International Journal of Mathematical Archive-3(5), 2012, 1915-1919
Available online through www.ijma.info ISSN 2229 – 5046

PATH RELATED ARITHMETIC GRAPHS

P. B. SARASIJA*
Department of Mathematics, Noorul Islam University, Kumaracoil, (T.N.), India

N. ADALIN BEATRESS
Department of Mathematics, All Saints College of Education, Kaliakkavilai, Malayadi, (T.N.), India

(Received on: 07-05-12; Revised & Accepted on: 31-05-12)

ABSTRACT
A (p, q) graph G = (V, E) is said to be (k,d) arithmetic, where k and d are positive integers if its p vertices admits a labeling of distinct non negative integers such that the values of the edges obtained as the sum of the labels of their end vertices form the set{ k, k+d, …,k+ (q-1)d}. In this paper we prove that P_n^2, $(P_n; K_1)$, *$(P_n ; S_1)$, $(P_n ; S_2)$ and $(P_n,; S_3)$ are arithmetic graphs.*

Key Words: *Arithmetic labeling, Arithmetic graphs, path, star.*

AMS Subject Classification (2000): *05C78.*

1. INTRODUCTION

For all terminology and notation in graph theory we follow [3]. Graph labeling where the vertices are assigned values subject to certain conditions. Labeled graphs serve as mathematical models for a broad range of applications such as coding theory, inducing the design of good radar type's codes. Labeled graphs have also been applied in x – ray crystallographic analysis, to design a communication network addressing system.

B.D. Acharya and S.M. Hedge [1, 3] have introduced the notion of (k, d) arithmetic labeling of graphs. For a non negative integer k and positive integer d, a (p, q) graph G = (V, E), a (k, d) arithmetic labeling is an injective mapping $f : V(G) \rightarrow \{0,1,2,\ldots\}$, where the induced edge function $f^*: E(G) \rightarrow \{k, k+d, k+2d,\ldots, k+(q-1)d\}$ such that $f^*(uv) = f(u) + f(v)$ for all $uv \in E(G)$ is also injective. If a graph G admits such a labeling then the graph G is called(k, d) arithmetic graph. The greatest integer less than or equal to the real number x is denoted by $\lfloor x \rfloor$. The greatest integer greater than or equal to the real number x is denoted by $\lfloor x \rfloor$.

Consider the following path related graphs. [2]

1. P_n^k, the k^{th} power of P_n is the graph obtained from the path P_n by adding edges that join all vertices u and v with degree of (u, v) = k.
2. The graph $(P_n; K_1)$ is obtained from a path P_n by joining a pendant edge at each vertex of the path.
3. Let S_m be a Star graph with vertices $v, w_1, \ldots w_m$ .Define $(P_n ; S_m)$ the graph obtained from n copies of S_m and the path P_n: $u_1u_2\ldots u_n$ by joining u_j the vertex v of the j^{th} copy of S_m by means of an edge, for $1 \le m \le 3$, $1 \le j \le n$.

For example $(P_4; S_2)$ is shown in Figure 1.

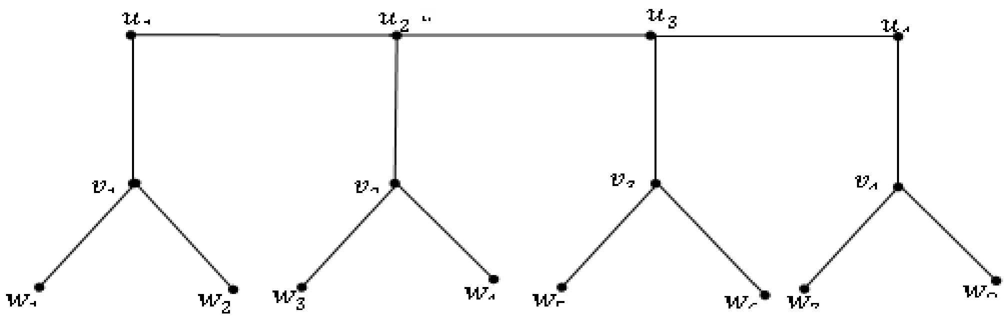

Figure 1.

