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On g^*s-closed sets in Bitopological Spaces

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ABSTRACT

In this paper we introduce g^*s-closed sets and g^*s-open sets in bitopological spaces and study some of their characteristics. Further we introduce and study g^*s-continuous maps and g^*s-irresolute maps in bitopological spaces.

Key words: *(i, j)-g^*s-closed sets, (i, j)-g^*s-open sets, (i, j)-gs-open sets, (i, j)-gs-closed sets (i, j)-σ_k-g^*s-continuous maps and pairwise g^*s-irresolute maps.*

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1. Introduction

A triple (X, τ_1, τ_2) where X is non empty set and τ_1, τ_2 are two topologies on X is called a bitopological space. Kelly[6] initiated the study of these spaces in 1963. Fukutake[5] introduced the concept of g-closed sets in bitopological spaces in 1985. Arya and Nour [1] defined gs-open sets using semi open sets. A. Pushpalatha and K. Anitha [10] introduced the concept of g^*s-closed sets in topological spaces.

In the present paper we introduce the concept of g^*s-closed sets, g^*s-open sets, g^*s-continuous maps and g^*s-irresolute maps in bitopological spaces.

2. Preliminaries

Throughout this paper X and Y always represent non-empty bitopological space (X, τ_1, τ_2) and (Y, σ_1, σ_2). For a subset A of X τ_j-scl(A) (resp. τ_j-cl(A) and τ_j-αcl(A),) denote the semi closure (resp. closure and α-closure) of X with respect to topology τ_j. In general by (i, j) we mean pair of topologies (τ_1, τ_2).

We recall the following definitions:

Definition 2.1: A subset A of a bitopological space (X, $\tau_{1,}$ τ_2) is called

(i) (i, j)αg-closed if τ_j-αcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-open
(ii) (i, j)-strongly g-closed if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τi-g-open in X.

Definition: 2.2: A subset of a bitopological space (X, $\tau_{1,}$ τ_2) is called

(i) (i, j)-g-closed [5] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-open.
(ii) (i, j)-sg-closed [9] if τ_j-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi open in τ_i.
(iii) (i, j)-ω-closed [4] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi open in τ_i.
(iv) (i, j)-wg-closed [3] if τ_j-cl(τ_i-int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-open.
(v) (i, j)-g^*-closed [11] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-generalized open.
(vi) (i, j)-gs-closed [8] if τ_j-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-open.
(vii) (i, j)-gα-closed [8] if τ_j-αcl (A) $\subseteq$ U whenever A$\subseteq$ U and U is τ_i-α-open.
(viii) (i, j)-preclosed[7] if and only if τ_i-cl(τ_j-int$_j$(A)) $\subseteq$ A.

Definition 2.3: A map f: (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is called

(i) τ_j-σ_k-continuous [2] if f^{-1}(V) $\in$ τ_j for every V $\in$ σ_k.
(ii) (i, j)-σ_k-sg-continous[9] if the inverse Image of every σ_k-closed set is (i, j)-sg closed.
(iii) (i, j)-σ_k-gs-continous[8] if the inverse Image of every σ_k-closed set is (i, j)-gs closed.

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3. g^*s-closed sets in Bitopological Spaces

Definition 3.1: A subset of a bitopological space (X, τ_1, τ_2) is said to be an (i, j)-g^*s-closed set if τ_j-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is gs-open in τ_i.

Theorem 3.2: Every τ_j-closed subset of a bitopological space (X, τ_1, τ_2) is (i, j)-g^*s-closed but the converse need not be true.

Proof: Let A be a τ_j-closed set in X. Let U be a gs-open in τ_i such that A $\subseteq$ U. Since A is τ_j-closed, τ_j-cl(A) = A, τ_j-cl(A) $\subseteq$ U. But τ_j-scl(A) $\subseteq$ τ_j-cl(A) $\subseteq$ U. Therefore τ_j-scl(A) $\subseteq$ U. Hence A is (i, j)-g^*s-closed set.

Example 3.3: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X} and τ_2 = {ϕ, {b, c}, {b}, X}. Then the set A= {c} is (1, 2)-g^*s-closed but not τ_2-closed in (X, τ_1, τ_2) .

Theorem 3.4: If A and B are (i, j)-g^*s-closed then A $\cup$ B is (i, j)-g^*s-closed.

