DISPERSION OF A SOLUTE IN MAGNETOHYDRODYNAMIC TWO FLUID FLOW WITH HOMOGENEOUS AND HETEROGENEOUS CHEMICAL REACTIONS.

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ABSTRACT

The effect of homogeneous and heterogeneous reactions on the dispersion of a solute in a magnetohydrodynamic two fluid flow between two parallel plates is studied. The fluids in both the regions of the channel are incompressible and transport properties are assumed to be constant. The results are tabulated for various values of viscosity ratio, pressure gradient and Hartmann number on the effective dispersion coefficient and volumetric flow rate. The effective dispersion coefficient decreases with increase in the Hartmann number in the absence of chemical reactions. It is found that for first order homogeneous and heterogeneous chemical reaction, the effective dispersion coefficient decreases as the reaction rate parameter and Hartmann number increases. The validity of the results obtained for magnetohydrodyanmic two fluid model is compared with the available one fluid model for clear viscous fluid and good agreement is found.

Key Words: Taylor dispersion, immiscible fluids, horizontal channel, MHD, homogeneous and heterogeneous chemical reaction.

1. INTRODUCTION

Taylor [22-24] initiated the investigation of the dispersion of a soluble matter in a non-conducting viscous fluid flowing through a circular tube under laminar conditions. His results show that the soluble matter can be regarded as dispersing along the tube with an apparent diffusion coefficient $R^2 \bar{V} x^2 / 48 D$, where R is the radius of the circular tube and D is the molecular diffusion coefficient. He has also shown that the condition under which his analysis is valid is $4L/R \gg \bar{V} x R / D \gg 6.4$, L being the length in the flow direction. Aris [1] extended Taylor's results and established that the rate of growth of variance of the solute distribution is proportional to the sum of the molecular diffusion coefficient and Taylor diffusion coefficient. His analysis removed the restrictions imposed by Taylor. Subsequently Taylor's analysis was extended to the case of different models of non-Newtonian fluids by Fan and Hwang [5], Fan and Wang [6], Ghoshal [7], and Ghoshal et al. [8]. Gupta and Chatterjee [9] investigated the dispersion of soluble matter in the hydromagnetic laminar flow between two parallel plates. All the investigations mentioned above deal with flows where the solute does not chemically react with the liquid through which it dispersed.

Chemical reactions can be classified as either homogenous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A reaction is said to be the first order if the rate of reaction is directly proportional to the concentration itself. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid does occur.

However, the diffusion process, combined with homogeneous and heterogeneous chemical reactions of a solute is important in hydrolysis of ester, gas absorption in an agitated tank with chemical reaction and so on (Bird et al. [2]). Cleland and Wilhelm [3] discussed the problem of a finite first-order homogeneous reaction in a pipe under laminar flow conditions by a finite difference method. He supported his results with experimental data. Katz [11] investigated the effect of homogeneous reaction at the wall on the concentration profiles. While combined first order heterogeneous and homogeneous reactions were studied by Walker [30] and Solomon and Hudson [20].

All the investigations on dispersion of a solute, however, carried out to chemical reactions under steady state conditions until when Gupta and Gupta [10] discussed for the unsteady case. They have analyzed the unsteady dispersion of solute with simultaneous chemical reaction in a liquid flowing in a channel by adopting Taylor's method. Dutta et al. [4] also

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discussed the non-Newtonian fluid with simultaneous chemical reaction. The influence of an applied magnetic field on the dispersion has been discussed by Narasimha Murthy and Krishna Murthy [15]. They found that the solute in an electrically conducting solvent can regulate rate of diffusion. In 1975, Narasimha Murthy and Ali [16] analyzed the problem including the effects of homogenous and heterogeneous chemical reactions of a solute in a porous medium.

All the above studies were analyzed for one fluid model. Most of the problems relating to petroleum industry, geophysics, plasma physics, magneto-fluid dynamics etc. involve multi-fluid flow situations. There have been studies on hydrodynamic aspects of the two-phase reported in the literature. The first investigations were associated with the LM-MFM generator project at the Argon National Laboratory. Packham and Shail [17] analyzed a stratified laminar flow of two immiscible liquids in a horizontal pipe. Shail [19] found that an increase of order 30% could be achieved in the flow rate with a layer of conducting fluid in the lower layer and a layer of non-conducting in the upper channel wall. Umavathi et al. [25-29] analyzed steady and unsteady flow and heat transfer of two immiscible fluids in a horizontal channel. Malashetty and Umavathi [12] and Malashetty et al. [13-14] studied the free convective flow and heat transfer of conducting two fluid flows in a vertical and inclined channels. Recently Prathap Kumar et al. [18] analyzed mixed convection of immiscible fluids in a vertical channel.

In the present paper we have studied the dispersion of a solute in the laminar flow of conducting and non-conducting immiscible liquids between two parallel plates in the absence and in the presence of first order chemical reactions.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The physical configuration considered in this study is shown in figure 1. Consider the laminar flow of two immiscible fluids between two parallel plates distant $2h$ apart, taking X-axis along the mid-section of the channel and Y-axis perpendicular to the walls. Region-1 $(-h \leq Y \leq 0)$ is filled with the conducting fluid with conductivity σ_e, density ρ_1, viscosity μ_1, under a uniform pressure gradient $\dfrac{dP_1}{dX}$ whereas region-2 $(0 \leq Y \leq h)$ is filled with non-conducting fluid of density ρ_2, viscosity μ_2, under a uniform pressure gradient $\dfrac{dP_2}{dX}$. The fluids in both the regions are Newtonian fluids.
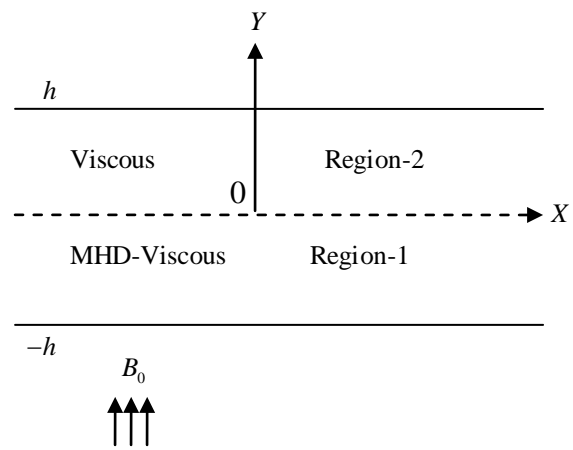

Fig. (1): Physical configuration and coordinate system

It is assumed that the flow is steady, laminar, fully developed, and that fluid properties are constant. The flow in both regions is assumed to be driven by a common constant pressure gradient. Under these assumptions, the governing equations of motion for incompressible fluids are

Region-1

$$\mu_1 \frac{d^2 U_1}{dY^2} - \frac{dP_1}{dX} - \sigma_e B_0^2 U_1 = 0 \qquad (1)$$

Region-2

$$\mu_2 \frac{d^2 U_2}{dY^2} - \frac{dP_2}{dX} = 0 \tag{2}$$

where U_i is the X-component of fluid velocity and P_i is the pressure. The subscripts 1 and 2 denote the values for region-1 and region-2 respectively.

