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FLOW AND HEAT TRANSFER OF AN EXPONENTIAL STRETCHING SHEET IN BOUSSINESQ-STOKES SUSPENSION

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ABSTRACT

An analysis is carried out to study the flow and heat transfer due to an exponentially stretching sheet in a Boussinesq-Stokes suspension. Two cases are studied in heat transfer, namely (i) the sheet with prescribed exponential order surface temperature (PEST-case) and (ii) the sheet with prescribed exponential order heat flux (PEHF-case). The governing coupled, non-linear, partial differential equations are converted into coupled, non-linear, ordinary differential equations by a similarity transformation and are solved numerically using shooting method. The classical explicit Runge-Kutta-Fehlberg 45 method is used to solve the initial value problem by the shooting technique. The effects of various parameters such as the couple stress parameter, Reynolds number and Prandtl number on velocity and temperature profiles are presented and discussed. The results have possible technological applications in the liquid-based systems involving stretchable materials.

1.1 INTRODUCTION

Boundary layer flow on continuous moving surface is an important type of flow occurring in a number of engineering processes. Aerodynamic extrusion of plastic sheets, cooling of an infinite metallic plate in a cooling path, the boundary layer along a liquid film in condensation process and a polymer sheet of filament extruded continuously from a die are examples of practical applications of continuous moving surfaces. Gas blowing, continuous casting and spinning of fibers also involve the flow due to a stretching surface.

Sakiadis [1-3] initiated the study of the boundary layer flow over a continuous solid surface moving with constant speed. Erickson et al. [4] extended the work of Sakiadis to account for mass transfer at the stretching sheet surface. Tsou et al. [5] reported both analytical and experimental results for the flow and heat transfer aspects developed by a continuously moving surface. Crane [6] studied the steady two dimensional boundary layer flow caused by the stretching sheet, which moves in its own plane with a velocity which varies linearly with the axial distance. Several researchers considered various aspects of momentum and heat transfer characteristics in boundary layer flow over a stretching boundary (Gupta and Gupta [7], Kumaran and Ramanaiah [8] and references therein [9-13]).

Magyari and Keller [14] studied the heat and mass transfer on the boundary layer flow due to an exponentially stretching surface. Elbashbeshy [15] added new dimension to the study on exponentially stretching surface. Partha et al. [16] have examined the mixed convection flow and heat transfer from an exponentially stretching vertical surface in quiescent liquid using a similarity solution. Heat and mass transfer in a viscoelastic boundary layer flow over an exponentially stretching sheet were investigated by Khan and Sanjayanand [17-18]. Sajid and Hayat [19] considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. The constitutive equations for couple stress fluids are given by Stokes [20]. The present work analyses the flow and heat transfer due to an exponentially stretching continuous surface in the presence of Boussinesq-Stokes suspension.

1.2 MATHEMATICAL FORMULATION

We consider a steady, two-dimensional boundary layer flow of an incompressible Boussinesq-Stokes suspension flow due to an exponentially stretching sheet .The flow is assumed to be generated by stretching of the sheet from a slit with a velocity which varies exponentially in the direction of x-axis. In this situation the governing boundary layer equations for momentum and heat transfer are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1.2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\nu'\frac{\partial^4 u}{\partial y^4}, \tag{1.2.2}$$

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$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}, \tag{1.2.3}$$

subject to the boundary conditions:

$$u=U_w(x)=U_0\,e^{\frac{x}{l}},\ v=0,\ \frac{\partial^2 u}{\partial y^2}=0,\ \left\{\begin{array}{ll} T=T_w=T_\infty+A_0\,e^{\frac{x}{l}} & in\ PEST\ case \\ -k\left(\frac{\partial T}{\partial y}\right)_w=A_1\,e^{\frac{3x}{2l}} & in\ PEHF\ case \end{array}\right\}\ at\ \ y=0, \tag{1.2.4}$$

$$u\to 0,\ \ \frac{\partial^2 u}{\partial y^2}\to 0, T\to T_\infty\quad as\quad y\to\infty.$$

where u and v are the velocity components of the fluid in x and y directions, ν is the kinematic coefficient of viscosity, ν′ is the couple stress viscosity, U_w stands for stretching velocity of the boundary, U_0 is a constant, l is the reference length, is the density, k is the thermal conductivity, T is the temperature, T_∞ is the temperature at the wall outside the dynamic region and C_p is the specific heat at constant pressure. Here A_0 and A_1 are the parameters of the temperature distribution on the stretching surface.

