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NEW FAMILIES OF 3-TOTAL PRODUCT CORDIAL GRAPHS

R. Ponraj[1*], M. Sivakumar[2] and M. Sundaram[3]

[1]Department of Mathematics, Sri Paramakalyani College, Alwarkurchi-627412, India
[2]Department of Mathematics, Unnamalai Institute of Technology, Kovilpatti-628502, India
[3]Associate Professor (Rtd.), Department of Mathematics, Sri Paramakalyani College Alwarkurchi-627412, India

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ABSTRACT

Let f be a map from V(G) to{0,1,...k-1}where k is an integer, $2 \le k \le |V(G)|$. For each edge uv assign the label f(u) f(v)(modk). f is called a k- Total Product cordial labeling if $|f(i)-f(j)| \le 1$, $i,j \in \{0,1,..k-1\}$, where f(x) denotes the total number of vertices and edges labelled with x(x=0,1,2....k-1). A graph that admits a k- Total Product cordial labelling is called a k- Total Product cordial graph. In this paper we investigate 3- Total Product cordial labeling behaviour of some standard graphs like Wheels, Helms, Dragons, etc.

Keywords: *Wheel, Helms, Dragon, $C_n\Theta 2K_1$.*

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1. INTRODUTION

The graphs considered here are finite, undirected and simple. The vertex set and edge set of a graph *G* are denoted by *V(G)* and *X(G)* respectively. The following definitions are used here.

- The corona $G_1\Theta G_2$ of two graphs G_1 and G_2 is defined as the graph *G* obtained by taking one copy of G_1(which has $p_1$vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy G_2.
- The graph $W_n = C_n + K_1$ is called a wheel.
- The Helms H_n is the graph obtained from wheel by attaching a pendant edge at each vertex of the cycle C_n.
- A Dragon is formed by identifying the end vertex of the path to the vertex of a cycle.
- m copies of the graph G is denoted by mG.

The notion of k-Product cordial labeling of graph was introduced in [2] where the k-Product cordial labeling behaviour of some standard graphs was studied. Also k-Total Product labeling of graphs was introduced in [4].Obviously 2-Total Product cordial labeling is simply a Total Product cordial labeling[5]. Also 3-Total Product cordial labeling behaviour of some standard graphs was studied in [3]. In this paper we investigate 3-Total Product cordial labeling behaviour of Helms, Wheel, Dragon, $C_n\Theta 2K_1$ and some standard graphs. Terms not defined here are used in the sense of Harary[1].

2. k-TOTAL PRODUCT CORDIAL LABELING

Definition 2.1:
Let f be a function from *V(G)* to {*0,1….k-1*} where *k* is an integer, $2 \le k \le |V(G)|$. For each edge *uv*, assign the label *f(u)f(v)(mod k)*. f is called a *k*- Total Product cordial labeling of *G* if $|f(i) - f(j)| \le 1$, $i,j \in \{0,1,..k-1\}$ where $f(x)$ denotes the total number of vertices and edges labelled with *x*(*x=0,1,2…k-1*).A graph with a *k*- Total Product cordial labelling is called a *k*- Total Product cordial graph.

Theorem 2.2: Let G be a (p, p) graph. Then mG is 3-Total Product cordial where $m \equiv 0 (\text{mod } 3)$.

Corresponding author: R. Ponraj[1*]
[1]Department of Mathematics, Sri Paramakalyani College, Alwarkurchi-627412, India

Proof: let m=3t. Clearly mG has 3pt vertices and 3pt edges. Assign the label 2 to all the vertices of first 2t copies of G. Then assign 0 to all the vertices of remaining t copies of G.

Then $f(0) = f(1) = f(2) = 2pt$. Therefore f is a 3-Total Product cordial labeling.

Corollary 2.3: If $m \equiv 0 (mod 3)$, then mC_n is 3- Total Product cordial.

Notation: Let G be any graph. Then the graph obtained from G by identifying the central vertex of $K_{1,p}$ to any vertex of G is denoted by $G * K_{1,p}$.