The boundary conditions on velocity are no-slip conditions requiring that the velocity must vanish at the walls. In addition, continuity of velocity and shear stress at the interface is assumed. With these assumptions, the boundary and interface conditions on velocity become

$$U_1 = 0 \quad at \quad Y = -h$$

$$U_2 = 0 \quad at \quad Y = h \tag{3}$$

$$U_1 = U_2 \quad at \quad Y = 0$$

$$\mu_1 \frac{dU_1}{dY} = \mu_2 \frac{dU_2}{dY} \quad at \quad Y = 0$$

Using the non-dimensional parameters,

$$\eta = \frac{Y}{h}, \ u_1 = \frac{\rho_1 h}{\mu_1} U_1, \ u_2 = \frac{\rho_2 h}{\mu_2} U_2, \ x = \frac{X}{h}, \ p_1^* = \frac{P_1}{\rho_1 (v_1 / h)^2}, \ p_2^* = \frac{P_2}{\rho_2 (v_2 / h)^2}, \ M = B_0 h \sqrt{\frac{\sigma_e}{\mu_1}}. \tag{4}$$

the equations (1) to (3) become

Region-1

$$\frac{d^2 u_1}{d\eta^2} - \frac{dp_1^*}{dx} - M^2 u_1 = 0 \tag{5}$$

Region-2

$$\frac{d^2 u_2}{d\eta^2} - \frac{dp_2^*}{dx} = 0 \tag{6}$$

$$u_1 = 0 \quad at \quad \eta = -1$$

$$u_2 = 0 \quad at \quad \eta = 1 \tag{7}$$

$$u_1 = mn u_2; \qquad \frac{du_1}{d\eta} = m^2 n \frac{du_2}{d\eta} \quad at \quad \eta = 0$$

3. SOLUTIONS

Solutions of equations (5) and (6) using boundary and interface conditions (7) become

$$u_1 = a_1 \cosh(M\eta) + a_2 \sinh(M\eta) - \frac{p_1}{M^2} \tag{8}$$

$$u_2 = \frac{p_2}{2} \eta^2 + a_3 \eta + a_4 \tag{9}$$

From equations (8) and (9) the average velocities become

$$\overline{u}_1 = \frac{1}{2}\int_{-1}^{0} u_1 \, d\eta \tag{10}$$

$$\overline{u}_2 = \frac{1}{2}\int_{0}^{1} u_2 \, d\eta \tag{11}$$

Case 1a: Diffusion of a tracer in the absence of homogeneous first-order chemical reaction.

The equation for the concentration C_1 of the solute for the region-1 satisfies

$$\frac{\partial C_1}{\partial t} + u_1 \frac{\partial C_1}{\partial X} = D_1 \left(\frac{\partial^2 C_1}{\partial X^2} + \frac{\partial^2 C_1}{\partial Y^2} \right) \tag{12}$$

Similarly, the equation for the concentration C_2 of the solute for the region-2 satisfies

$$\frac{\partial C_2}{\partial t} + u_2 \frac{\partial C_2}{\partial X} = D_2 \left(\frac{\partial^2 C_2}{\partial X^2} + \frac{\partial^2 C_2}{\partial Y^2} \right) \tag{13}$$

in which D_1 and D_2 are the molecular diffusion coefficients (assumed constants) for the region-1 and region-2 respectively .

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane the fluid velocities are given by

Region-1

$$u_{1x} = u_1 - \overline{u} = a_1 \cosh(M\eta) + a_2 \sinh(M\eta) + l_1 \tag{14}$$

Region-2

$$u_{2x} = u_2 - \overline{u} = \frac{p_2 \eta^2}{2} + a_3 \eta + l_2 \tag{15}$$

where $\overline{u}$ is the sum of average velocities of region-1 and region-2. Introducing the dimensionless quantities

$$\theta_1 = \frac{t_1}{\overline{t}_1}, \ \overline{t}_1 = \frac{L_1}{\overline{u}_1}, \ \xi_1 = \frac{x_1 - \overline{u}_1 t}{L}, \ \theta_2 = \frac{t_2}{\overline{t}_2}, \ \overline{t}_2 = \frac{L_2}{\overline{u}_2}, \ \xi_2 = \frac{x_2 - \overline{u}_2 t}{L} \tag{16}$$

and using equations (14) and (15), the equations (12) and (13) become (using the assumption that

$$\frac{\partial^2 C_1}{\partial X^2} \ll \frac{\partial^2 C_1}{\partial Y^2} \quad \text{and} \quad \frac{\partial^2 C_2}{\partial X^2} \ll \frac{\partial^2 C_2}{\partial Y^2} \bigg)$$

Region-1

$$\frac{1}{t}\frac{\partial C_1}{\partial \theta_1} + \frac{u_{1x}}{L}\frac{\partial C_1}{\partial \xi_1} = \frac{D_1}{h^2}\frac{\partial^2 C_1}{\partial \eta^2} \tag{17}$$

Region-2

$$\frac{1}{t}\frac{\partial C_2}{\partial \theta_2} + \frac{u_{2x}}{L}\frac{\partial C_2}{\partial \xi_2} = \frac{D_2}{h^2}\frac{\partial^2 C_2}{\partial \eta^2} \tag{18}$$

where L is the typical length along the flow direction. Following Taylor [22], we now assume that partial equilibrium is established in any cross-section of the channel so that the variations of C_1 and C_2 with η are calculated from equations (17) and (18) as

Region-1

$$\frac{\partial^2 C_1}{\partial \eta^2} = \frac{h^2}{D_1 L} u_{1x} \frac{\partial C_1}{\partial \xi_1} \tag{19}$$

Region-2

$$\frac{\partial^2 C_2}{\partial \eta^2} = \frac{h^2}{D_2 L} u_{2x} \frac{\partial C_2}{\partial \xi_2} \tag{20}$$

To solve these equations we use the following boundary and interface conditions,

$$\frac{\partial C_1}{\partial \eta} = 0 \quad \text{at} \quad \eta = -1 \quad \text{and} \quad \frac{\partial C_2}{\partial \eta} = 0 \quad \text{at} \quad \eta = 1 \tag{21}$$

Equations (19) and (20) are solved for C_1 and C_2 which are given by

Region-1

$$C_1 = Z_1 \left(\frac{a_1}{M^2} \cosh(M\eta) + \frac{a_2}{M^2} \sinh(M\eta) + \frac{l_1}{2}\eta^2 \right) + b_1 \eta + b_2 \tag{22}$$

Region-2

$$C_2 = Z_2 \left(\frac{P_2 \eta^4}{24} + \frac{a_3 \eta^3}{6} + \frac{l_2}{2}\eta^2 \right) + b_3 \eta + b_4 \tag{23}$$

where b_2 and b_4 being constants to be determined using the entry conditions.

The volumetric flow rates at which the solute is transported across a section of the channel of unit breadth Q_1 (region-1) and Q_2 (region-2) using equations (14), (15) and (22), (23), respectively are given by

$$Q_1 = h \int_{-1}^{0} C_1 u_{1x} \, d\eta = -Z_1 h \int_{-1}^{0} C_{11} u_{1x} \, d\eta \tag{24}$$

$$Q_2 = h \int_{0}^{1} C_2 u_{2x} \, d\eta = -Z_2 h \int_{0}^{1} C_{22} u_{2x} \, d\eta \tag{25}$$

where $C_{11} = -\dfrac{a_1 \cosh(M\eta)}{M^2} - \dfrac{a_2 \sinh(M\eta)}{M^2} + \dfrac{l_1 \eta^2}{2} - b_1 \eta$, $C_{22} = -\dfrac{P_2 \eta^4}{24} - \dfrac{a_3 \eta^3}{6} - \dfrac{l_2 \eta^2}{2} - b_3 \eta$.

Following Taylor [22], we assume that the variations of C_1 and C_2 with η are small compared with those in the longitudinal direction, and if C_{m1} and C_{m2} are the mean concentration over a section, $\partial C_1 / \partial \xi_1$ and $\partial C_2 / \partial \xi_2$ are indistinguishable from $\partial C_{m1} / \partial \xi_1$ and $\partial C_{m2} / \partial \xi_2$ respectively so that equations (24) and (25) may be written as

Region-1

$$Q_1 = -D_1^* \frac{\partial C_{m1}}{\partial \xi_1} \tag{26}$$

Region-2

$$Q_2 = -D_2^* \frac{\partial C_{m2}}{\partial \xi_2}$$ (27)

The fact that no material is lost in the process is expressed by the continuity equation for C_{m1} and C_{m2}, namely

Region-1

$$\frac{\partial Q_1}{\partial \xi_1} = -2 \frac{\partial C_{m1}}{\partial t}$$ (28)

Region-2

$$\frac{\partial Q_2}{\partial \xi_2} = -2 \frac{\partial C_{m2}}{\partial t}$$ (29)

Equations (28) and (29) using equations (24) and (25) become

Region-1

$$\frac{\partial C_{m1}}{\partial t} = \frac{D_1^*}{2} \frac{\partial^2 C_{m1}}{\partial \xi_1^2}$$ (30)

Region-2

$$\frac{\partial C_{m2}}{\partial t} = \frac{D_2^*}{2} \frac{\partial^2 C_{m2}}{\partial \xi_2^2}$$ (31)

which are the equations governing the longitudinal dispersion, where

$$D_1^* = \frac{h^2}{2D_1} \int_{-1}^{0} C_{11} u_{1x} \, d\eta = \frac{h^2}{2D_1} F_1(M, p_1, p_2, m, n); \quad D_2^* = \frac{h^2}{2D_2} \int_{-1}^{0} C_{12} u_{1x} \, d\eta = \frac{h^2}{2D_2} F_2(M, p_1, p_2, m, n).$$

Equations (30) and (31) are the well known heat equations which can be solved easily for a given initial conditions.