Proof: Let U be a gs-open in τ_i. such that A$\cup$B $\subseteq$U. Then A $\subseteq$ U and B $\subseteq$ U. Since A and B are (i, j)-g^*s-closed set τ_j-scl(A) $\subseteq$ U and τ_j-scl(B) $\subseteq$ U. Hence τ_j-scl(A $\cup$ B) $\subseteq$ τ_j-scl(A) $\cup$ τ_j-scl(B) $\subseteq$ U. Therefore A $\cup$ B is (i, j)-g^*s-closed.

Theorem 3.5: In a bitopological space (X, τ_1, τ_2), every (i, j)-g^*s-closed set is (i, j) gs-closed but the converse need not be true.

Proof: Let A$\subseteq$ U and U is open in τ_i. Since every τ_i-open is τ_i- gs-open and A is (i, j)-g^*s-closed, we have τ_j-scl(A) $\subseteq$ U. Therefore A is (i, j) gs-closed.

Example 3.6: Let X = {a, b, c}, τ_1 = { ϕ, {b}, {b, c}, X} and τ_2 = { ϕ, {a }, {a, c }, X}. Then the subset A= {a} is (1, 2) gs-closed but not (1, 2)-g^*s-closed.

Theorem 3.7: In a bitopological space (X, τ_1, τ_2), every (i, j)-g^*s-closed set is (i, j) sg-closed but the converse need not be true.

Proof: Let U be a τ_i- semi open and A $\subseteq$ U. Since every τ_i- semi open set is τ_i- gs-open and A is (i, j)-g^*s-closed, we have τ_j-scl(A) $\subseteq$ U. Therefore A is (i, j)-sg-closed.

Example 3.8: In Example 3. 6, the set A = {a, c} is (1, 2)-sg-closed but not (1, 2)-g^*s-closed.

Theorem 3.9: In a bitopological space (X, τ_1, τ_2), every τ_j-semi closed is (i, j)-g^*s-closed but the converse need not be true.

Proof: Let A be a (i, j)-semi closed. Let U be a τ_i-gs-open such that A $\subseteq$ U. Since A is τ_j-semi closed, we have τ_j-scl(A) =A. Therefore τ_j-scl(A) $\subseteq$ U. Hence A is (i, j)-g^*s-closed.

Example 3.10: Let X= {a, b, c}, τ_1 = {ϕ, {b}, {b, c}, X} and τ_2 = {ϕ, {c}, X}. Then the subset A = {a, c} is (1, 2)-g^*s-closed but not τ_2-semi closed.

Remark 3.11: The following example shows that (i, j)-g^*s-closed set is independent of (i, j)-g-closed set, (i, j)-ω-closed set and (i, j)-g^*-closed set.

Example 3.12: Let X = {a, b, c, d}, τ_1 = {ϕ, {d}, {a, c}, {a, c, d}, X} and τ_2 = {ϕ, {b}, {a, b}, {b, d}, {a, b, d}, X}. Then the subset {d} is (1, 2)-g^*s-closed set but not (1, 2)-g-closed set and the subset {b} is (1, 2)-g-closed set but not (1, 2)-g^*s-closed set and the subset {a, b, d} is (1, 2)-ω-closed set but not (1, 2)-g^*s-closed set and the subset {d} is (1, 2)-g^*s-closed set but not (1, 2)-ω-closed set and the subset {b} is (1, 2)-g^*-closed set but not (1, 2)-g^*s-closed set and the subset {a} is (1, 2)-g^*s-closed set but not (1, 2)-g^*-closed set.

Remark 3.13: The following example shows that (i, j)-g^*s-closed set is independent of (i, j)-strongly g-closed set.

Example 3.14: In Example 3.6, the subset {c} is (1, 2)-g^*s-closed set but not (1, 2)-strongly g- closed set and the subset {a} is (1, 2)-strongly g-closed set but not (1, 2)-g^*s-closed set.

Remark 3.15: The following example shows that (i, j)-g^*s-closed set is independent of (i, j)-αg-closed set and (i, j)-gα-closed set.

Example 3.16: Let X = {a, b, c}, τ_1 = {ϕ, {c}, {a, b}, X} and τ_2 = {ϕ, {a}, X}. Then the subset A = {a, c} is (1, 2)-αg-closed set and (1, 2)-gα-closed set but not (1, 2)- g^*s-closed set.