In the boundary conditions (1.2.4), the third and the sixth conditions pertain to the couple stress. The finite value of the couple stress at the sheet depends on the flow under question. At the sheet and far away from the sheet the velocity is zero and hence the second order derivative vanishes.

We introduce the stream function $\psi(x, y)$ defined by

$$u=\frac{\partial\psi}{\partial y},\ \ v=-\frac{\partial\psi}{\partial x} \tag{1.2.5}$$

The above set of partial differential equations is converted in to ordinary differential equations using the following similarity transformation.

$$X=\frac{x}{l}, Y=\frac{y}{l}, \Psi(X,Y)=\frac{\psi(x,y)}{\nu}=\sqrt{2\,Re}\ f(\eta)e^{\frac{X}{2}}, \eta=Y\sqrt{\frac{Re}{2}}\,e^{\frac{X}{2}},$$
$$\left\{\begin{array}{ll} \theta(\eta)=\dfrac{T-T_w}{T_w-T_\infty} & in\ \ PEST\ \ case \\ \phi(\eta)=\dfrac{T-T_w}{\frac{A_1}{k}\sqrt{\frac{2}{Re}}\,e^X} & in\ \ PEHF\ case \end{array}\right\}. \tag{1.2.6}$$

where η is the similarity variable and $Re=\dfrac{U_0\,l}{\nu}$ is the Reynold's number.

Using the similarity transformation (1.2.6) in the equation (1.2.2), one immediately obtains the governing partial differential equations (1.2.2) and (1.2.3) reduces to a set of ordinary differential equations as

$$C^2\,Re\,f^{v}-2f'''-2f\ f''+4(f')^2=0, \tag{1.2.7}$$

The boundary conditions (1.2.4) for velocity can be written as:

$$f(0)=0,\ \ f'(0)=1,\ \ f'''(0)=0,$$
$$f'(\infty)\to 0,\ \ f'''(\infty)\to 0. \tag{1.2.8}$$

Here, $C^2=\dfrac{\nu'}{\nu\,l^2}$ is the couple stress parameter.

Using equation (1.2.6) in equations (1.2.3) and (1.2.4), we get:

(i) PEST:

$$\theta'' - Pr\left(2 f'\theta - f\theta'\right) = 0, \tag{1.2.9}$$

$$\theta(0) = 1, \theta(\infty) \to 0. \tag{1.2.10}$$

(ii) PEHF:

$$\phi'' - Pr\left(2 f'\phi - f\phi'\right) = 0, \tag{1.2.11}$$

$$\phi'(0) = -1, \phi(\infty) \to 0. \tag{1.2.12}$$

We now outline the procedure for solving two boundary value problems (1.2.9)-(1.2.10) and (1.2.11)-(1.2.12) which are coupled with (1.2.7)-(1.2.8).

1.3 METHOD OF SOLUTION

We adopt the shooting method with Runge-Kutta-Fehlberg 45 scheme to solve the initial value problems in PEST and PEHF cases mentioned in the previous section. The coupled non-linear equations (1.2.7)-(1.2.10) in PEST case are transformed in to a system of seven first order ordinary differential equations as follows.

$$\begin{aligned}
\frac{dy_1}{dY} &= y_2, \\
\frac{dy_2}{dY} &= y_3, \\
\frac{dy_3}{dY} &= y_4, \\
\frac{dy_4}{dY} &= y_5, \\
\frac{dy_5}{dY} &= \frac{1}{C^2\, Re}\left(2y_4 + 2y_1y_3 - 4y_2^{\,2}\right), \\
\frac{dy_6}{dY} &= y_7, \\
\frac{dy_7}{dY} &= Pr\left(2y_2y_6 - y_1y_7\right),
\end{aligned} \tag{1.2.13}$$

with the boundary conditions

$$\begin{aligned}
&y_1(0) = 0,\ y_2(0) = 1,\ y_4(0) = 0, y_6(0) = 1, \\
&y_2(\infty) = 0,\ y_4(\infty) = 0,\ y_6(\infty) = 0.
\end{aligned} \tag{1.2.14}$$

Here, $y_1 = f(\eta)$ and $y_6 = \theta(\eta)$.