Case 1b: Diffusion of a tracer in the absence of first order chemical reaction for purely viscous fluid (two fluid model)

To validate the results of the present model, the problem is solved in the absence of magnetic field and compared with the results of Gupta and Gupta [10].

The non-dimensional equations of motion for incompressible, viscous fluids are

Region-1

$$\frac{d^2 u_1}{dy^2} - \frac{dp_1^*}{dx} = 0$$ (32)

Region-2

$$\frac{d^2 u_2}{dy^2} - \frac{dp_2^*}{dx} = 0$$ (33)

The boundary and interface conditions are defined as in equation (7). Using equation (7) in equations (32) and (33), the solutions become

$$u_1 = \frac{p_1\eta^2}{2} + a_1\eta + a_2 \tag{34}$$

$$u_2 = \frac{p_2\eta^2}{2} + a_3\eta + a_4 \tag{35}$$

The average velocities become

$$\bar{u}_1 = \frac{1}{2}\left(\frac{p_1}{6} - \frac{a_1}{2} + a_2 \right) \tag{36}$$

$$\bar{u}_2 = \frac{1}{2}\left(\frac{p_1}{6} + \frac{a_3}{2} + a_4 \right) \tag{37}$$

The solutions of equations (19) and (20) in the absence of magnetic field B_0 yields

$$C_1 = \frac{h^2}{D_1 L}\frac{\partial C_1}{\partial \xi_1}\left(\frac{p_1\eta^4}{24} + \frac{a_1\eta^3}{6} + \frac{lc_1\eta^2}{2} + b_1\eta \right) + C_{01} \tag{38}$$

$$C_2 = \frac{h^2}{D_2 L}\frac{\partial C_2}{\partial \xi_2}\left(\frac{p_2\eta^4}{24} + \frac{a_3\eta^3}{6} + \frac{lc_2\eta^2}{2} + b_3\eta \right) + C_{02} \tag{39}$$

where C_{01} and C_{02} being constants to be determined using entry conditions.

The volumetric rates at which the solute is transported across a section of the channel of unit breadth Q_1 (region-1) and Q_2 (region-2) and the evaluation of effective dispersion coefficients F_{ii} are evaluated as explained in the case 1a. The values of $F_i\left(p_1, p_2, m, n \right)$ are computed for different values of the dimensionless parameters p_i and m and are shown in Table-2.

Case 1c: Diffusion of a tracer in the absence of first order chemical reaction for purely viscous fluid (one fluid model)

The non-dimensional equation of motion is

$$\frac{d^2u}{d\eta^2} = \frac{dp}{dx} \tag{40}$$

along with boundary conditions

$$u = 0 \quad at \quad \eta = \pm 1 \tag{41}$$

The solution of equation (40) is

$$u = -p(1-\eta^2)/2$$

The average velocity is given by

$$\bar{u} = -p/3$$

The concentration equation for one fluid model using Taylor [22] become

$$\frac{\partial^2 C}{\partial \eta^2} = \frac{h^2}{DL}\frac{\partial C}{\partial \xi}u_x \tag{42}$$

where $u_x = \frac{p\eta^2}{2} - \frac{p}{6}$

The solution of equation (42) using boundary conditions $\dfrac{\partial C}{\partial \eta} = 0$ at $\eta = \pm 1$ is

$$C = \frac{h^2}{DL}\frac{\partial C}{\partial \xi}\left(\frac{p}{24}\eta^4 - \frac{p}{12}\eta^2\right) + C_0 \tag{43}$$

where C_0 being a constant to be determined using entry conditions.

The volumetric flow rate in which the solute is transported across a section of the channel of unit breadth is

$$Q = h\int_{-1}^{1} C u_x d\eta = -\frac{h^2 p^2}{D}\frac{\partial C}{\partial \xi}\left(\frac{2}{945}\right) \tag{44}$$

so that the value for D^* can be written as $\dfrac{h^2 p^2}{D}\dfrac{2}{945}$ by comparing with Fick's law of diffusion which agrees with the results of Wooding [31] where p is the non-dimensional pressure gradient. D^* is also the effective dispersion co-efficient obtained by Gupta and Gupta [10] in the absence of chemical reactions.

Case 2a: Diffusion of a tracer in the presence of homogeneous first-order chemical reaction.

The physical model and the assumptions made in case1 are true here, except that we have the chemical reaction. In this case we assume that the chemical reaction is first order and it occurs under such conditions that the gas film resistance is negligible. This means that the reaction term is $-KC_1$ (region-1- mol cm^{-3} s^{-1}) and $-KC_2$ (region-2- mol cm^{-3} s^{-1}), which represents the volume rate of disappearance of the solute due to chemical reaction. Here K represents the first-order reaction rate constant.

The velocity and average velocity are exactly the same as in equations (8)-(11). The equations for concentration, instead of equations (12) and (13), are

Region-1

$$\frac{\partial C_1}{\partial t} + u_1\frac{\partial C_1}{\partial X} = D_1\left(\frac{\partial^2 C_1}{\partial X^2} + \frac{\partial^2 C_1}{\partial Y^2}\right) - K_1 C_1 \tag{45}$$

Region-2

$$\frac{\partial C_2}{\partial t} + u_2\frac{\partial C_2}{\partial X} = D_2\left(\frac{\partial^2 C_2}{\partial X^2} + \frac{\partial^2 C_2}{\partial Y^2}\right) - K_2 C_2 \tag{46}$$

Along with boundary conditions (21), the continuity of concentration and continuity of mass flux at the interface is considered to evaluate the integrating constants i.e.,

$$C_1 = C_2 \text{ and } \frac{\partial C_1}{\partial \eta} = \frac{D_2}{D_1}\frac{\partial C_2}{\partial \eta} \quad \text{at} \quad \eta = 0 \tag{47}$$

Following the analysis of case 1, the non-dimensional form of equations (45) and (46) are

Region-1

$$\frac{\partial^2 C_1}{\partial \eta^2} - \alpha_1{}^2 C_1 = \frac{h^2}{D_1 L}u_{1x}\frac{\partial C_1}{\partial \xi_1} \tag{48}$$

Region-2

$$\frac{\partial^2 C_2}{\partial \eta^2} - \alpha_2{}^2 C_2 = \frac{h^2}{D_2 L}u_{2x}\frac{\partial C_2}{\partial \xi_2} \tag{49}$$

where $\alpha_1 = h\sqrt{K_1/D_1}$ and $\alpha_2 = h\sqrt{K_2/D_2}$.

The solutions of equations (48) and (49) using boundary and interface conditions as in equations (21) and (47) become

Region-1

$$C_1 = b_1 \cosh(\alpha_1\eta) + b_2 \sinh(\alpha_1\eta) + Z_1\left(\frac{a_1}{M^2 - \alpha_1^2}\cosh(M\eta) + \frac{a_2}{M^2 - \alpha_1^2}\sinh(M\eta) + \frac{l_1}{\alpha_1^2}\eta^2\right) \qquad (50)$$

Region-2

$$C_2 = b_3 \cosh(\alpha_2\eta) + b_4 \sinh(\alpha_2\eta) + Z_2\left(l_3\eta^2 + l_4\eta + l_5\right) \qquad (51)$$

The expressions for C_1 and C_2 can also written as

$$C_1 = \frac{h^2}{D_1 L}\frac{\partial C_1}{\partial \xi_1}C_{11} + \frac{h^2}{D_2 L}\frac{\partial C_2}{\partial \xi_2}C_{12}$$

$$C_2 = \frac{h^2}{D_1 L}\frac{\partial C_1}{\partial \xi_1}C_{21} + \frac{h^2}{D_2 L}\frac{\partial C_2}{\partial \xi_2}C_{22}$$

The volumetric flow rates at which the solute is transported across a section of the channel of unit breadth Q_1 (region-1) and Q_2 (region-2) using equations (14), (15) and (50), (51), respectively are given by

Region-1

$$Q_1 = h\int_{-1}^{0} C_1 u_{1x}\,d\eta = -\left(Q_{11} + Q_{12}\right) \qquad (52)$$

Region-2

$$Q_2 = h\int_{0}^{1} C_2 u_{2x}\,d\eta = -\left(Q_{21} + Q_{22}\right) \qquad (53)$$

where $Q_{11} = -Z_1 h\int_{-1}^{0} C_{11} u_{1x}\,d\eta$, $Q_{12} = -Z_2 h\int_{-1}^{0} C_{12} u_{1x}\,d\eta$, $Q_{21} = -Z_1 h\int_{0}^{1} C_{21} u_{2x}\,d\eta$, $Q_{22} = -Z_2 h\int_{0}^{1} C_{22} u_{2x}\,d\eta$.

