example supports that a g^{**} – closed set need not be closed in general.

Example 3.3: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$. Let $A = \{b\}$. A is a g^{**} -closed set but not a closed set of (X, τ) .

So, the class of g^{**} -closed sets properly contains the class of closed sets.

Next we show that the class of g^{**} -closed sets is properly contained in the class of g -closed, rg -closed, gpr -closed and wg -closed sets.

Proposition 3.4: Every g^{**} -closed set is

- (i) g -closed
- (ii) rg -closed
- (iii) gpr -closed
- (iv) wg -closed.

Proof: Let A be a g^{**} -closed set

(i) Let $A \subseteq U$ and U be open then U is g^* -open. Since A is g^{**} -closed, $cl(A) \subseteq U$ therefore A is g -closed.

(ii) Let $A \subseteq U$ and U be regular open. Then U is open and hence U is g^* -open. Since A is g^{**} -closed, $cl(A) \subseteq U$, therefore A is rg -closed.

(iii) Let $A \subseteq U$ where U is regular open. Then U is g^* -open. Since A is g^{**} -closed, $cl(A) \subseteq U$ which implies $pcl(A) \subseteq cl(A) \subseteq U$. Therefore A is gpr -closed.

(iv) Let $A \subseteq U$ where U is open. Then U is g^* -open. $Int(A) \subseteq A$ implies $cl(int(A)) \subseteq cl(A) \subseteq U$. Therefore A is wg -closed.

The converse of the above proposition need not be true in general as it can be seen from the following example.

Example 3.5: In example (3.3), $A = \{a\}$ is gpr -closed but not g^{**} -closed.

Example 3.6: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$. $A = \{c\}$ is g -closed and hence it is rg -closed and αg -closed. That is A is gsp -closed, rg -closed and wg -closed but it is not g^{**} -closed.

Proposition 3.7: Every g^* -closed set is gpr -continuous.

Proof: Let A be a g^* -closed set. Let $A \subseteq U$ and U be g^* -open. Then $A \subseteq U$ and U is g -open. Hence $cl(A) \subseteq U$, and U is g^* -open. Therefore A is g^{**} -closed.

The converse of the above proposition need not be true.

Example 3.8: In example (3.3), $\{a, c\}$ is g^{**} -closed but not gsp -continuous. So the class of g^{**} -closed sets properly contains the class of g^* -closed sets.

Proposition 3.9: Every g^{**} -closed set is αg -closed set and hence gs -closed, gsp -closed, gp -closed and also $\alpha^{**}g$ -closed but not conversely.

Proof: Let A be a g^{**} -closed set of (X, τ) . By proposition (3.2), A is g -closed. By the implications (2.4) in Maki.et.al. [18], A is αg -closed and $\alpha^{**}g$ -closed. Therefore every g^{**} -closed set is αg -closed and $\alpha^{**}g$ -closed. From the investigations of Dontchev [11] and Gnanambal [15], we know that every

g -closed set is gs -closed, gsp -closed and gp -closed. Therefore every g^{**} -closed set is gs -closed, gsp -closed and gp -closed.

Example 3.10: In example (3.6), $\{c\}$ is g -closed and hence it is gs -closed, gsp -closed and gp -closed, αg -closed and $\alpha^{**}g$ -closed but it is not g^{**} -closed.

So the class of g^{**} -closed sets is properly contained in the class of αg -closed sets, the class of gs -closed sets, the class of gsp -closed sets, the class of gp -closed sets, the class of αg -closed sets and the class of $\alpha^{**}g$ -closed sets.

Proposition 3.11: If A and B are g^{**} -closed sets, then $A \cup B$ is also a g^{**} -closed set.

Proof follows from the fact that $cl(A \cup B) = cl(A) \cup cl(B)$.

Proposition 3.12: If A is both g^* -open and g^{**} -closed, then A is closed.

Proof follows from the definition of g^{**} -closed sets.

Proposition 3.13: A is a g^{**} -closed set of (X, τ) if $cl(A) \setminus A$ does not contain any non-empty g^* -closed set.

Proof: Let F be a g^* -closed set of (X, τ) such that $F \subseteq cl(A) \setminus A$. Then $A \subseteq X \setminus F$. Since A is g^{**} -closed and $X \setminus F$ is g^* -open, $cl(A) \subseteq X \setminus F$. This implies $F \subseteq X \setminus cl(A)$. So $F \subseteq (X \setminus cl(A)) \cap (cl(A) \setminus A) \subseteq (X \setminus cl(A)) \cap cl(A) = \emptyset$. Therefore $F = \emptyset$.