Corresponding author: P. B. SARASIJA*
Department of Mathematics, Noorul Islam University, Kumaracoil, (T.N.), India

2. ARITHMETIC GRAPH OF P_n^2

In [5] Jinnah and Singh noted that P_n^2 is additively graceful, that is k = 1 and d = 1.

In this paper we prove that P_n^2 is a (k,d) arithmetic graph for all $k \geq d$.

Theorem 2.1: *For all positive integer n, $k \geq d$, k and d are either both even or odd positive integers, P_n^2 is a (k, d) arithmetic graph.*

Proof: Denote the vertices of the path P_n as $u_1, u_2, \ldots, u_n$.

Define a labeling f: V (P_n) → {0, 1…} such that

$$f(u_1) = \frac{k-d}{2},\ \ f(u_2) = k - u_1,\ \ f(u_i) = u_{i-1} + d,\ \ 3 \leq i \leq n.$$

$$f^*(E(P_n^2)) = \{k, k+d, \ldots, k+(q-1)d\}.$$

Hence P_n^2 is (k, d) arithmetic graph.

For example, (6,4) arithmetic labeling of P_5^2 using the above theorem is shown in Figure 2.

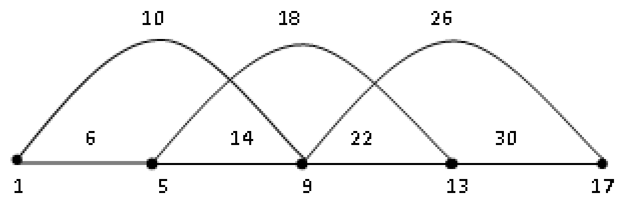

Figure 2.

$$f^*(E\ (P_5^2)) = \{6, 10, 14, 18, 22, 26, 30\}.$$

3. ARITHMETIC GRAPH OF (P_n; k_1)

Theorem 3.1: *For all positive integer k, d and n (P_n; K_1) is a (k,d) arithmetic if*

$$\begin{cases} k < d; & for\ all\ k\ and\ d \\ k > d; & k\ is\ not\ a\ multiple\ of\ d \end{cases}$$

Proof: Denote the vertices of the path P_n as $u_1, u_2, \ldots, u_n$. Let $v_1, v_2, \ldots, v_n$ be the vertices of the pendant edges.

Define a labeling f: $V(P_n; K_1) \rightarrow$ {0, 1,…} such that

$$f(u_1) = 0,\ f(u_{2i}) = k + (2i-1)d,\ \ 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f(u_{2i+1}) = 2id,\ \ 1 \leq i < \lceil \tfrac{n}{2} \rceil \text{ and}$$
$$f(v_1) = k,\ \ f(v_{2i}) = (2i-1)d,\ \ 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor,$$
$$f(v_{2i+1}) = k + 2id,\ 1 \leq i < \lceil \tfrac{n}{2} \rceil$$

Hence $(P_n; K_1)$ is (k, d) arithmetic graph.

For example, (5, 3) arithmetic labeling of $(P_4;\ K_1)$ is shown in Figure 3.

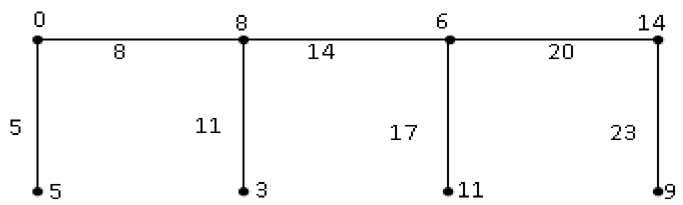

Figure 3.

4 ARITHMETIC GRAPH OF (P_n ; S_1)

Theorem 4.1: *For all positive integer k, d and n (P_n ; S_1)is a (k,d) arithmetic if*

$$\begin{cases} k < d; & for\ all\ k\ and\ d \\ k > d; & k\ is\ not\ a\ multiple\ of\ d \end{cases}$$

Proof: Denote the vertices of the path P_n as $u_1, u_2,\ldots,u_n$. Let $v_1, v_2,\ldots,v_n$ and $w_1, w_2,\ldots,w_n$ be the vertices of the Star S_1.