Example 3.17: Let X = {a, b, c, d, e}, τ_1 = {ϕ, {a}, {b}, {a, b}, {a, c}, {a, b, c}, X} and τ_2 = {ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}, X}. Then the subset {a} is (1, 2)- g^*s-closed set but not (1, 2)-αg-closed set and (1, 2)-gα-closed set.

Remark 3.18: The following example shows that (i, j)-g^*s-closed set is independent of (i, j)-pre closed set.

Example 3.19: In Example 3.17, the subset {a} is (1, 2)- g^*s-closed set but not (1, 2)-pre closed.

Example 3.20: Let X = {a, b, c, d}, τ_1 = {ϕ, {b}, {a, b}, {b, c}, {a, b, c}, X} and τ_2 = {ϕ, {a, b}, {a, b, c}, {a, b, d}, X}. Then the subset {a} is (1, 2)-pre closed set but not (1, 2)- g^*s-closed set.

Theorem 3.21: A subset A of (X, τ_1, τ_2) is (i, j)-g^*s-closed set iff τ_j-scl(A) – A contains no non empty τ_i-gs-closed set.

Proof: Suppose that F is a non empty τ_i-gs-closed subset of τ_j-scl(A) – A. Now F $\subseteq$ τ_j-scl(A) – A. Then F $\subseteq$ τ_j-scl(A) $\cap$ A^c. Therefore, F $\subseteq$ τ_j-scl(A) and F $\subseteq$ A^c. Since Fc is τ_i-gs-open set and A is (i, j)-g^*s-closed, τ_j-scl(A) $\subseteq$ F^c. That is F $\subseteq$ $(\tau_j\text{-scl}(A))^c$. Hence F $\subseteq$ τ_j-scl(A) $\cap$ $(\tau_j\text{-scl}(A))^c$ = ϕ. That is F = ϕ. Thus τ_j-scl(A) – A contains no nonempty τ_i-gs-closed set.

Conversely, Assume τ_j-scl(A) – A contains no nonempty τ_i-gs-closed set. Let A $\subseteq$ U, U is τ_i-gs-open set. Suppose that τ_j-scl(A) is not contained in U. Then τ_j-scl(A) $\cap$ U^c is a nonempty τ_i-gs-closed set and contained in τ_j-scl(A) – A, which is a contradiction. Therefore τ_j-scl(A) $\subseteq$ U and hence A is (i, j)-g^*s-closed set.

Theorem 3.22: For each element x of (X, τ_1, τ_2), {x} is τ_i-gs-closed or $\{x\}^c$ is (i, j)-g^*s-closed.

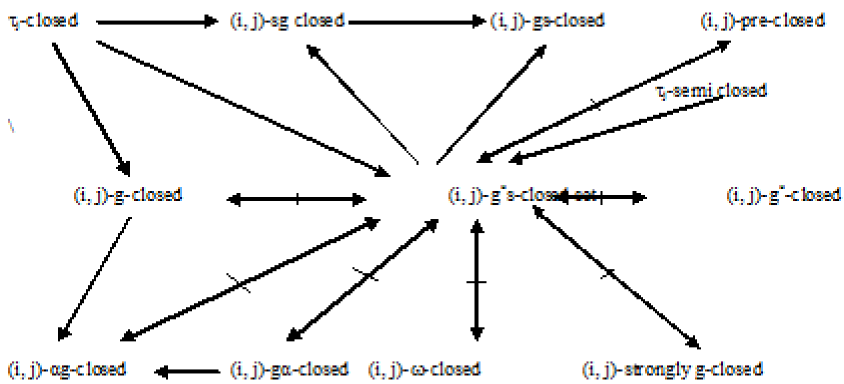
Proof: If {x} is not τ_i-gs-closed, then $\{x\}^c$ is not τ_i-gs-open and a τ_i-gs-open set containing $\{x\}^c$ is X only. Also τ_j-scl($\{x\}^c$) $\subseteq$ X. Therefore $\{x\}^c$ is (i, j)-g^*s-closed.

Theorem 3.23: If A is an (i, j)-g^*s-closed set of (X, τ_1, τ_2) such that A $\subseteq$ B $\subseteq$ τ_j-scl(A), then B is also an (i, j)-g^*s-closed set of (X, τ_1, τ_2).