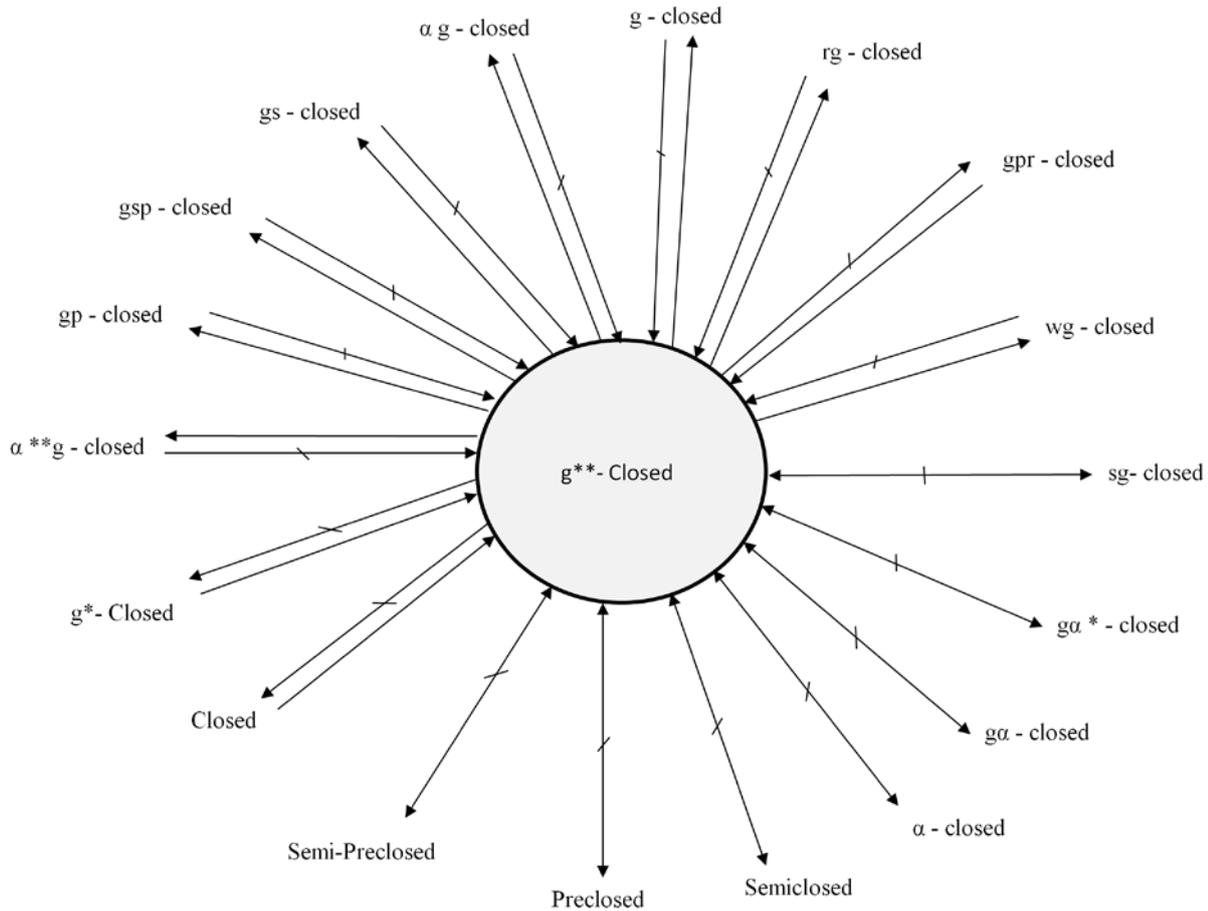
Remark 3.14: g^{**} -closedness is independent from α -closedness, semi-closedness, $g : (Y, \sigma) \rightarrow (Z, \eta)$, $g \circ f$, g^{**} -continuous, pre-closedness and semi-preclosedness.

Proof: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$. Let $A = \{a, b\}$ and $B = \{c\}$. A is g^{**} -closed. A is neither α -closed nor semi-closed, in fact, it is not even a semi-preclosed set. Also it is not preclosed. B is α -closed and hence semi-closed, preclosed, semi-preclosed, sg -closed, $g\alpha$ -closed and $g\alpha^*$ -closed but it is not g^{**} -closed.

Proposition 3.15: If A is a g^{**} -closed set of (X, τ) such that $A \subseteq B \subseteq cl(A)$, then B is also a g^{**} -closed set of (X, τ) .

Proof: Let U be a g^* -open set of (X, τ) such that $B \subseteq U$. Then $A \subseteq U$, since A is g^{**} -closed, then $cl(A) \subseteq U$. Now $cl(B) \subseteq cl(cl(A)) = cl(A) \subseteq U$. Therefore B is also a g^{**} -closed set of (X, τ) .

The above results can be represented in the following figure.



Where $A \rightarrow B$ (resp. $A \not\leftrightarrow B$) represents A implies B (resp. A and B are independent).

4. g^{} -continuous and g^{**} -irresolute maps.**

We introduce the following definitions.

Definition 4.1: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called g^{**} -continuous if $f^{-1}(V)$ is a g^{**} -closed set of (X, τ) for every closed set of (Y, σ) .