Following the procedure explained in case 1 and using the fact that no material is lost in the process expressed by the continuity equation for C_1 and C_2, given by equations (52) and (53), we obtain an effective dispersion coefficient D^* in the form

$$D_{11}^* = \frac{h^2}{2D_1}\int_{-1}^{0} C_{11} u_{1x}\,d\eta = \frac{h^2}{2D_1}F_{11}\left(M, p_1, p_2, \alpha_1, \alpha_2, m, n\right),$$

$$D_{12}^* = \frac{h^2}{2D_2}\int_{-1}^{0} C_{12} u_{1x}\,d\eta = \frac{h^2}{2D_2}F_{12}\left(M, p_1, p_2, \alpha_1, \alpha_2, m, n\right),$$

$$D_{21}^* = \frac{h^2}{2D_1}\int_{-1}^{0} C_{21} u_{2x}\,d\eta = \frac{h^2}{2D_1}F_{21}\left(M, p_1, p_2, \alpha_1, \alpha_2, m, n\right),$$

$$D_{22}^* = \frac{h^2}{2D_2}\int_{-1}^{0} C_{22} u_{2x}\,d\eta = \frac{h^2}{2D_2}F_{22}\left(M, p_1, p_2, \alpha_1, \alpha_2, m, n\right). \qquad (54)$$

Values of F_{ii} are computed for different values of dimensionless parameters such as Hartman number M, viscosity ratio m and pressure gradients p_1, p_2 for variations of α_1 and α_2. Volumetric flow rate is also computed for variations of Hartman number, viscosity ratio, pressure gradients and height of the channel.

Case 2b: Diffusion of a tracer with combined homogeneous and heterogeneous first-order chemical reaction.

We now discuss the problem of diffusion in a channel with a first-order chemical reaction taking place both in the bulk of the fluid as well as at the walls which are assumed to be catalytic. In this case the diffusion equations remain the same as defined in equations (48) and (49) subject to the dimensionless boundary and interface conditions as

$$\frac{\partial C_1}{\partial \eta} - \beta_1 C_1 = 0 \quad at \quad \eta = -1$$

$$\frac{\partial C_2}{\partial \eta} + \beta_2 C_2 = 0 \quad at \quad \eta = 1 \tag{55}$$

$$C_1 = C_2 \qquad at \quad \eta = 0$$

$$D_1 \frac{\partial C_1}{\partial \eta} = D_2 \frac{\partial C_2}{\partial \eta} \quad at \; \eta = 0$$

where $\beta_1 = f_1 h$ and $\beta_2 = f_2 h$ are the heterogeneous reaction rate parameters corresponding to catalytic reaction at the walls.

The solutions of equations (48) and (49) are same as in equations (50) and (51). The integrating constants b_1, b_2, b_3 and b_4 are obtained using boundary and interface conditions as defined in equation (55) and given as follows

$$b_1 = Z_1 b_{11} + Z_2 b_{12} \,,\; b_2 = Z_1 b_{21} + Z_2 b_{22} \,,\; b_3 = Z_1 b_{31} + Z_2 b_{32} \,,\; b_4 = Z_1 b_{41} + Z_2 b_{42}\,.$$

The procedure of evaluating the volumetric flow rate and effective dispersion coefficient is same as in equations (52) to (55).

Case 2c: Diffusion of a tracer in the presence of homogeneous first-order chemical reaction in the absence of magnetic field for purely viscous fluid (two fluid model).

We justify our results by comparing with the results obtained by Gupta and Gupta [10] (one fluid model) with first order chemical reaction for purely viscous fluid.

The solutions of velocities and average velocities are same as in equations (34) to (37).

The solutions of equations (48) and (49) for purely viscous fluid yields

$$C_1 = b_1 \cosh(\alpha_1 \eta) + b_2 \sinh(\alpha_1 \eta) + Z_1 \left(l_1 \eta^2 + l_2 \eta + l_3 \right) \tag{56}$$

$$C_2 = b_3 \cosh(\alpha_2 \eta) + b_4 \sinh(\alpha_2 \eta) + Z_2 \left(l_4 \eta^2 + l_5 \eta + l_6 \right) \tag{57}$$

The volumetric rates at which the solute is transported across a section of the channel of unit breadth Q_1 (region-1) and Q_2 (region-2) and the evaluation of effective dispersion coefficients F_{ii} are evaluated as explained in the case1a. The values of $F_{ii}(\alpha_1, \alpha_2, p_1, p_2, m, n)$ are computed for different values of the dimensionless reaction rate parameters α_i, p_i and m and are shown in Table-2.

Case 2d: The channel filled with only viscous fluid (one fluid model) for homogeneous chemical reaction.

The solutions of velocities and average velocities are given in case 1c.
The concentration equation for one fluid model using Taylor [22] become

$$\frac{\partial^2 C}{\partial \eta^2} - \alpha^2 C = \frac{h^2}{DL} \frac{\partial C}{\partial \xi} u_x \tag{58}$$

where $u_x = \dfrac{p \eta^2}{2} - \dfrac{p}{6}$

The solution of equation (58) using boundary conditions $\dfrac{\partial C}{\partial \eta} = 0$ at $\eta = \pm 1$ is

$$C = A \cosh(\alpha \eta) - \frac{h^2}{\alpha^2 DL} \frac{\partial C}{\partial \xi} \left(\frac{p}{2} \eta^2 - \frac{p}{6} + \frac{p}{\alpha^2} \right) \qquad (59)$$

The volumetric flow rate in which the solute is transported across a section of the channel of unit breadth is

$$Q = h \int_{-1}^{1} C u_x d\eta = \frac{h^2 p^2}{\alpha^2 D} \frac{\partial C}{\partial \xi} \left(\frac{1}{\alpha^4} + \frac{1}{3\alpha^2} - \frac{\coth(\alpha)}{\alpha} - \frac{1}{45} \right) \qquad (60)$$

Comparing equation (60) with Fick's law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* given by

$$D^* = \frac{h^2 p^2}{D} F(\alpha)$$

where $F(\alpha) = \dfrac{1}{\alpha^2} \left(\dfrac{\coth(\alpha)}{\alpha} - \dfrac{1}{\alpha^4} - \dfrac{1}{3\alpha^2} + \dfrac{1}{45} \right) \qquad (61)$

Values of $F(\alpha)$ are computed for different values of the dimensionless reaction rate parameter α and are shown in Table 2. When $\alpha \to 0$, Eqn. (61) gives

$$\lim_{\alpha \to 0} F(\alpha) = \frac{2}{945}$$

so that the value for D^* can be written as $\dfrac{h^2 p^2}{D} \dfrac{2}{945}$ which agrees with the results of Wooding [31] where p is the non-dimensional pressure gradient.

The solution for heterogeneous chemical reaction is also found for two fluid and one fluid model for purely viscous fluid and the results are shown in Table-2.

The constants appeared in all the above equations are given in the Appendix.

4. RESULTS AND DISCUSSION

The problem under the study is the longitudinal dispersion of a solute subject to molecular diffusion when it is introduced into a channel filled with conducting and non-conducting immiscible fluid. The dispersion of a solute is analyzed in the absence and in the presence of an irreversible first-order chemical reaction following Taylor diffusion model.

The average velocities in both the regions are evaluated using no-slip conditions at the boundaries and continuity of velocity and shear stress at the interface. The volumetric flow rate and effective Taylor dispersion coefficient in each region is evaluated for various governing parameters.