Aforementioned boundary value problem is converted in to an initial value problem by choosing the values of $y_3(0)$, $y_5(0)$ and $y_7(0)$ appropriately. Resulting initial value problem is integrated using Runge-Kutta-Fehlberg 45 order method. Newton-Raphson method is used to correct the guess values of $y_3(0)$, $y_5(0)$ and $y_7(0)$. In solving equations (1.2.13) subjected to the boundary conditions (1.2.14) the approximate '∞' is determined through actual computation. Same procedure is adopted to solve (1.2.7)-(1.2.8) and (1.2.11) and (1.2.12). The results are presented in several graphs.

1.4 RESULTS AND DISCUSSION

The boundary layer flow and heat transfer of a stretching sheet in the presence of Boussinesq-Stokes suspension is analyzed. The effects of the parameters *C*, *Re* and *Pr* are shown in several graphs in figures 1.1 to 1.9.

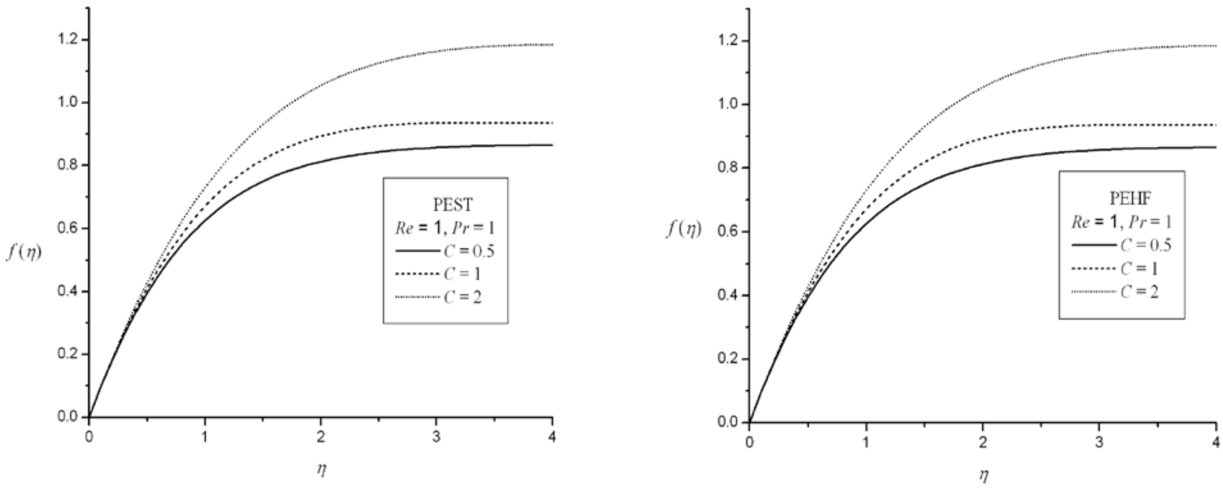
Figure 1.1-1.4 illustrates the effect of couple stress parameter *C* on the flow and heat transfer in PEST and PEHF cases. It is observed from these plots that $f(\eta)$ and $f'(\eta)$ increases with increasing values of *C*, where as $\theta(\eta)$ decreases with increasing values of *C*. This means that the increasing values of *C* results in thickening of the momentum boundary layer and thinning of thermal boundary layer.

Figure 1.4-1.6 demonstrates the effect of *Re* on the flow and heat transfer. The effect of *Re* is similar to that of *C* in both PEST and PEHF cases.