Theorem 4.2: Every continuous map is g^{**} -continuous.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be continuous and let F be any closed set of Y . Then $f^{-1}(F)$ is closed in X . Since every closed set is g^{**} -closed, $f^{-1}(F)$ is g^{**} -closed. Therefore f is g^{**} -continuous.

The following example shows that the converse of the above theorem need not be true in general.

Example 4.3: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$, $\sigma = \{\emptyset, Y, \{b\}\}$, $f : (X, \tau) \rightarrow (Y, \sigma)$ is defined as the identity map. The inverse image of all the closed sets of (Y, σ) are g^{**} -closed in (X, τ) . Therefore f is g^{**} -continuous but not continuous.

Thus the class of all g^{**} -continuous maps properly contains the class of continuous maps.

Theorem 4.4: Every g^{**} -continuous map is g -continuous and hence an αg -continuous, $\alpha^{**}g$ -continuous, gs -continuous, gsp -continuous, gp -continuous, rg -continuous, gpr -continuous and wg -continuous maps.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g^{**} -continuous map. Let V be a closed set of (Y, σ) .

Since f is g^{**} -continuous, then $f^{-1}(V)$ is a g^{**} -closed set in (X, τ) .

By proposition (3.4) and (3.9), $f^{-1}(V)$ is g -closed, rg -closed, gpr -closed, wg -closed, αg -closed, gs -closed, gsp -closed, gp -closed and $\alpha^{**}g$ -closed set of (X, τ) .

The converse of the above theorem need not be true as seen in the following example.

Example 4.5: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then $f^{-1}(\{c\}) = \{c\}$ is not g^{**} -closed in X . But $\{c\}$ is g -closed and hence rg -closed, gpr -closed, wg -closed, αg -closed, gs -closed, gsp -closed, gp -closed and $\alpha^{**}g$ -closed. Therefore f is g -continuous and hence rg -continuous, gpr -continuous, wg -continuous, αg -continuous, gs -continuous, gsp -continuous, gp -continuous and $\alpha^{**}g$ -continuous but f is not g^{**} -continuous.

Thus the class of all g^{**} -continuous maps is properly contained in the classes of g -continuous, rg -continuous, gpr -continuous, wg -continuous, αg -continuous, gs -continuous, gsp -continuous, gp -continuous and $\alpha^{**}g$ -continuous maps.

The following example shows that the compositions of two g^{**} -continuous maps need not be a g^{**} -continuous map.

Example 4.6: Let $X = Y = Z = \{a, b, c\}$ and let $f : (X, \tau) \rightarrow (Y, \sigma)$, $g : (Y, \sigma) \rightarrow (Z, \eta)$, be the identity maps. $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$, $\sigma = \{\emptyset, Y, \{a\}\}$, $\eta = \{\emptyset, Z, \{b\}\}$. $(f \circ g)^{-1}(\{a, c\}) = f^{-1}(g^{-1}(\{a, c\})) = f^{-1}(\{a, c\}) = \{a, c\}$ is not g^{**} -closed in (X, τ) . But f and g are g^{**} -continuous maps.

Theorem 4.7: Every g^* -continuous map is g^{**} -continuous.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be g^* -continuous and let V be a closed set of Y . Then $f^{-1}(V)$ is g^* -closed and hence by proposition (3.7), it is g^{**} -closed. Hence f is g^{**} -continuous.

The following example shows that the converse of the above theorem is not true in general.

Example 4.8: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$, $\sigma = \{\emptyset, X, \{b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. $A = \{a, c\}$ is closed in (Y, σ) and is g^{**} -closed in (X, τ) but not g^* -closed in (X, τ) . Therefore f is g^{**} -continuous but not g^* -continuous.

Definition 4.9: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called g^{**} -irresolute if $f^{-1}(V)$ is a g^{**} -closed set of (X, τ) for every g^{**} -closed set V of (Y, σ) .

Theorem 4.10: Every g^{**} -irresolute function is g^{**} -continuous.

Proof follows from the definition.

Theorem 4.11: Every g^* -irresolute function is g^{**} -continuous.

Proof follows from the definition.

Converse of theorems (4.10) and (4.11) need not be true in general as seen in the following example.

Example 4.12: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$.

Define $h : (X, \tau) \rightarrow (Y, \sigma)$ by $h(a) = b$, $h(b) = c$ and $h(c) = a$. $\{c\}$ is the only closed set of Y . $h^{-1}(\{c\}) = \{b\}$ is g^{**} -closed in X . Therefore h is g^{**} -continuous. $\{b, c\}$ is a g^* -closed set of Y but $h^{-1}\{b, c\} = \{a, b\}$ is not g^* -closed in X . Therefore h is not g^* -irresolute. Therefore h is g^{**} -continuous but not g^* -irresolute. $\{b, c\}$ is a g^{**} -closed set in Y but $h^{-1}\{b, c\} = \{a, b\}$ is not g^{**} -closed in X .

Therefore h is not g^{**} -irresolute. Therefore h is g^{**} -continuous but not g^{**} -irresolute.