Case 1: Diffusion of a tracer in the absence of homogeneous first-order chemical reaction:

The effect of Hartman number M on the velocity is shown in figure 2. We observe that an increase in the value of the Hartman number M decreases the velocity. This is the classical Hartman effect. The fluid in region-1 is being conducting experiences the effect of magnetic field more. The fluid in region-2 is electrically non-conducting, and the flow in this region is affected by the applied fields through the coupling.

Table-1 illustrate the effects of Hartman number M, viscosity ratio m and pressure gradient $p \left(= p_1 = p_2 \right)$ on the effective Taylor dispersion coefficient (ETDC) in each region. It is evident from Table-1 that the ETDC decreases with increase in the Hartman number M. The results are compatible with the physics of the problem. In Hartman flow, it is seen that the velocity profile becomes flatter (Fig. 2) with the increase of Hartman number M, when compared to the profiles of non-magnetic case. This means for a fixed pressure gradient and viscosity ratio the flow rate decreases with increase in M.

As the viscosity ratio m increases ETDC decreases for values of $m < 1$ and increases in magnitude for values of $m > 1$. This is due to the fact that viscosity ratio m has significant effect on velocity in region-2 i.e., the region containing non-conducting fluid. As m becomes small, the velocity profiles become flat in region-1 and parabolic in region-2 which causes for reduction in F for values of $m < 1$ and increases F for values of $m > 1$. The values of ETDC (F) is symmetric for pressure gradient $p > 0$ and for $p < 0$. Further as p increases, F increases for values of $p > 0$ and decreases as p increases for values of $p < 0$. This is due to the fact that as p increases for values of $p > 0$, velocity increases which causes an increase in F. $\left(\text{ It should be noted here that } p \text{ is defined as } -\dfrac{dp^*}{dx}\right)$.

The effects of Hartmann number M, viscosity ratio m, pressure gradient p and height of the channel h on the volumetric flow rate Q is shown in figure 3. As the Hartmann number increases, volumetric flow rate decreases for $M = 3$ and remains constant for $M \geq 3$. The viscosity ratio increases the flow rate in magnitude for values of m up to 0.9 (approximately) and remains invariant. Volumetric flow rate is symmetric for negative and positive values of pressure gradient p and the optimal flow rate is attained in the absence of pressure gradient. As the height ratio h increases, flow rate decreases in magnitude.

The results obtained (two-fluid model) in the absence of first order chemical reactions agree with the results obtained by Gupta and Chatterjee [9] for the effect of Hartmann number on effective Taylor dispersion co-efficient. That is, as M increases F decreases. Letting $M \to 0$ and fixing $m = 1, p = 1$ and $h = 1$ (i.e., considering same fluid in both the regions) we obtain Gupta and Gupta [10] results in the absence of first order chemical reactions which are also the results of Wooding [31] as shown in Table-2.

Case 2: Diffusion of a tracer with combined homogeneous and heterogeneous first-order chemical reaction.

The ETDC F_1 (region-1) and F_2 (region-2) for different values of viscosity ratio m, pressure gradient p and Hartmann number M for homogeneous reaction is shown in Table-3. As the reaction rate parameter $\alpha (= \alpha_1 = \alpha_2)$ increases, F_1 and F_2 decreases in both regions for all values of m, p and M. This is due to the fact that, increase in α signifies that increasing number of moles of solute undergoing chemical reaction resulting in a drop in dispersion coefficient.

As the viscosity ratio m increases, the total effective dispersion co-efficient $F (= F_1 = F_2)$ decreases for values of $m \leq 1$ and increases for $m \geq 1$. F decreases as p increases for $p < 1$ and increases as p increases for values of $p > 1$. As the Hartmann number M increases ETDC decreases in both the regions. However the values of F_1 are less when compared to F_2, which again signifies the compatibility with the physics of the problem. In the Hartmann flow, the velocity profiles become flatter, when compared to the profiles of non-magnetic case.

From Table-4, we find the variations of ETDC and the wall catalytic parameter β, on the viscosity ratio m, pressure gradient p and Hartmann number M for fixed value of homogeneous reaction rate parameter $\alpha (= \alpha_1 = \alpha_2)$. As the wall catalytic parameter $\beta (= \beta_1 = \beta_2)$ increases, ETDC decreases for all values of m, p and M (similar results observed for homogeneous chemical reaction). Further the effects of m, p and M on ETDC are the similar results observed for homogeneous chemical reaction.

The effect of viscosity ratio m, pressure gradient p and height of the channel on the volumetric flow rate for both homogeneous and heterogeneous chemical reaction shows the similar result as observed for the diffusion without chemical reaction (caase-1), whereas the Hartmann number increases, volumetric flow rate increases for $M = 4$ and remains constant for $M > 4$ as shown in figure 4.

The effects of Hartmann number on the ETDC, reaction rate parameter and wall catalytic parameter for homogeneous and heterogeneous chemical reactions of the present model (two-fluid) agree with Sudhanshu et al [21] (one fluid model). That is, as the Hartman number M increases, the reaction rate parameter α, wall catalytic parameter β and ETDC decreases. Letting $M \to 0, m = 1$ and $p = 1$ agree with the results of Gupta and Gupta [10] as shown in Table-2.

ACKNOWLEDGEMENT

5. CONCLUSIONS

1. The ETDC decreases with increase in Hartman number with or without chemical reactions.

2. The ETDC decreases for $m \leq 1$ and $p < 1$ whereas it increases for $m > 1$ and $p > 1$ as m and p increases in the presence or in the absence of chemical reactions. The results for two fluid model agree with the results for one fluid models of Gupta and Chatterjee [9], Sudhanshu et al [21] and Gupta and Gupta [10].

3. As the homogeneous reaction rate parameter and wall catalytic parameter increases, the ETDC decreases for all values of viscosity ratio, pressure gradient and Hartman number.

Fig. (2): Velocity profiles for different values of Hartman number M.

Fig. (3): Volumetric flow rate Q versus Hartman number M, viscosity ratio m, pressure gradient p and height of the channel h in the absence of first order chemical reaction.