Theorem 2.4: If G is a (p, p) graph.
(i) If p is even then $G * K_{1,\frac{p}{2}}$ is 3-Total Product cordial.
(ii) If p is odd then $G * K_{1,\frac{p-1}{2}}$ and $G * K_{1,\frac{p+1}{2}}$ are 3-Total Product cordial .

Proof: Case (i): p is even.
Assign the label 2 to all the vertices of G and 0 to all pendant vertices of $K_{1,\frac{p}{2}}$. Clearly $f(0) = f(1) = f(2) = p$.

Case (ii): p is odd.
Assign label as in case (i) Clearly $f(0) = p-1$, Therefore $f(1) = f(2) = p$ for the graph $G * K_{1,\frac{p-1}{2}}$. and $f(0) = p+1$, $f(1) = f(2) = p$ for the graph $G * K_{1,\frac{p+1}{2}}$. Therefore f is a 3-Total Product cordial labeling.

Theorem 2.5: The Wheel W_n is 3-Total Product Cordial.

Proof: Let C_n be the cycle $u_1u_2\ldots u_nu_1$ and let $V(W_n) = V(C_n) \cup \{u\}$ $E(W_n) = E(C_n) \cup \{uu_i : 1 \le i \le n\}$.

Define $f(u) = 0$,
$f(u_i) = 2,\ 1 \le i \le n$.

Here $f(0) = n+1$ and $f(1) = f(2) = n$ .Hence f is a 3-Total Product cordial labeling.

Theorem 2.6: The Helms H_n is 3-Total Product cordial.

Proof: Let the vertex set and edge set of the wheel W_n be defined as in theorem2.5.

Let $V(H_n) = V(W_n) \cup \{v_i, 1 \le i \le n\}$ and $E(H_n) = E(W_n) \cup \{u_iv_i : 1 \le i \le n\}$.

Case (i): $n \equiv 0 (mod\ 3)$

Let n=3t

Define $f(u) = 0$,
$f(u_i) = 2,\ 1 \le i \le n$
$f(v_i) = 2,\ 1 \le i \le 2t$
$f(v_{2t+i}) = 0,\ 1 \le i \le t$

Here $f(0) = 5t+1$ and $f(1) = f(2) = 5t$ .Hence f is a 3-Total Product cordial labeling.

Case (ii): $n \equiv 1 (mod\ 3)$

Let n=3t+1. Assign labels to the vertices u, u_i, v_i $(1 \le i \le n-1)$ as in case(i). Then assign the labels 2, 2 to the vertices u_n, v_n respectively. Here $f(0) = f(1) = f(2) = 5t+2$. Hence f is a 3-Total Product cordial labeling.

Case (iii): n≡2(mod 3)

Let n=3t+2. Assign labels to the vertices u, u_i, v_i $(1 \le i \le n-1)$ as in case (ii). Then assign the labels 2, 2 to the vertices u_n, v_n respectively. Here $f(0) = 5t+3$ and $f(1) = f(2) = 5t+4$. Hence f is a 3-Total Product cordial labeling.

Illustration 2.7: A 3-Total Product cordial labeling of H_7 is

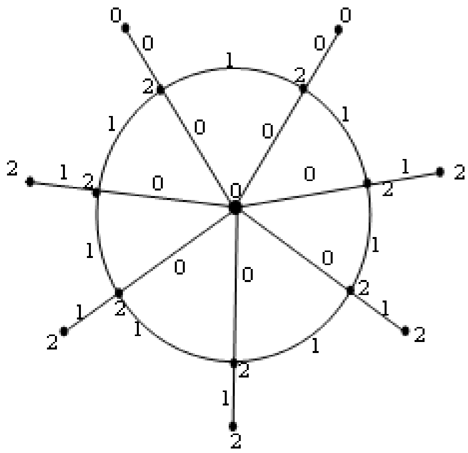

Figure (i)

Notation 2.8: Let C_n be the cycle $u_1, u_2, \ldots, u_n, u_1$. Let G_n denotes the graph with $V(G_n) = V(C_n) \cup \{v_i, w_i, 1 \le i \le n\}$ and $E(G_n) = E(C_n) \cup \{u_iv_i, u_iw_i, v_iw_i : 1 \le i \le n\}$

Theorem 2.9: G_n is 3-Total Product cordial.