Theorem 4.13: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions then

- (i) $g \circ f$ is g^{**} -continuous if g is continuous and f is g^{**} -continuous.
- (ii) $g \circ f$ is g^{**} -irresolute if both f and g are g^{**} -irresolute.
- (iii) $g \circ f$ is g^{**} -continuous if g is g^{**} -continuous and f is g^{**} -irresolute.

5. Applications of g^{**} -closed sets

As applications of g^{**} -closed sets, new spaces, namely, $T_{1/2}^{**}$ space, ${}_a T_c^*$ spaces and ${}^{**}T_{1/2}$ spaces, ${}^*T_{1/2}$ and T_c^* spaces are introduced.

We introduce the following definition.

Definition 5.1: A space (X, τ) is called a $T_{1/2}^{**}$ space if every g^{**} -closed set is closed.

Theorem 5.2: Every $T_{1/2}$ space is $T_{1/2}^{**}$ space.

A $T_{1/2}^{**}$ space need not be $T_{1/2}$ space, which can be seen from the following example.

Example 5.3: Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}\}$. (X, τ) is a $T_{1/2}^{**}$ space but not a $T_{1/2}$ space since $A = \{b\}$ is g -closed but not closed.

Therefore the class of $T_{1/2}^{**}$ space properly contains the class of $T_{1/2}$ - spaces.

Theorem 5.4: Every $T_{1/2}^{**}$ space is $T_{1/2}^*$ space

The converse need not be true in general as seen in the following example.

Example 5.5: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$, $G^*C(X, \tau) = \{\emptyset, X, \{b, c\}\} = C(X, \tau)$. Therefore (X, τ) is a $T_{1/2}^*$ - space but not $T_{1/2}^{**}$ space since $\{a, b\}$ is a g^{**} -closed set but not a closed set of (X, τ) .

Theorem 5.6: Every T_b - space is a $T_{1/2}^{**}$ - space.

The converse need not be true in general as seen in the following example.

Example 5.7: Let, $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. (X, τ) is a $T_{1/2}^{**}$ space but not a T_b space since $A = \{a\}$ is g -closed but not closed.

Remark 5.8: T_d - ness is independent of $T_{1/2}^{**}$ - ness as it can be seen from the following example.

Example 5.9: In example (5.7), (X, τ) is a $T_{1/2}^{**}$ space but not a ${}_a T_d$ space since $A = \{a\}$ is g -closed but not g -closed.

Example 5.10: $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. (X, τ) is a T_d space but not a $T_{1/2}^{**}$ space since $A = \{b\}$ is g^{**} -closed but not closed.

Theorem 5.11: Every ${}_aT_b - space$ is a $T_{1/2}^{**} - space$.

Example 5.12: In example (5.3). (X, τ) is a $T_{1/2}^{**}$ space but not a ${}_aT_b$ space since $A = \{b\}$ is αg -closed but not closed.

Theorem 5.13: The following conditions are equivalent in a topological space (X, τ) .

- (i) (X, τ) is a $T_{1/2}^{**} - space$.
- (ii) Every singleton of X is either $g^* - closed$ or open.

Proof:

(i) = (ii): Let (X, τ) be a $T_{1/2}^{**} - space$. Let $x \in X$ and suppose $\{x\}$ is not $g^* - closed$. Then $X \setminus \{x\}$ is not $g^* - open$. This implies that X is the only $g^* - open$ set containing $X \setminus \{x\}$. Therefore $X \setminus \{x\}$ is a $g^{**} - closed$ set of (X, τ) . Therefore $X \setminus \{x\}$ is closed since (X, τ) is a $T_{1/2}^{**} - space$. Therefore $\{x\}$ is open in (X, τ) .

(ii) = (i): Let A be a $g^{**} - closed$ set of (X, τ) $A \subseteq cl(A)$ and let $x \in cl(A)$. By (ii) $\{x\}$ is $g^* - closed$ or open.

Case (i): Let $\{x\}$ be $g^* - closed$. If $x \notin A$, then $cl(A) \setminus A$ contains a non-empty $g^* - closed$ set $\{x\}$. But it is not possible by proposition (3.13). Therefore $x \in A$.

Case (ii): Let $\{x\}$ be open. Now $x \in cl(A)$, then $\{x\} \cap A \neq \emptyset$. Therefore $x \in A$ and so $cl(A) \subseteq A$ and hence $A = cl(A)$ or A is closed. Therefore (X, τ) is a $T_{1/2}^{**} - space$.

We introduce the following definition.

Definition 5.14: A space (X, τ) is called an ${}_aT_c^* - space$ if every $\alpha g - closed$ set of (X, τ) is $g^{**} - closed$. We show that the class of ${}_aT_c^* - spaces$ properly contains the class of ${}_aT_b - spaces$ and is properly contained in the class of ${}_aT_d - spaces$. Moreover we prove that ${}_aT_c^* - ness$ and $T_{1/2}^{**} - ness$ are independent from each other.