Proof: Let U be an τ_i-gs-open set of (X, τ_1, τ_2) such that B $\subseteq$ U. Then A $\subseteq$ U. Since A is (i, j)-g^*s-closed, τ_j-scl(A) $\subseteq$ U. We have τ_j-scl(B) $\subseteq$ τ_j-scl(τ_j-scl(A)) = τ_j-scl(A) $\subseteq$ U. Thus B is also an (i, j)-g^*s-closed set of (X, τ_1, τ_2).

Remark 3.24: (1, 2)- g^*s-closed set is generally not equal to (2, 1)-g^*s-closed set. For Example, (1, 2)- g^*s-closed set $\neq$ (2,1)-g^*s-closed set in Example 3.6.

The relations between the previous classes of sets are shown in the following diagram



4. g^*s-open sets in Bitopological Spaces

Definition 4.1: A subset A of a bitopological space (X, τ_1, τ_2) is called (i, j)-g^*s-open if A^c is (i, j)-g^*s-closed.

Theorem 4.2: In a bitopological space (X, τ_1, τ_2)

i) Every τ_j-open sets (i, j)-g^*s-open but not conversely.
ii) Every (i, j)-g^*s-open is (i, j)-gs-open and (i, j)-sg-open

Theorem 4.3: If A and B are (i, j)-g^*s-open sets in (X, τ_1, τ_2) then A ∩ B is also an (i, j)-g^*s-open set in (X, τ_1, τ_2).

Proof: Let A and B be two (i, j)-g^*s-open sets. Then A^c and B^c are (i, j)- g^*s-closed sets. By Theorem 3.4, $A^c \cup B^c$ is a (i, j)-g^*s-closed set in (X, τ_1, τ_2). That is $(A \cap B)^c$ is a (i, j)-g^*s-closed set. Therefore (A ∩ B) is (i, j)-g^*s-open set in (X, τ_1, τ_2).

5. g^*s-continuous Maps and g^*s-irresolute Maps in Bitopological Spaces

In this section we introduce g^*s-continuous maps from a bitopological space (X, τ_1, τ_2) into a bitopological space (Y, σ_1, σ_2) and study some of their properties.

Definition 5.1: A map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called (i, j)-σ_k-g^*s-continuous map if the inverse image of every σ_k-closed set is an (i, j)-g^*s-closed set.

Definition: 5.2: A map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called Pairwise g^*s-irresoulte, if the inverse image of every (e, k)-g^*s-closed sets in Y is an (i, j)- g^*s-closed sets in X.

Definition: 5.3: A bitopological space (X, τ_i, τ_j) is called (i, j)-T_{g^*s}-space if every (i, j)-g^*s-closed set is τ_j-closed.

Theorem 5.4: If a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is τ_j-σ_k-continuous, then it is an (i, j)-σ_k-g^*s-continuous map but not conversely.

Proof: Let V be σ_k-closed set in Y, then $f^{-1}(V)$ is τ_j-closed set, since f is τ_j-σ_k-continuous. By Theorem 3.2, $f^{-1}(V)$ is (i, j)- g^*s-closed. Therefore f is (i, j)-σ_k-g^*s-continuous map.

Example 5.5: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = {ϕ, {b}, {b, c}, X}, let = {p, q, r}, σ_1 = {ϕ, {q}, {q, r}, Y}, σ_2 = {ϕ, {p}, {p, r}, Y}. Define a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(c) = p, f(b) =q, f(a) = r. Then f is (1, 2)-σ_1- g^*s-continuous map but not τ_2-σ_1-continuous.

Theorem 5.6: If a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (i, j)-σ_k-g^*s continuous, then it is an (i, j)-σ_k-gs-continuous map but not conversely.

Proof: Let V be σ_k-closed set in Y, then $f^{-1}(V)$ is (i, j)-g^*s-closed in X, since f is (i, j)-σ_k-g^*s continuous. By Theorem 3.5, $f^{-1}(V)$ is (i, j)- gs-closed in X and so f is (i, j)-σ_k-gs-continuous map.

Example 5.7: Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = {ϕ, {b}, {b, c}, X}, let Y = {p, q, r}, σ_1 = {ϕ, {p}, {p, r}, Y}, σ_2 = {ϕ, {p}, {p, q}, Y}. Define a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = p, f(c) = r, f(b) = q. Then f is (1, 2)-σ_2-gs-continuous map but not (1, 2)-σ_2- g^*s-continuous map.

Theorem 5.8: If a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (i, j)-σ_k-g^*s continuous, then it is an (i, j)-σ_k-sg-continuous map but not conversely.