Proof: Let the vertex set and edge set of the graph G_n be as defined above.

Case (i): n≡0(mod 3)

Let n=3t. Define $f(v_i) = f(c_i) = 0,\ 1 \le i \le t$

$$f(w_i) = 0,\ 1 \le i \le t-1$$
$$f(w_t) = 0.$$
$$f(u_{t+i}) = f(v_{t+i}) = f(w_{t+i}) = 2, \quad 1 \le i \le t$$
$$f(u_{2t+i}) = 2 \ , \quad 1 \le i \le t$$
$$f(v_{2t+i}) = 1 \ , \quad 1 \le i \le t$$
$$f(w_{2t+i}) = 2 \ , \quad 1 \le i \le t$$

Then $f(0) = f(1) = f(2) = 7t.$ Hence f is a 3-Total Product cordial labeling.

Case (ii): n≡1(mod 3)

Let n=3t+1. Assign labels to the vertices $u_i, v_i, w_i,\ 1 \le i \le n-1$ as in case (i). Then assign the labels 2, 2 and 0 to the vertices u_n, v_n, w_n respectively .Here $f(0) = 7t+3$ and $f(1) = f(2) = 7t+2$.Hence f is 3-Total Product cordial labeling.

Case (iii): n≡2(mod 3)

Let n=3t+2. Assign labels to the vertices $u_i, v_i, w_i,\ 1 \le i \le n-2$ as in case (i). Then assign the labels 0, 2, 0, 2, 2 and 2 to the vertices $v_{n-1}, v_n, w_{n-1}, w_n, u_{n-1}, u_n$ respectively .Here $f(0) = 7t+5$ and $f(1) = f(2) = 7t+4$.Hence f is 3-Total Product cordial labeling.

Illustration 2.10: A 3-Total Product cordial labelling of G_6 is

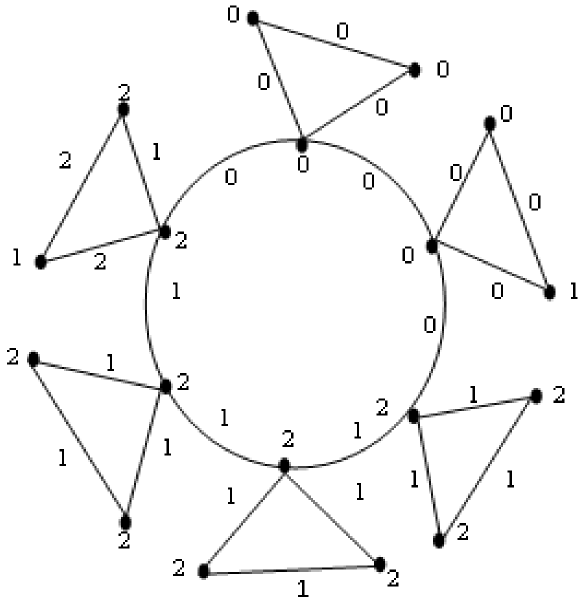

Figure (ii)

Theorem 2.11: $C_n\Theta 2K_1$ is 3-Total Product cordial.

Proof: Let V $(C_n\Theta 2K_1) = \{u_i, v_i, w_i, \quad 1 \le i \le n,$ } and
$E(C_n\Theta 2K_1)=\{ u_iu_{i+1}, u_nu_1 : 1 \le i \le n-1\} \cup \{u_iv_i, u_iw_i, 1 \le i \le n\}$

Case (i): n is even.