Theorem 5.15: Every ${}_aT_b - space$ is an ${}_aT_c^* - space$ but not conversely.

Example 5.16: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$, (X, τ) is an ${}_aT_c^* - space$ but not ${}_aT_b - space$ since $\{a, c\}$ is $\alpha g - closed$ but not closed.

Theorem 5.17: Every ${}_aT_c^* - space$ is an ${}_aT_d - space$ but not conversely.

Example 5.18: In example (5.3). (X, τ) is a ${}_aT_d$ space but not a ${}_aT_c^*$ space since $A = \{b\}$ is αg -closed but not g^{**} -closed.

Theorem 5.19: A space (X, τ) is an ${}_aT_b - space$ if and only if it is a ${}_aT_c^* - space$ and a $T_{1/2}^{**} - space$.

Proof: Necessity – Follows from theorem 5.4 and 5.10

Sufficiency: Suppose (X, τ) is ${}_aT_c^* - space$ and $T_{1/2}^{**} - space$. Let A be $\alpha g - closed$. Since (X, τ) is an ${}_aT_c^* - space$, A is $g^{**} - closed$ and since (X, τ) is an $T_{1/2}^{**} - space$, A is closed. Therefore (X, τ) is an ${}_aT_b - space$.

Remark 5.20: ${}_aT_c^* - ness$ is independent from $T_{1/2}^{**} - ness$, as it is clear from the following examples.

Example 5.21: In example (5.16), (X, τ) is an ${}_aT_c^*$ - space but not $T_{1/2}^{**}$ since, $\{a, c\}$ is g^{**} -closed but not closed.

Example 5.22: In example (5.3). (X, τ) is a $T_{1/2}^{**}$ space but not a ${}_aT_c^*$ space since $A = \{b\}$ is ag -closed but not g^{**} -closed.

Definition 5.23: A subset A of a space (X, τ) is called a g^{**} -open set if its complement is a g^{**} -closed set of (X, τ) .

Theorem 5.24: If (X, τ) is an ${}_aT_c^*$ - space for each $x \in X$, $\{x\}$ is either ag - closed or g^{**} - open .

Proof: Let $x \in X$ suppose that $\{x\}$ is not an ag - closed set of (X, τ) . Then $\{x\}$ is not a closed set since every closed set is an ag - closed set. Therefore $X \setminus \{x\}$ is not open. Therefore $X \setminus \{x\}$ is an ag - closed set since X is the only open set which contains $X \setminus \{x\}$. Since (X, τ) is an ${}_aT_c^*$ - space, $X \setminus \{x\}$ is a g^{**} - closed set or $\{x\}$ is g^{**} - open .

Remark 5.25: The converse of the above theorem is not true as it can be seen from the following example.

Example 5.26: $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. (X, τ) is not a ${}_aT_c^*$ space but $\{c\}$ and $\{b\}$ are ag -closed and $\{a\}$ is g^{**} -open.

We introduce the following definition.

Definition 5.27: A space (X, τ) is called an $^{**}T_{1/2}$ - space if every g^{**} - closed set of (X, τ) is a g^* - closed set.

Remark 5.28: $T_{1/2}^*$ - ness is independent from $^{**}T_{1/2}$ - ness as it is clear from the following example.

Example 5.29: In example (5.26), (X, τ) is a $^{**}T_{1/2}$ - space but not a $T_{1/2}^*$ - space since $A = \{a, c\}$ is g^* -closed but not closed.

Example 5.30: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$. (X, τ) is a $T_{1/2}^*$ space but not a $^{**}T_{1/2}$ since $A = \{a, c\}$ is g^{**} -closed but not g^* -closed.

Theorem 5.31: Every $T_{1/2}^{**}$ - space is $^{**}T_{1/2}$ - space .

Proof: Let (X, τ) be an $T_{1/2}^{**}$ - space . Let A be a g^{**} - closed set of (X, τ) . Since (X, τ) is a $T_{1/2}^{**}$ - space , A is closed. By theorem (3.2) of [24], A is g^* - closed . Therefore (X, τ) is a $^{**}T_{1/2}$ - space .

The converse of the above theorem need not be true as seen in the following example.

Example 5.32: In example (5.26), (X, τ) is a $^{**}T_{1/2}$ - space but not a $T_{1/2}^{**}$ - space since $A = \{a, c\}$ is g^{**} -closed but not closed.

Theorem 5.33: Every T_b - space is a $^{**}T_{1/2}$ - space .

Proof: Let (X, τ) be a T_b - space . Then by theorem 5.6, it is a $T_{1/2}^{**}$ - space . Therefore by theorem 5.31, it is a $^{**}T_{1/2}$ - space .

The converse of the above theorem need not be true as seen in the following example.

Example 5.34: In example (5.26), (X, τ) is a $^{**}T_{1/2}$ - space but not a T_b - space since $A = \{a, c\}$ is gs -closed but not closed.