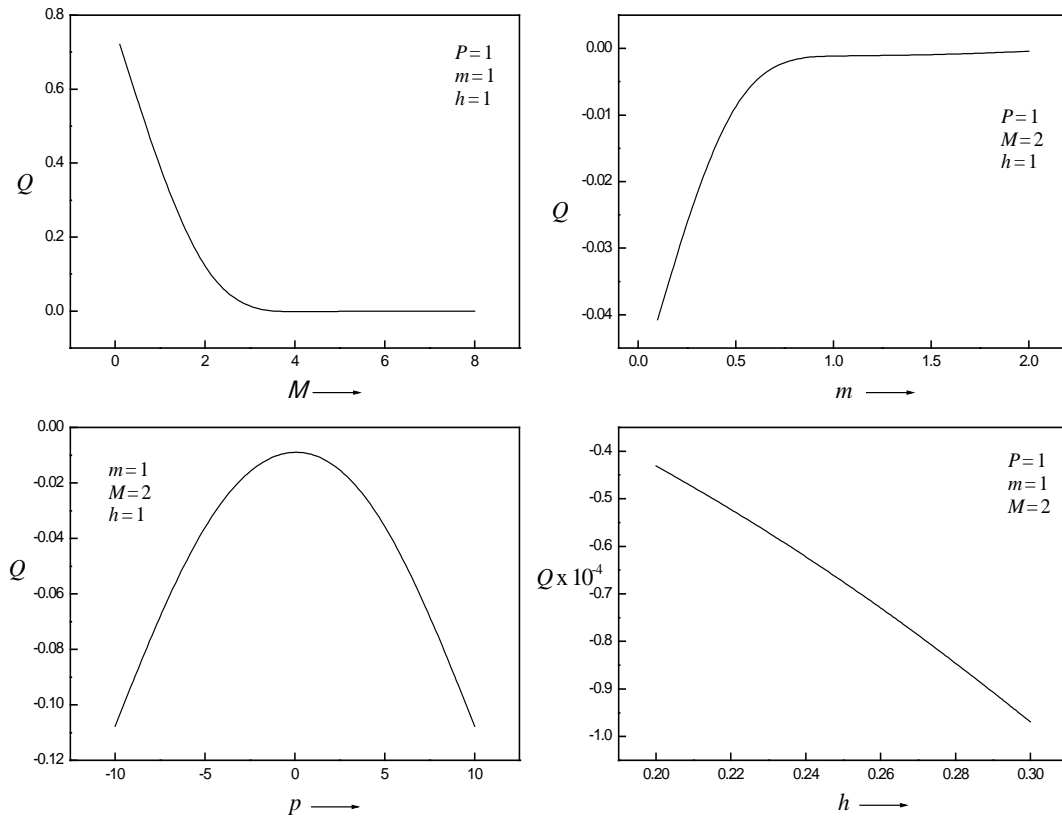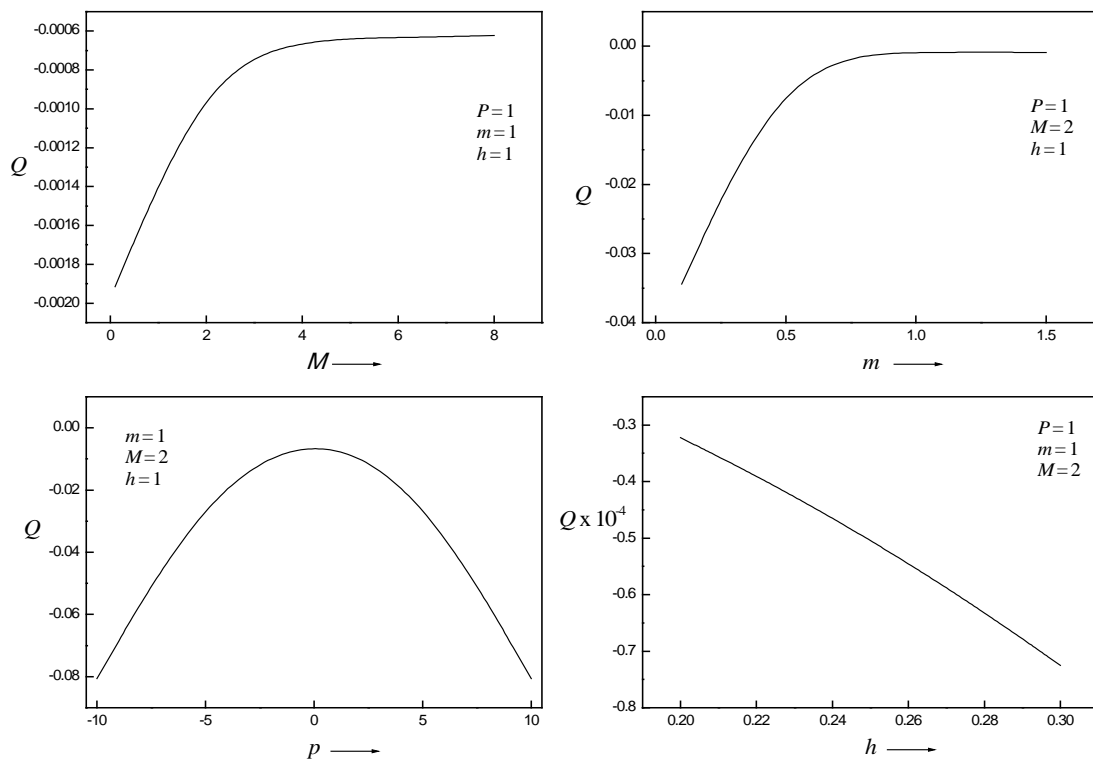**Fig. (4):** Volumetric flow rate Q versus Hartman number M, viscosity ratio m, pressure gradient p and height of the channel h in the presence of first order chemical reaction.

Table-1: Values of effective dispersion coefficient for variations of Hartman number, viscosity ratio, and pressure gradients in the absence of first order chemical reaction.

M	$F_1(M, m, p)$	$F_2(M, m, p)$	$F(M, m, p)$
0.1	-0.722669	0.001051	-0.721618
2	8.8431E-4	1.9282E-4	0.001077
4	7.3578E-4	2.2678E-4	9.6255E-4
6	5.8713E-4	3.1839E-4	9.0552E-4
8	5.1204E-4	3.6054E-4	8.7257E-4
m			
0.1	0.028582	0.012176	0.040758
0.5	0.001054	4.5133E-4	0.001505
1	8.8431E-4	1.9282E-4	0.001077
2	-3.4833E-4	7.7197E-4	4.2364E-4
3	-0.006858	0.002919	-0.003939
4	-0.020934	0.007164	-0.013770
p			
-15	0.198970	0.043385	0.242355
-10	0.088431	0.019282	0.107713
-5	0.022108	0.004821	0.026928
0.1	8.8431E-6	1.9282E-6	1.0771E-5
5	0.022108	0.004821	0.026928
10	0.088431	0.019282	0.107713
15	0.198970	0.043385	0.242355

Table-2: Values of effective dispersion coefficient.

	Two fluid model (present model)			One fluid model Gupta and Gupta [10]
$m = p$	$F_1(m, p)$	$F_2(m, p)$	$F(m, p)$	$F(m, p)$
In the absence of first-order chemical reaction				
1	0.0010582	0.0010582	0.0021164	0.0021164
In the presence of first-order homogeneous chemical reaction				
α	$F_1(\alpha_1, \alpha_2)$	$F_2(\alpha_1, \alpha_2)$	$F(\alpha_1, \alpha_2)$	$F(\alpha)$
0.4	0.0010099	0.0010099	0.0020199	0.00201987
0.8	9.1846E-4	9.1846E-4	0.0018369	0.00183692
1.2	8.2952E-4	8.2952E-4	0.0016590	0.00165904
1.6	7.474E-4	7.474E-4	0.0014948	0.00149480
2	6.70579E-4	6.70579E-4	0.0013412	0.00134116
In the presence of first-order combined homogeneous and heterogeneous chemical reaction				
β	$F_1(\alpha_i, \beta_i)$	$F_2(\alpha_i, \beta_i)$	$F(\alpha_i, \beta_i)$	$F(\alpha, \beta)$
2	8.48463E-4	8.48463E-4	0.00169693	0.0016969
4	8.30289E-4	8.30289E-4	0.00166058	0.0016606
6	8.22866E-4	8.22866E-4	0.00164573	0.0016457
8	8.18832E-4	8.18832E-4	0.00163766	0.0016377
10	8.16298E-4	8.16298E-4	0.00163260	0.0016326

Table-3: Values of effective dispersion coefficient for variations of reaction rate parameter α, Hartman number M, viscosity ratio m, and pressure gradients p in the presence of first order chemical reaction.

	$m = 0.1$			$m = 1$		
α	$F_1(\alpha_1, \alpha_2)$	$F_2(\alpha_1, \alpha_2)$	$F(\alpha_1, \alpha_2)$	$F_1(\alpha_1, \alpha_2)$	$F_2(\alpha_1, \alpha_2)$	$F(\alpha_1, \alpha_2)$
0.4	0.00372143	0.00319035	0.00691178	4.780674E-4	3.225716E-4	8.00639E-4
0.8	0.00316900	0.00286012	0.00602911	4.103112E-4	2.881178E-4	6.98429E-4
1.2	0.00254787	0.00247189	0.00501976	3.333973E-4	2.477554E-4	5.811527E-4
1.6	0.00200757	0.00211124	0.00411881	2.65561E-4	2.104516E-4	4.760121E-4
2.0	0.00158479	0.00180496	0.00338975	2.11585E-4	1.789711E-4	3.905567E-4
	$m = 2$			$p_1 = p_2 = -5$		