The impact of Prandtl number *Pr* on the momentum and heat transfer is depicted in figures 1.7-1.9. Increasing values of *Pr* does not effect the profiles of $f(\eta)$ and $f'(\eta)$, where as the temperature at a given point decreases with an in increase in the Prandtl number *Pr*. This is in agreement with the physical fact that the thermal boundary layer thickness decreases with increasing Prandtl number.

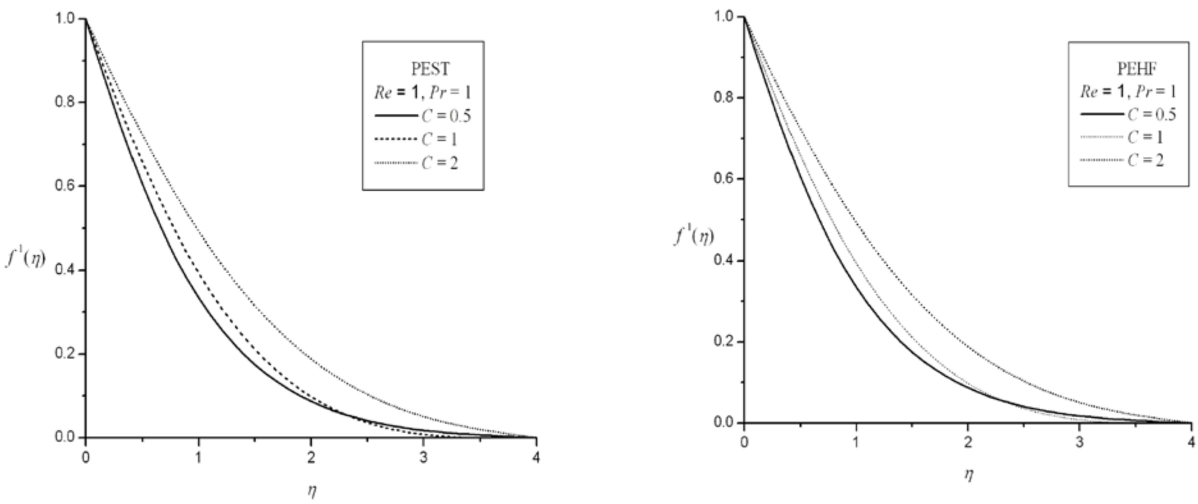
1.5 CONCLUSIONS

1. Increasing values of couple stress parameter *C* results in thickening of the momentum boundary layer and thinning of thermal boundary layer. The same effect is observed for increasing values of *Re*.
2. The effect of increasing values of Prandtl number *Pr* is to decrease the magnitude of heat transfer.
3. Comparison of results of PEST and PEHF boundary conditions reveals that PEHF is better suited for effective cooling of the stretching sheet.



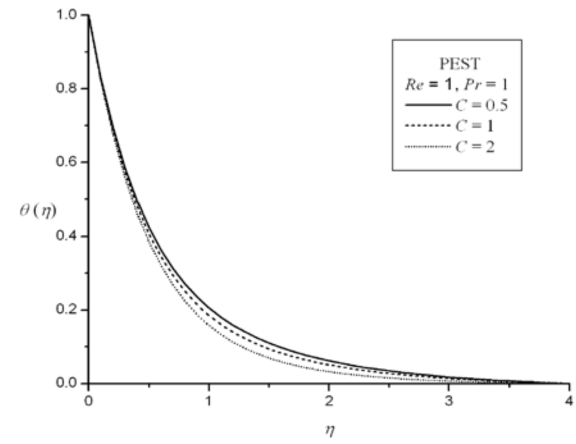
1.1(a) **1.1(b)**

Fig. 1.1: Plot of $f(\eta)$ versus η for values of couple stress Parameter (*C*).