Define

$f(u_i) = 2, 1 \le i \le n$

$f(v_i) = f(w_i) = 0, 1 \le i \le \dfrac{n}{2}$

$f(v_{\frac{n}{2}+i}) = f(w_{\frac{n}{2}+i}) = 2, 1 \le i \le \dfrac{n}{2}$

Then $f(0) = f(1) = f(2) = 2n.$ Hence f is a 3-Total Product cordial labeling.

Case (ii): n is odd.

Define $f(u_i) = 2, 1 \le i \le n$, $f(v_i) = 0, 1 \le i \le \dfrac{n+1}{2}$

$f(w_i) = 0, 1 \le i \le \dfrac{n-1}{2}, \ f(v_{\frac{n+1}{2}+i}) = 2, 1 \le i \le \dfrac{n-1}{2}$

$f(w_{\frac{n-1}{2}+i}) = 2, 1 \le i \le \dfrac{n+1}{2}$

Then $f(0) = f(1) = f(2) = 2n.$ Hence f is a 3-Total Product cordial labeling.

Illustration 2.12: A 3-Total Product cordial labeling of $C_7\Theta 2K_1$ is

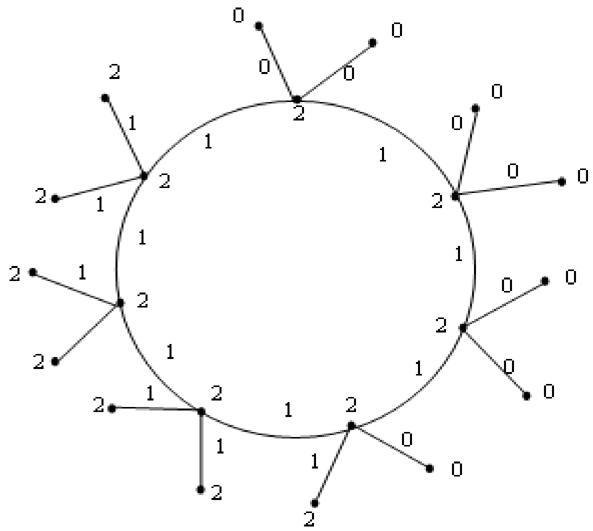

Figure (iii)

Theorem 2.13: The Dragon $C_m@P_n$ is 3-Total Product cordial.

Proof: Let C_m be the cycle $u_1, u_2...u_m,u_1$ and P_n be the path $v_1,v_2,\ldots.v_n.$ identify the vertex u_1 with v_1.

Case (i): *$m\equiv 0(mod\ 3)$ and $n\equiv 0(mod\ 3)$*

Let $m=3t_1$ and $n=3t_2$

Define

$$f(u_i)=0,\ 1\le i\le t_1$$
$$f(u_{t_1+i})=2,\ 1\le i\le 2t_1$$
$$f(v_i)=0,\ 2\le i\le t_2-1$$
$$f(v_{t_2})=f(v_{t_2+1})=1,$$
$$f(v_{t_2+1+i})=2,\ 1\le i\le 2t_2-1$$

Then $f(0)=2t_1+2t_2$ and $f(1)=f(2)=2t_1+2t_2-1.$ Hence f is a 3-Total Product cordial labeling.

Case (ii): *$m\equiv 0(mod\ 3)$ and $n\equiv 1(mod\ 3)$*

Let $m=3t_1$ and $n=3t_2+1$. Assign label 1 to the vertex v_n and assign labels to all the remaining vertices as in case (i). In this case $f(0)=f(1)=f(2)=2t_1+2t_2$. Hence f is a 3-Total Product cordial labeling.

Case (iii): *$m\equiv 0(mod\ 3)$ and $n\equiv 2(mod\ 3)$*

Let $m=3t_1$ and $n=3t_2+2$. Assign labels 2, 1 to the vertices v_{n-1}, v_n respectively. Then assign the labels to all the remaining vertices as in case(i).Here $f(0)=2t_1+2t_2$ and $f(1)=f(2)=2t_1+2t_2+1.$

Hence f is a 3-Total Product cordial labeling.