Proof: Let V be σ_k-closed set in Y, then $f^{-1}(V)$ is (i, j)-g^*s-closed in X, since f is (i, j)-σ_k-g^*s continuous. By Theorem 3.7, $f^{-1}(V)$ is (i, j)-sg-closed set in X and so f is (i, j)-σ_k-sg-continuous map.

Example 5.9: Let X = {a, b, c}, τ_1 = {ϕ, {b}, {b, c}, X}, τ_2 = {ϕ, {a}, {a, c}, X}, let Y = {p, q, r}, σ_1 = {ϕ, {p}, {p, r}, Y}, σ_2 = {ϕ, {q}, {q, r}, Y}. Define a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = r, f(b) =q, f(c) = p. Then f is (1, 2)-σ_2-sg-continuous map but not (1, 2)-σ_2- g^*s-continuous map.

Theorem 5.10: If a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is a pairwise g*s-irresolute, then it is an (i, j)-σ_k-g^*s-continuous map but not conversely.

Proof: Assume that f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is a pairwise g*s-irresolute. Let V be σ_k-closed set in Y. So it is (e, k)-g^*s-closed in Y by Theorem 3.2. By our assumption, $f^{-1}(V)$ is (i, j)-g^*s-closed set in X and so f is (i, j)-σ_k-g^*s-continuous map.

Example 5.11: Let X = Y = {a, b, c}, τ_1 = {ϕ, {b}, {b, c}, X}, τ_2 = {ϕ, {a}, {a, c}, X}, σ_1 = {ϕ, {b}, {b, c}, Y}, σ_2 = {ϕ, {a}, Y}. Define a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = b, f(b) =a, f(c) = c. Then f is (1, 2)-σ_1-g^*s-continuous map but not pairwise g^*s-irresolute.

Theorem 5.12: Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a map, then the following statements are equivalent

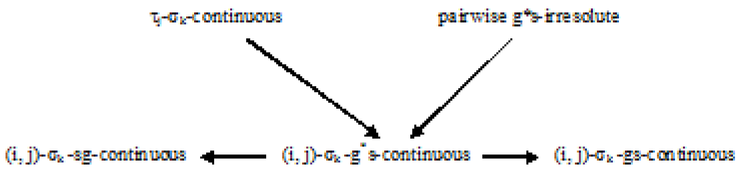
a) f is (i, j)-σ_k-g^*s-continuous map
b) the inverse image of each σ_k-open se in Y is (i, j)-σ_k-g^*s-continuous map.

Proof: Assume that f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (i, j)-σ_k-g^*s-continuous map. Let G be σ_k-open in Y. Then G^c is σ_k-closed in Y. Since f is (i, j)-σ_k-g^*s-continuous map, $f^{-1}(G^c)$ is (i, j)-g^*s-closed in X. But $f^{-1}(G^c)$ = X- $f^{-1}(G)$. Thus $f^{-1}(G)$ is (i, j)- g^*s-open in X.

Conversely, assume that the inverse image of each σ_k-open set in Y is (i, j)-g^*s-open in X. Let F be any σ_k-closed set in Y, then $f^{-1}(F^c)$ is (i, j)- g^*s-open. But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $f^{-1}(F)$ is (i, j)- g^*s-closed in X. Therefore f is (i, j)-σ_k-g^*s-continuous map.

Theorem 5.13: Let (X, τ_1, τ_2) and (Z, μ_1, μ_2) be any bitopological spaces and Y be a (e, k)-T_{g^*s}-space, then the composition $g \circ f$: X →Z is (i, j)-μ_p-g*s-continuous map, if f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (i, j)-σ_k-g^*s-continuous map and g : (Y, σ_1, σ_2) → (Z, μ_1, μ_2) is (e, k)-μ_p-g*s-continuous map.

Proof: Let F be μ_p-closed in Z. Since g is (e, k)-μ_p- g*s-continuous map, $g^{-1}(F)$ is (e, k)- g*s-closed in Y. But Y is (e, k)-T_{g^*s}-space and so $g^{-1}(F)$ is σ_k-closed in Y. Since f is (i, j)-σ_k-g^*s-continuous map, $f^{-1}(g^{-1}(F))$ is (i, j)- g^*s-closed in X. But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}$ (F). Therefore $(g \circ f)^{-1}$ (F) is (i, j)-g*s-closed. Hence $g \circ f$ is (i, j)-μ_p-g*s-continuous map.



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