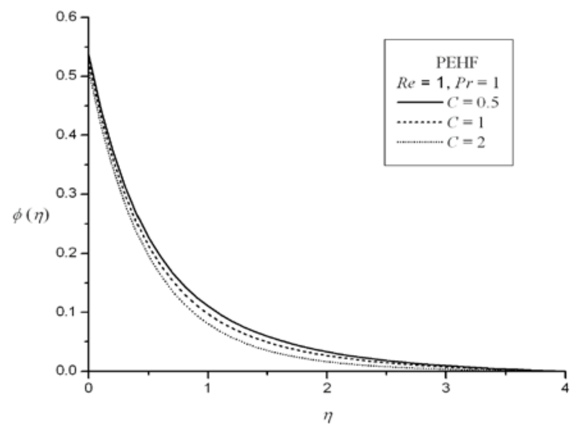


1.2(a) **1.2(b)**

Fig. 1.2: Plot of $f'(\eta)$ versus η for values of *C*.

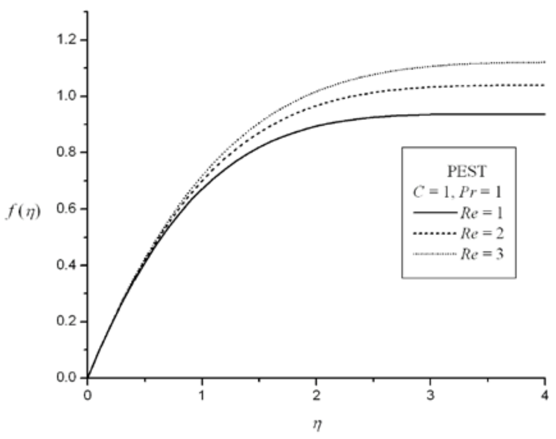


1.3(a)

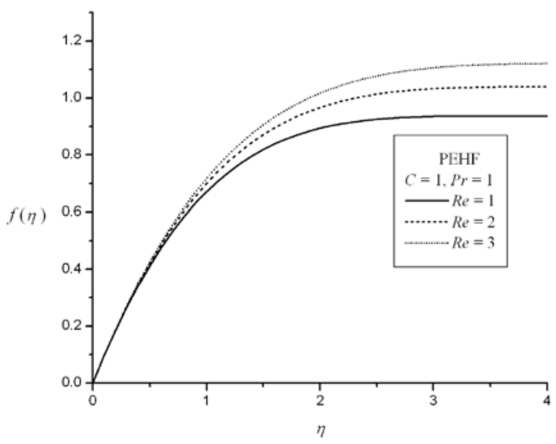


1.3(b)

Fig. 1.3: Plot of temperature profiles for values of couple stress Parameter (C)

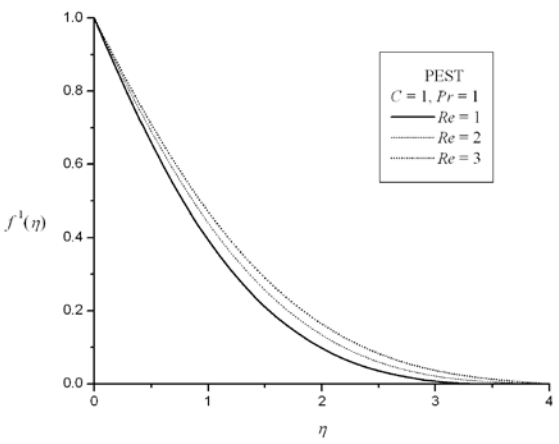


1.4(a)

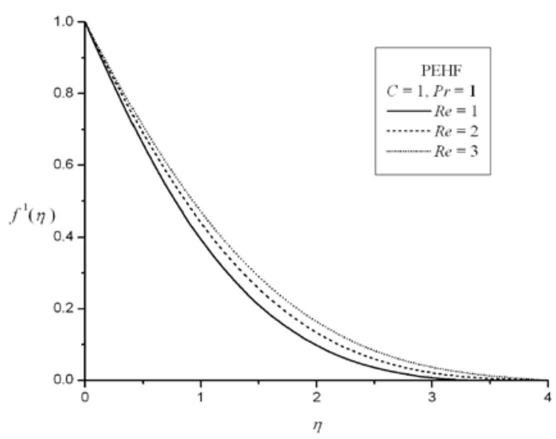


1.4(b)

Fig. 1.4: Plot of $f(\eta)$ versus η for values of Re.