0.4	6.96851E-4	2.410362E-4	9.378872E-4	0.01195168	0.00806429	0.02001598
0.8	6.19891E-4	2.273203E-4	8.472109E-4	0.01025778	0.00720295	0.01746073
1.2	5.28997E-4	2.087085E-4	7.377051E-4	0.00833493	0.00619388	0.01452882
1.6	4.44242E-4	1.882975E-4	6.325399E-4	0.00663901	0.00526129	0.01190030
2.0	3.7227E-4	1.680401E-4	5.403097E-4	0.00528964	0.00447428	0.00976392
	$p_1 = p_2 = 0.1$			$p_1 = p_2 = 5$		
0.4	4.780674E-6	3.225716E-6	8.00639E-6	0.01195168	0.00806429	0.02001598
0.8	4.10311E-6	2.881178E-6	6.98429E-6	0.01025778	0.00720295	0.01746073
1.2	3.333973E-6	2.477554E-6	5.811527E-6	0.00833493	0.00619388	0.01452882
1.6	2.655605E-6	2.104516E-6	4.760121E-6	0.00663901	0.00526129	0.01190030
2.0	2.1158E-6	1.789711E-6	3.905566E-6	0.00528964	0.00447428	0.00976392
	$M = 0.1$			$M = 5$		
0.4	0.001038	0.001036	0.002074	4.624221E-4	3.475333E-4	8.099554E-4
0.8	9.91158E-4	9.89435E-4	0.001981	3.945474E-4	3.053233E-4	6.998707E-4
1.2	9.21798E-4	9.20548E-4	0.001842	3.179332E-4	2.568271E-4	5.747602E-4
1.6	8.39725E-4	8.38873E-4	0.001679	2.50917E-4	2.132239E-4	4.641409E-4
2.0	7.53668E-4	7.53113E-4	0.001507	1.981353E-4	1.776012E-4	3.757365E-4
	$M = 10$			$M = 15$		
0.4	4.048575E-4	3.828985E-4	7.87756E-4	3.72265E-4	3.782103E-4	7.504753E-4
0.8	3.436994E-4	3.298954E-4	6.735949E-4	3.15931E-4	3.248135E-4	6.407445E-4
1.2	2.750242E-4	2.700146E-4	5.450388E-4	2.527028E-4	2.646192E-4	5.173221E-4
1.6	2.154077E-4	2.175091E-4	4.329169E-4	1.978524E-4	2.120171E-4	4.098695E-4
2.0	1.688915E-4	1.759473E-4	3.448388E-4	1.550896E-4	1.70565E-4	3.256546E-4

Table-4: Values of effective dispersion coefficient for variations of wall catalytic parameter β, Hartman number M, viscosity ratio m, and pressure gradients p in the presence of first order chemical reaction.

	$m = 0.1$			$m = 1$		
β	$F_1(\alpha_i, \beta_i)$	$F_2(\alpha_i, \beta_i)$	$F(\alpha_i, \beta_i)$	$F_1(\alpha_i, \beta_i)$	$F_2(\alpha_i, \beta_i)$	$F(\alpha_i, \beta_i)$
2	0.001209	0.002834	0.004043	1.60494E-4	2.92623E-4	4.53117E-4
4	8.55809E-4	0.002830	0.003686	1.15092E-4	2.93554E-4	4.08646E-4
6	7.00217E-4	0.002824	0.003524	9.50742E-5	2.93553E-4	3.88627E-4
8	6.12538E-4	0.002820	0.003433	8.37906E-5	2.93447E-4	3.77238E-4
10	5.56254E-4	0.002817	0.003373	7.65464E-5	2.93341E-4	3.69887E-4
	$m = 2$			$p_1 = p_2 = -5$		
2	3.34959E-4	2.80205E-4	6.15164E-4	0.004012	0.007316	0.011328
4	2.84299E-4	2.91222E-4	5.75521E-4	0.002877	0.007339	0.010216
6	2.62049E-4	2.95864E-4	5.57913E-4	0.002377	0.007339	0.009716
8	2.49528E-4	2.98427E-4	5.47955E-4	0.002095	0.007336	0.009431
10	2.41497E-4	3.00052E-4	5.41549E-4	0.001914	0.007334	0.009247
	$p_1 = p_2 = 0.1$			$p_1 = p_2 = 5$		
2	1.60494E-6	2.92623E-6	4.53117E-6	0.004012	0.007316	0.011328
4	1.15092E-6	2.93554E-6	4.08646E-6	0.002877	0.007339	0.010216
6	9.50742E-7	2.93553E-6	3.88627E-6	0.002377	0.007339	0.009716
8	8.37906E-7	2.93447E-6	3.77238E-6	0.002095	0.007336	0.009431
10	7.65464E-7	2.93341E-6	3.69887E-6	0.001914	0.007334	0.009247
	$M = 0.1$			$M = 5$		
2	8.44147E-4	8.45444E-4	0.001690	1.61016E-4	2.77462E-4	4.38478E-4
4	8.25776E-4	8.27622E-4	0.001653	1.18231E-4	2.722E-4	3.90431E-4
6	8.18267E-4	8.2035E-4	0.001639	9.92856E-5	2.69455E-4	3.68741E-4
8	8.14184E-4	8.16399E-4	0.001631	8.85857E-5	2.678E-4	3.56386E-4
10	8.11618E-4	8.13918E-4	0.001626	8.17084E-5	2.66698E-4	3.48407E-4
	$M = 10$			$M = 15$		
2	1.61621E-4	2.42691E-4	4.04311E-4	1.55111E-4	2.26121E-4	3.81231E-4
4	1.27764E-4	2.26304E-4	3.54067E-4	1.25052E-4	2.07607E-4	3.32659E-4
6	1.12633E-4	2.1869E-4	3.31323E-4	1.11579E-4	1.99085E-4	3.10664E-4
8	1.04053E-4	2.143E-4	3.18352E-4	1.03928E-4	1.9419E-4	2.98118E-4
10	9.85249E-5	2.11444E-4	3.09969E-4	9.89956E-5	1.91013E-4	2.90009E-4

NOMENCLATURE

B_0 applied magnetic field

C_i concentration of the solute

D_i molecular diffusion coefficient

D ratio of molecular diffusion coefficient (D_2/D_1)

h distance between the plates

K_i first-order reaction rate constant

L typical length along the flow direction

Q_i volumetric flow rate

U_i velocity

$\bar{u}_i$ non-dimensional average velocity

u_i non-dimensional velocity

$\dfrac{dP_i}{dX_i}$ pressure gradient

M Hartman number

m viscosity ratio (μ_2/μ_1)

n density ratio (ρ_1/ρ_2)

p_i non-dimensional pressure gradient

GREEK SYMBOLS

α_i dimensionless reaction rate parameters

β_i wall catalytic parameter

μ_i dynamic viscosity

σ_e electrical conductivity

η dimensionless length

ρ_i density of the fluid

SUBSCRIPTS

$i = 1, 2$ where 1, 2 –quantities for region-1 and region-2, respectively.

Appendix

Case 1a: Diffusion of a tracer in the absence of homogeneous first-order chemical reaction.

$$p_1 = \frac{dp_1^*}{dx}, \quad p_2 = \frac{dp_2^*}{dx}, \quad a_3 = \frac{\dfrac{p_1}{M}\big(\cosh(M)-1\big) - \dfrac{p_2 M}{2} mn\cosh(M)}{M\, mn\cosh(M) + m^2 n\, \sinh(M)}, \quad a_4 = -a_3 - \frac{p_2}{2}, \quad a_2 = \frac{\big(M^2 a_1 \cosh(M) - p_1\big)}{M^2 \sinh(M)},$$

$$a_1 = \frac{p_1}{M^2} + mn\, a_4, \quad l_1 = -\frac{p_1}{M^2} + lc_1, \quad l_2 = a_4 + lc_1, \quad lc_1 = -\left(\frac{a_2}{2M} + \frac{a_1\sinh(M)}{2M} - \frac{a_2\cosh(M)}{2M} - \frac{p_1}{2M^2} + \frac{p_2}{12} + \frac{a_3}{4} + \frac{a_4}{2}\right),$$

$$Z_1 = \frac{h^2}{D_1 L}\frac{\partial C_1}{\partial \xi_1}, \quad Z_2 = \frac{h^2}{D_2 L}\frac{\partial C_2}{\partial \xi_2}, \quad b_1 = Z_1\left(\frac{a_1}{M}\sinh(M) - \frac{a_2}{M}\cosh(M) + l_1\right), \quad b_3 = -Z_2\left(\frac{P_2}{6} + \frac{a_3}{2} + l_2\right).$$

Case 1b: Diffusion of a tracer in the absence of first order chemical reaction for purely viscous fluid (two fluid model)

$$a_3 = \frac{p_1 - p_2 mn}{2(m+1)mn}, \quad a_4 = -\frac{p_2}{2} - a_3, \quad a_2 = mn a_4, \quad a_1 = m^2 n a_3.$$