Theorem 5.35: Every ${}_aT_b$ - space is a ${}^{**}T_{1/2}$ - space .

Proof: Let (X, τ) be a ${}_aT_b$ - space . Then by theorem (5.11), it is a $T_{1/2}^{**}$ - space . Therefore by theorem (5.31), (X, τ) is a ${}^{**}T_{1/2}$ - space .

The converse of the above theorem need not be true as seen in the following example.

Example 5.36: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$, (X, τ) is an ${}^{**}T_{1/2}$ - space but not a ${}_aT_b$ - space since $A = \{a, c\}$ is a αg - closed set but not a closed set of (X, τ) .

Theorem 5.37: Every ${}_aT_c$ - space is a ${}^{**}T_{1/2}$ - space .

Proof: Let (X, τ) be a ${}_aT_c$ - space . Let A be a g^{**} - closed set. Then by proposition (3.9), A is αg - closed . Since (X, τ) is an ${}_aT_c$ - space , A is g^* - closed . Therefore it is a ${}^{**}T_{1/2}$ - space .

The converse of the above theorem need not be true as seen in the following example.

Example 5.38: In example (5.36), (X, τ) is an ${}^{**}T_{1/2}$ - space but not a ${}_aT_c$ - space since $A = \{b\}$ is a αg - closed set but not a g^* - closed .

Theorem 5.39: A space (X, τ) is a ${}_aT_c$ - space if and only if it is ${}_aT_c^*$ - space and ${}^{**}T_{1/2}$ space .

Proof: Necessity follows from theorem (5.37) and theorem (5.15).

Sufficiency: Let A be αg - closed . Then it is g^{**} - closed , since (X, τ) is an ${}_aT_c^*$ - space . Since (X, τ) is ${}^{**}T_{1/2}$ - space , A is g^* - closed . Therefore (X, τ) is an ${}_aT_c$ - space .

Definitions 5.40: A space (X, τ) is called a ${}^*T_{1/2}^*$ - space if every g - closed set of (X, τ) is g^{**} - closed .

Theorem 5.41: Every $T_{1/2}$ - space is a ${}^*T_{1/2}^*$ - space .

Proof: Let (X, τ) be a $T_{1/2}$ - space . Let A be a g - closed set of (X, τ) . Then A is closed since (X, τ) is a $T_{1/2}$ - space . But by proposition (3.2), A is g^{**} - closed . Therefore (X, τ) is a ${}^*T_{1/2}^*$ - space .

The converse of the above theorem need not be true as seen in the following example.

Example 5.42: In example (5.36), (X, τ) is a ${}^*T_{1/2}^*$ - space but not a $T_{1/2}$ - space since $A = \{a, c\}$ is g -closed but not closed.

Remark 5.43: ${}^*T_{1/2}^*$ - ness and $T_{1/2}^*$ - ness are independent as it is shown in the following examples.

Example 5.44: In example (5, 36), (X, τ) is a ${}^*T_{1/2}^*$ - space but not a $T_{1/2}^*$ - space since $A = \{a, c\}$ is a g^* -closed set but not a closed set. $A = \{a, b\}$ is a g^* - closed set but not a closed set.

Example 5.45: In example (5.3), (X, τ) is a $T_{1/2}^*$ space but not a ${}^*T_{1/2}^*$ since $A = \{c\}$ is a g -closed set but not a g^{**} -closed set.

Theorem 5.46: Every ${}_aT_c$ - space is a ${}^*T_{1/2}^*$ - space .

Proof: Let (X, τ) be an ${}_aT_c$ - space. Let A be a g - closed set of (X, τ) . Then A is also an αg - closed set. Since (X, τ) is an ${}_aT_c$ - space, A is g^* - closed. Then by proposition (3.8), A is g^{**} - closed. Therefore (X, τ) is a ${}^*T_{1/2}^*$ - space.

The converse need not be true in general as seen in the following example.

Example 5.47: In example (5.36), (X, τ) is a ${}^*T_{1/2}^*$ space but not a ${}_aT_c$ since $A = \{b\}$ is a αg -closed set but not a g^* -closed set.

Theorem 5.48: Every ${}^*T_{1/2}^*$ - space is a ${}^*T_{1/2}^*$ - space.

Proof: Let (X, τ) be an ${}^*T_{1/2}^*$ - space. Let A be a g - closed set of (X, τ) . Then A is g^* - closed. (X, τ) is a ${}^*T_{1/2}^*$ - space. Then by proposition (3.8), A is g^{**} - closed. Therefore (X, τ) is a ${}^*T_{1/2}^*$ - space.

The converse need not be true in general as seen in the following example.

Example 5.49: In example (3.3), (X, τ) is a ${}^*T_{1/2}^*$ - space but not a ${}^*T_{1/2}^*$ - space since $A = \{b\}$ is a g - closed but not a g^* - closed.

Theorem 5.50: The space (X, τ) is a $T_{1/2}^{**}$ - space and a ${}^*T_{1/2}^*$ - space if and only if it is a $T_{1/2}^*$ - space.