Case (iv): *$m\equiv 1(mod\ 3)$ and $n\equiv 0(mod\ 3)$*

Let $m=3t_1+1$ and $n=3t_2$. Assign label 1 to the vertex u_m .Then assign the labels to all the remaining vertices as in case (i). In this case $f(0)=f(1)=f(2)=2t_1+2t_2$. Hence f is a 3-Total Product cordial labeling.

Case (v): $m \equiv 1 (mod\ 3)$ and $n \equiv 1 (mod\ 3)$

Let $m=3t_1+1$ and $n=3t_2+1$. Assign label 1 to the vertex v_n. Then assign the labels to all the remaining vertices as in case (iv).Here $f(0) = 2t_1 + 2t_2$ and $f(1) = f(2) = 2t_1 + 2t_2 + 1$.Hence f is a 3-Total Product cordial labeling.

Case (vi): $m \equiv 1 (mod\ 3)$ and $n \equiv 2 (mod\ 3)$

Let $m=3t_1+1$ and $n=3t_2+2$. Assign label 0 to the vertex v_n. Then assign the labels to all the remaining vertices as in case (v).In this case $f(0) = 2t_1 + 2t_2 + 2$ and $f(1) = f(2) = 2t_1 + 2t_2 + 1$.Hence f is a 3-Total Product cordial labeling.

Case (vii): $m \equiv 2 (mod\ 3)$ and $n \equiv 0 (mod\ 3)$

Let $m=3t_1+2$ and $n=3t_2$. Assign labels 2,0 to the vertices u_{m-1} u_m respectively. Then assign the labels to the all the remaining vertices as in case (i). In this case $f(0) = 2t_1 + 2t_2$ and $f(1) = f(2) = 2t_1 + 2t_2 + 1$. Hence f is a 3-Total Product cordial labeling.

Case (viii): $m \equiv 2 (mod\ 3)$ and $n \equiv 1 (mod\ 3)$

Let $m=3t_1+2$ and $n=3t_2+1$.Assign label 0 to the vertex v_n. Then assign the labels to all the remaining vertices as in case (vii).Here $f(0) = 2t_1 + 2t_2 + 2$ and $f(1) = f(2) = 2t_1 + 2t_2 + 1$. Hence f is a 3-Total Product cordial labeling.

Case (ix): $m \equiv 2 (mod\ 3)$ and $n \equiv 2 (mod\ 3)$

Let $m=3t_1+2$ and $n=3t_2+2$. Assign labels 2, 0 to the vertex v_{n-1} ,v_n respectively. Then assign the labels to the all the remaining vertices as in case(vii).Here $f(0) = f(1) = f(2) = 2t_1 + 2t_2 + 2$ .Hence f is a 3-Total Product cordial labeling.

Illustration 2.14: A 3-Total Product cordial labeling of the dragon $C_{12}@P_8$ is

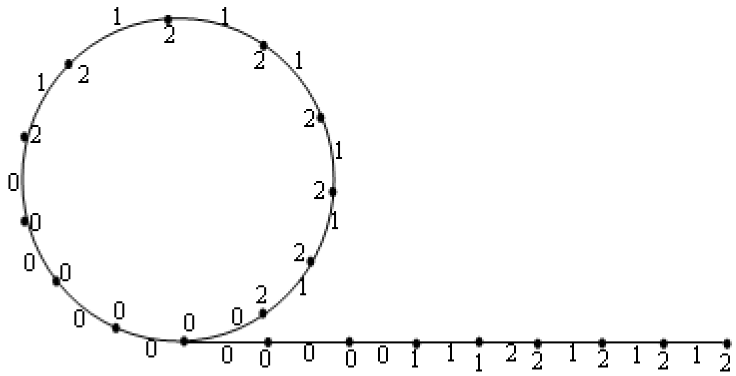

Figure (iv)

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