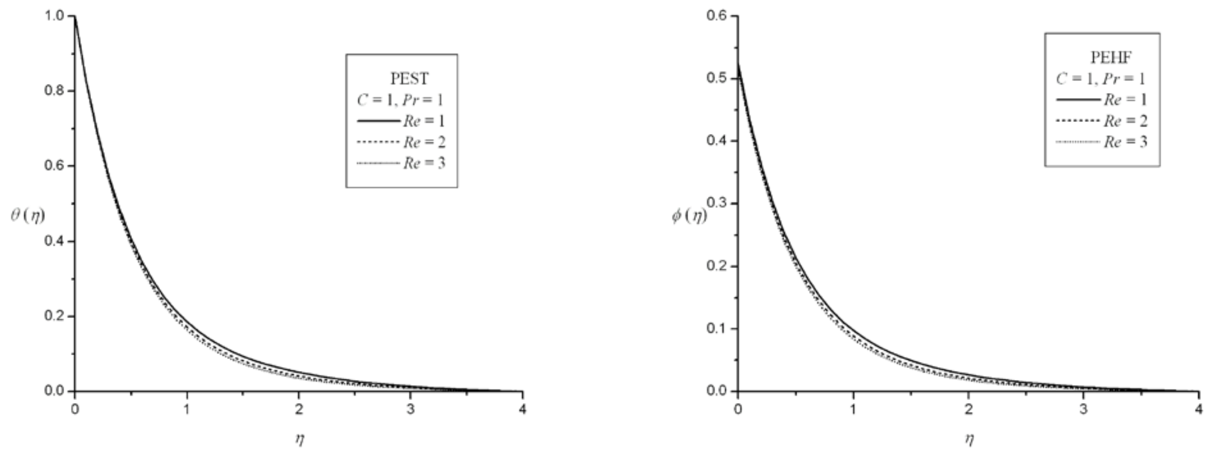


1.5(a)



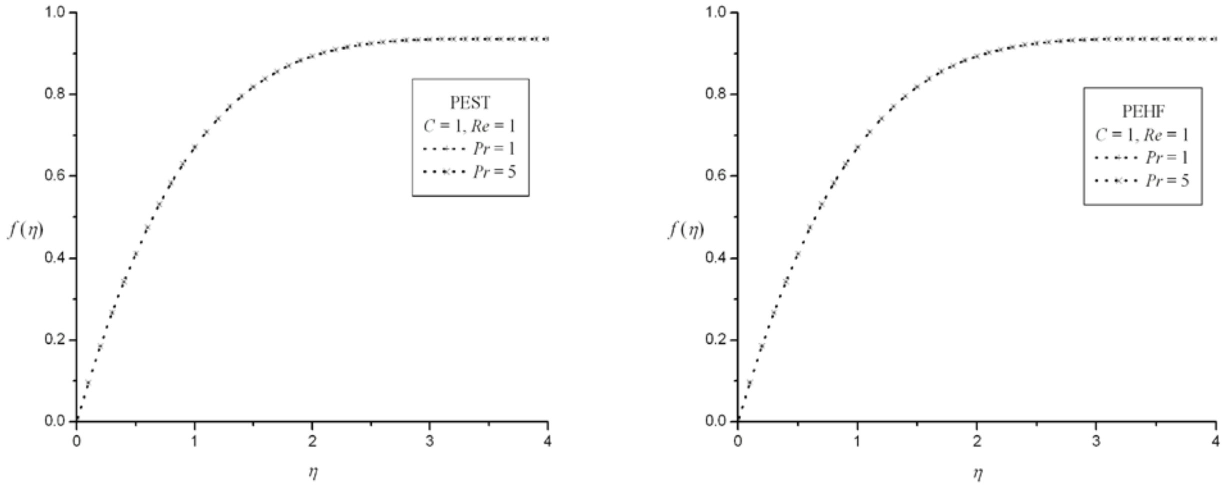
1.5(b)

Fig. 1.5: Plot of $f'(\eta)$ versus η for values of Re.