Proof Necessity: Let (X, τ) be a $T_{1/2}^{**}$ - space and a ${}^*T_{1/2}^*$ - space. Let A be a g - closed set of (X, τ) . Then A is g^{**} - closed since (X, τ) is a ${}^*T_{1/2}^*$ - space. Also since (X, τ) is a $T_{1/2}^{**}$ - space, A is a closed set. Therefore (X, τ) is a $T_{1/2}^*$ - space.

Sufficiency: Let (X, τ) be a $T_{1/2}^*$ - space. Then by theorem (5.2) and (5.41), (X, τ) is a ${}^*T_{1/2}^*$ - space and a $T_{1/2}^{**}$ - space.

Theorem 5.51: If (X, τ) is a ${}^*T_{1/2}^*$ - space, then for each $x \in X$, $\{x\}$ is either closed or g^{**} - open.

Proof: Suppose (X, τ) is a ${}^*T_{1/2}^*$ - space. Let $x \in X$ and let $\{x\}$ be not closed. Then $X \setminus \{x\}$ is not open set. Therefore $X \setminus \{x\}$ is a g - closed set since X is the only open set which contains $X \setminus \{x\}$. Since (X, τ) is a ${}^*T_{1/2}^*$ - space, $X \setminus \{x\}$ is g^{**} - closed. Therefore $\{x\}$ is g^{**} - open.

We introduce the following definition.

Definition 5.52: A space (X, τ) is called a T_c^* - space if every gs - closed set of (X, τ) is g^{**} - closed.

Theorem 5.53: Every T_c - space is a T_c^* - space.

Proof: Let (X, τ) be a T_c - space. Let A be a gs - closed set of (X, τ) . Then A is g^* - closed since (X, τ) is a T_c - space. But by proposition (3.8), A is g^{**} - closed. Therefore (X, τ) is a T_c^* - space.

The converse need not be true in general as seen in the following example.

Example 5.54: Let $X = \{a, b, c\}$ $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. (X, τ) is a T_c^* space but not a T_c - space since $A = \{b\}$ is a gs - closed but not g^* - closed.

Theorem 5.55: Every $T_b - space$ is a $T_c^* - space$.

Proof: Let (X, τ) be a $T_b - space$. Let A be a $gs - closed$ set of (X, τ) . Then A is also closed. But by proposition (3.3), A is $g^{**} - closed$. Therefore (X, τ) is a $T_c^* - space$.

The converse need not be true in general as seen in the following example.

Example 5.56: In example (5.54), (X, τ) is a $T_c^* - space$ but not a $T_b - space$ since $A = \{a, b\}$ is a $gs - closed$ set but not a closed set.

Theorem 5.57: Every $T_c^* - space$ is a $T_d - space$.

Proof: Let (X, τ) be a $T_c^* - space$. Let A be a $gs - closed$ set of (X, τ) . Since (X, τ) is a $T_c^* - space$, A is $g^{**} - closed$. But by proposition (3.4), A is $g - closed$. Therefore (X, τ) is a $T_d - space$.

The converse need not be true in general as seen in the following example.

Example 5.58: In example (5.3), (X, τ) is a T_d space but not a T_c^* space since $A = \{a, b\}$ is $gs - closed$ but not $g^{**} - closed$.

Theorem 5.59: Every $T_c^* - space$ is a ${}_aT_d - space$.

Proof: Let (X, τ) be a $T_c^* - space$. Let A be a $\alpha g - closed$ set of (X, τ) . Then A is also $gs - closed$ a set. Since (X, τ) is a $T_c^* - space$, A is $g^{**} - closed$. But by proposition (3.2), A is $g - closed$. Therefore (X, τ) is a ${}_aT_d - space$.

The converse need not be true in general as seen in the following example.

Example 5.60: In example (3.3), (X, τ) is a ${}_aT_d - space$ but not a $T_c^* - space$ since $A = \{a\}$ is a $gs - closed$ set but not a $g^{**} - closed$ set.

Theorem 5.61: If (X, τ) is a $T_c^* - space$ and a $T_{1/2}^{**} - space$, then it is a ${}_aT_b - space$.

Proof: Let (X, τ) be a $T_c^* - space$ and a $T_{1/2}^{**} - space$. Let A be a $\alpha g - closed$ set of (X, τ) . Then it is also a $gs - closed$ set. Hence A is $g^{**} - closed$ since (X, τ) is a $T_c^* - space$. But every $g^{**} - closed$ set is closed since (X, τ) is a $T_{1/2}^{**} - space$. Therefore A is closed and hence (X, τ) is a ${}_aT_b - space$.

Theorem 5.62: Every $T_c^* - space$ is a ${}^*T_{1/2} - space$.

Proof: Let (X, τ) be a $T_c^* - space$. Let A be a $g - closed$ set of (X, τ) . Then A is also $gs - closed$ a set. Then A is $g^{**} - closed$, since (X, τ) is a $T_c^* - space$. Therefore (X, τ) is a ${}^*T_{1/2} - space$.