Case 2a: Diffusion of a tracer in the presence of homogeneous first-order chemical reaction.

$$C_{11} = b_{11}\cosh(\alpha_1 \eta) + b_{21}\sinh(\alpha_1 \eta) + \frac{a_1}{M^2 - \alpha_1^2}\cosh(M\eta) + \frac{a_2}{M^2 - \alpha_1^2}\sinh(M\eta) + \frac{l_1}{\alpha_1^2}\eta^2,$$

$$C_{12} = b_{12}\cosh(\alpha_1 \eta) + b_{22}\sinh(\alpha_1 \eta), \quad C_{21} = b_{31}\cosh(\alpha_2 \eta) + b_{41}\sinh(\alpha_2 \eta),$$

$$C_{22} = b_{32}\cosh(\alpha_2 \eta) + b_{42}\sinh(\alpha_2 \eta) + l_3\eta^2 + l_4\eta + l_5, \qquad l_3 = -\frac{p_2}{2\alpha_2^2}, \qquad l_4 = -\frac{a_3}{\alpha_2^2}, \qquad l_5 = -\frac{1}{\alpha_2^2}\left(l_2 + \frac{p_2}{\alpha_2^2}\right),$$

$$g_1 = \frac{-a_1 M \sinh(M)}{M^2 - \alpha_1^2} + \frac{a_2 M \cosh(M)}{M^2 - \alpha_1^2}, \quad g_2 = \frac{a_1}{M^2 - \alpha_1^2} - \frac{l_1}{\alpha_1^2}, \quad g_3 = \frac{a_2 M}{M^2 - \alpha_1^2}$$

$$Dr = -D\alpha_2^2\cosh(\alpha_1)\sinh(\alpha_2) - \alpha_1\alpha_2\sinh(\alpha_1)\cosh(\alpha_2),$$

$$b_{41} = \frac{1}{Dr}\big(g_2\alpha_1\alpha_2\sinh(\alpha_2) - g_3\alpha_2\cosh(\alpha_1)\sinh(\alpha_2) + b_1\alpha_2\sinh(\alpha_2)\big),$$

$$b_{42} = \frac{1}{Dr}\big(-l_5\alpha_1\alpha_2\sinh(\alpha_1)\sinh(\alpha_2) + D l_4\alpha_2\cosh(\alpha_1)\sinh(\alpha_2) + (2l_2 + l_3)\alpha_1\sinh(\alpha_1)\big),$$

$$b_{31} = -\frac{b_{41}\cosh(\alpha_2)}{\sinh(\alpha_2)}, \quad b_{32} = \frac{-b_{42}\alpha_2\cosh(\alpha_2)-2l_3-l_4}{\alpha_2\sinh(\alpha_2)}, \quad b_{11} = b_{31}-g_2, \quad b_{12} = b_{32}+l_5, \quad b_{21} = \frac{b_{11}\alpha_1\sinh(\alpha_1)-g_1}{\alpha_1\cosh(\alpha_1)},$$

$$b_{22} = \frac{b_{12}\sinh(\alpha_1)}{\cosh(\alpha_1)}.$$

Case 2b: Diffusion of a tracer with combined homogeneous and heterogeneous first-order chemical reaction.

$$g_4 = -\alpha_1\sinh(\alpha_1)-\beta_1\cosh(\alpha_1), \qquad\qquad g_5 = \alpha_1\cosh(\alpha_1)+\beta_1\sinh(\alpha_1),$$

$$g_6 = -\frac{a_1 M\sinh(M)}{M^2-\alpha_1^2}+\frac{a_2 M\cosh(M)}{M^2-\alpha_1^2}-\beta_1\left(\frac{a_1\cosh(M)}{M^2-\alpha_1^2}-\frac{a_2\sinh(M)}{M^2-\alpha_1^2}-\frac{l_1}{\alpha_1^2}\right), \qquad g_7 = \alpha_2\sinh(\alpha_2)+\beta_2\cosh(\alpha_2),$$

$$g_8 = \alpha_2\cosh(\alpha_2)+\beta_2\sinh(\alpha_2), \quad g_9 = 2l_3+l_4+\beta_2(l_3+l_4+l_5),$$

$$b_{41} = \frac{\alpha_1}{g_5 g_7 D\alpha_2 - g_4 g_8\alpha_1}\left(g_4 g_7\left(\frac{a_1}{M^2-\alpha_1^2}-\frac{l_1}{\alpha_1^2}\right)-\frac{g_5 g_7 Ma_2}{\alpha_1(M^2-\alpha_1^2)}-g_6 g_7\right),$$

$$b_{42} = \frac{\alpha_1}{g_5 g_7 D\alpha_2 - g_4 g_8\alpha_1}\left(-g_4 g_7 l_5 - \frac{g_5 g_7 D l_4}{\alpha_1}+g_4 g_9\right), \quad b_{31} = -\frac{g_8 b_{41}}{g_7}, \quad b_{32} = \frac{-g_8 b_{42}-g_9}{g_7}, \quad b_{11} = b_{31}-\frac{a_1}{M^2-\alpha_1^2}+\frac{l_1}{\alpha_1^2},$$

$$b_{12} = b_{32}+l_5, \quad b_{21} = \frac{-g_4 b_{11}-g_6}{g_5}, \quad b_{22} = -\frac{g_4 b_{12}}{g_5}.$$

Case 2c: Diffusion of a tracer in the presence of homogeneous first-order chemical reaction in the absence of magnetic field for purely viscous fluid (two fluid model).

$$lc_1 = -\frac{1}{2}\left(\frac{p_1}{6}-\frac{a_1}{2}-a_2+\frac{p_2}{6}+\frac{a_3}{2}+a_4\right), \quad lc_2 = -\frac{1}{2}\left(\frac{p_1}{6}-\frac{a_1}{2}+a_2+\frac{p_2}{6}+\frac{a_3}{2}-a_4\right), \quad l_1 = \frac{-p_1}{2\alpha_1^2}, \quad l_2 = \frac{-a_1}{\alpha_1^2}, \quad l_3 = -\frac{lc_1}{\alpha_1^2}-\frac{p_1}{\alpha_1^4},$$

$$l_4 = \frac{-p_2}{2\alpha_2^2}, \quad l_5 = \frac{-a_3}{\alpha_2^2}, \quad l_6 = -\frac{lc_2}{\alpha_2^2}-\frac{p_2}{\alpha_2^4}, \quad Z_1 = \frac{h^2}{D_1 L}\frac{\partial C_1}{\partial\xi_1}, \quad Z_2 = \frac{h^2}{D_2 L}\frac{\partial C_2}{\partial\xi_2},$$

$$\text{Dr}=\alpha_1\alpha_2\sinh(\alpha_1)\cosh(\alpha_2)+\alpha_2^2 D\sinh(\alpha_2)\cosh(\alpha_1),$$

$$b_{41} = \frac{-1}{\text{Dr}}\left(l_3\alpha_1\alpha_2\sinh(\alpha_1)\sinh(\alpha_2)-l_2\alpha_2\cosh(\alpha_1)\sinh(\alpha_2)+\alpha_2\sinh(\alpha_2)(l_2-2l_1)\right),$$

$$b_{42} = \frac{-1}{\text{Dr}}\left(\alpha_1\sinh(\alpha_2)(l_5+2l_4)-l_6\alpha_1\alpha_2\sinh(\alpha_1)\sinh(\alpha_2)+l_5 D\alpha_2\cosh(\alpha_1)\sinh(\alpha_2)\right),$$

$$b_{31} = \frac{-b_{41}\cosh(\alpha_2)}{\sinh(\alpha_2)}, \quad b_{32} = \frac{-b_{42}\alpha_2\cosh(\alpha_2)-2l_4-l_5}{\alpha_2\sinh(\alpha_2)}, \quad b_{11} = b_{31}-l_3, \quad b_{12} = b_{32}+l_6, \quad b_{21} = \frac{b_{11}\alpha_1\sinh(\alpha_1)+2l_1-l_2}{\alpha_1\cosh(\alpha_1)},$$

$$b_{22} = \frac{b_{12}\sinh(\alpha_1)}{\cosh(\alpha_1)}.$$

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