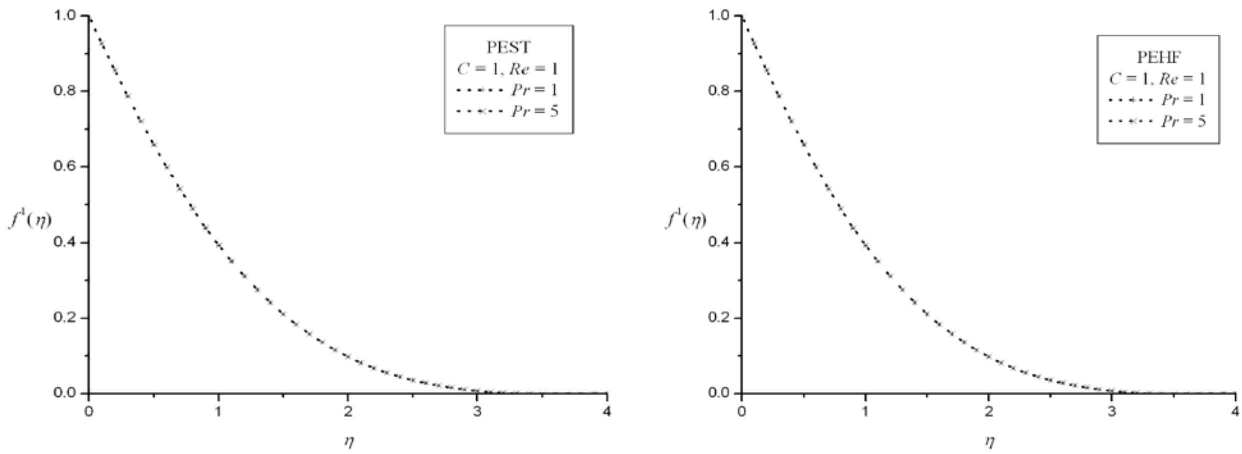
1.6(a) **1.6(b)**

Fig. 1.6: Plot of temperature profiles for different values of *Re*,



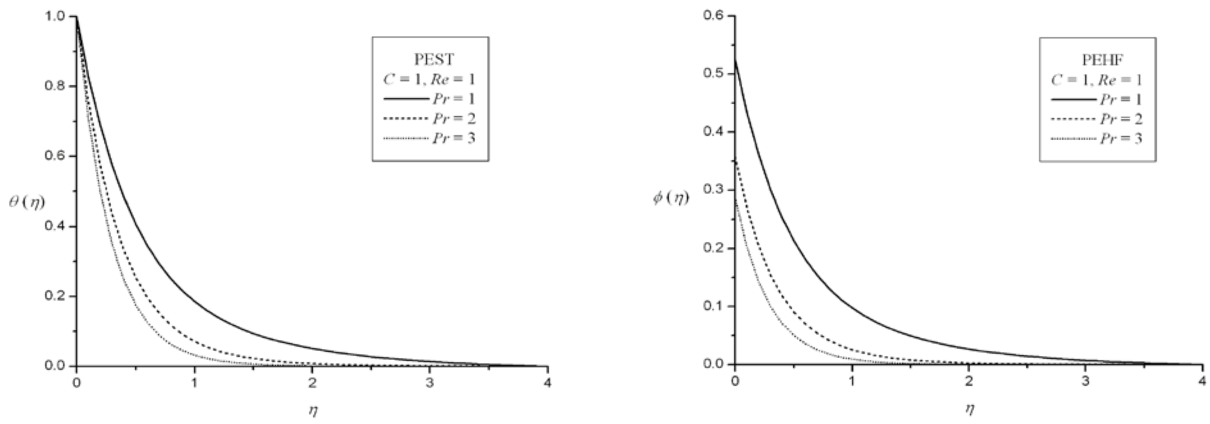
1.7 (a) **1.7 (b)**

Fig. 1.7: Plot of $f(\eta)$ versus η for values of *Pr*.



1.8 (a) **1.8 (b)**

Fig. 1.8: Plot of $f^{1}(\eta)$ versus η for values of *Pr*.



1.9(a) **1.9(b)**

Fig. 1.9: Plot of temperature profiles for different *Pr*.

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