The converse need not be true in general as seen in the following example.

Example 5.63: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. (X, τ) is a ${}^*T_{1/2} - space$ but not a $T_c^* - space$ since $A = \{a\}$ is $gs - closed$ but not $g^{**} - closed$.

Theorem 5.64: If (X, τ) is a $T_c^* - space$, then for each $x \in X$, $\{x\}$ is either semi-closed or $g^{**} - open$ in (X, τ) .

Proof: Let (X, τ) be a T_c^* -space and Let $x \in X$ suppose $\{x\}$ is not semi-closed, then by the proposition 6.4 (ii) of [7], $X \setminus \{x\}$ is sg -closed. Also $X \setminus \{x\}$ is gs -closed. Since (X, τ) is a T_c^* -space, then $X \setminus \{x\}$ is g^{**} -closed. Therefore $\{x\}$ is g^{**} -open.

Theorem 5.65: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g^{**} -continuous map. If (X, τ) is $T_{1/2}^{**}$ then f is continuous.

Theorem 5.66: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an αg -continuous map. If (X, τ) is ${}_a T_c^*$, then f is g^{**} -continuous.

Theorem 5.67: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g -continuous map. If (X, τ) is ${}^* T_{1/2}^*$, then f is g^{**} -continuous.

Theorem 5.68: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a gs -continuous map. If (X, τ) is T_c^* , then f is g^{**} -continuous.

Theorem 5.69: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g^* -irresolute map and a closed map. Then $f(A)$ is a g^{**} -closed set of (Y, σ) for every g^{**} -closed set A of (X, τ) .

Proof: Let A be a g^{**} -closed set of (X, τ) . Let U be a g^* -open set of (Y, σ) such that $f(A) \subseteq U$. Since f is g^* -irresolute, $f^{-1}(U)$ is g^* -open in (X, τ) . Now $f^{-1}(U)$ and A is g^{**} -closed set of (X, τ) , then $cl(A) \subseteq f^{-1}(U)$. Then $f(cl(A)) = cl[f(cl(A))]$.

Therefore

$cl[f(A)] \subseteq cl[f(cl(A))] = f(cl(A)) \subseteq U$. Therefore $f(A)$ is a g^{**} -closed set of (Y, σ) .

Theorem 5.70: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be onto, g^{**} -irresolute and closed. If (X, τ) is $T_{1/2}^{**}$, then (Y, σ) is also a $T_{1/2}^{**}$ -space.

We introduce the following definition.

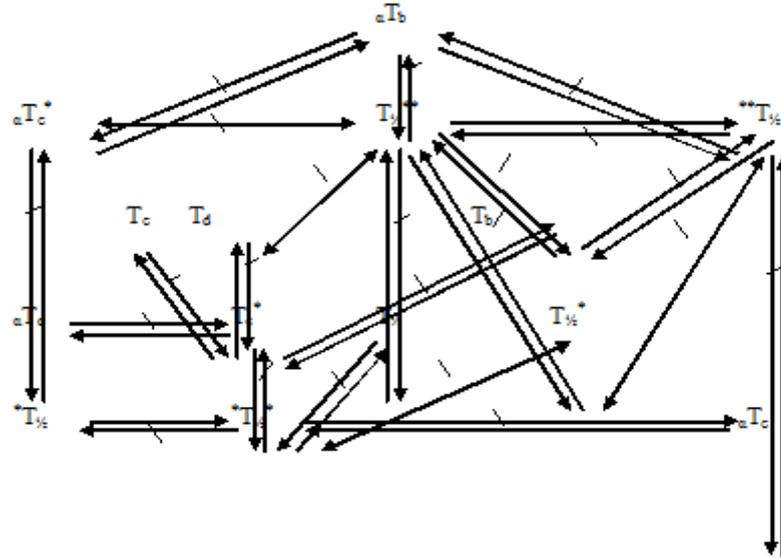
Definition 5.71: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a pre - g^{**} -closed set if $f(A)$ is a g^{**} -closed set of (Y, σ) for every g^{**} -closed set of (X, τ) .

Theorem 5.72: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be onto, αg -irresolute and pre - g^{**} -closed. If (X, τ) is a ${}_a T_c^*$ -space, then (Y, σ) is also an ${}_a T_c^*$ -space.

Theorem 5.73: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be onto, g^{**} -irresolute and pre - g^* -closed. If (X, τ) is ${}^{**} T_{1/2}$, then (Y, σ) is also a ${}^{**} T_{1/2}$ -space.

Theorem 5.74: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be onto, gs -irresolute and pre - g^{**} -closed. If (X, τ) is T_c^* -space, then (Y, σ) is also a T_c^* -space.

The above results can be represented in the following figure.



Where $A \rightarrow B$ (resp. $A \leftrightarrow B$) represents A implies B (resp. A and B are independent).

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