Define a labeling f: V(P_n ; S_1) → {0,1,…} such that

$$f(u_{2i}) = (3i-2)d,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$
$$f(u_{2i-1}) = k + (3i-2)d,\ 1 \le i \le \lceil \tfrac{n}{2} \rceil,$$
$$f(v_1) = 0,\ f(v_{2i}) = k + (3i-1)d,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor,$$
$$f(v_{2i+1}) = 3id,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$
$$f(w_1) = k,\ f(w_{2i}) = (3i-1)d,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor, \text{ and}$$
$$f(w_{2i+1}) = k + 3id,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$

Hence (P_n ; S_1) is (k, d) arithmetic graph.

For example, (3, 5) arithmetic labeling of (P_4; S_1) is shown in Figure 4.

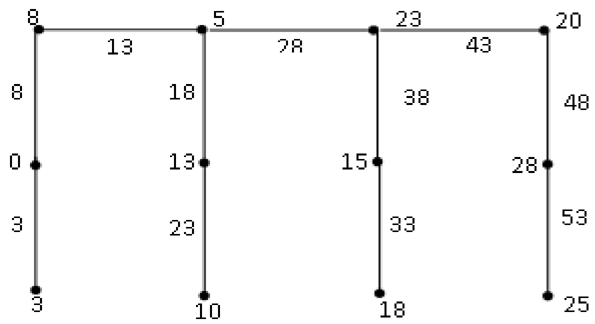

Figure 4.

5. Arithmetic graph of (P_n ; S_2)

Theorem 5.1: *For all **positive** integer k,d, and n, the graph (P_n ; S_2)is (k, d) arithmetic if*

$$\begin{cases} k < d; & for\ all\ k\ and\ d \\ k > d; & k\ is\ not\ a\ multiple\ of\ d \end{cases}$$

Proof: Denote the vertices of the path P_n as $u_1, u_2,\ldots,u_n$. Let $v_1,v_2,\ldots,v_n$ and $w_1,w_2,\ldots,w_{2n}$ be the vertices of the Star S_2.

Define a labeling f : V((P_n ; S_2) → {0,1,…} as follows

$$f(u_i) = \begin{cases} 2id, & if\ \ i\ \ is\ odd,\ 1 \le i \le n \\ k + (2i-3)d, & if\ i\ is\ even,\ 2 \le i \le n \end{cases}$$
$$f(v_i) = \begin{cases} (2i-1)d, & if\ \ i\ \ is\ even,\ 2 \le i \le n \\ k + 2(i-1)d, & if\ \ i\ \ is\ \ odd,\ 1 \le i \le n \end{cases}$$
$$f(w_i) = \begin{cases} (i-1)d & if\ \ i \equiv 1,2\ (mod\ 4), \\ k + (i-1)d, & if\ \ i \equiv 0,3\ (mod\ 4)\ ,1 \le i \le 2n \end{cases}$$

Hence (P_n; S_2) is (k, d) arithmetic graph.

For example, (2, 3) arithmetic labeling of (P_4; S_2) is shown in Figure 5.

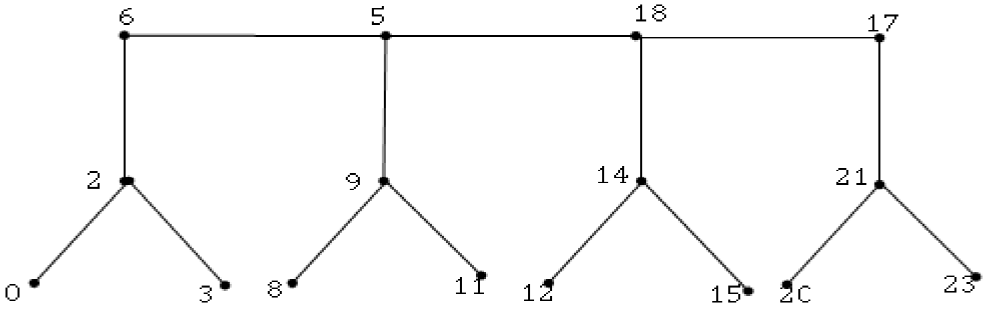

Figure 5.

$f^*(E\ (P_4;\ S_2))\ = \{2,5,8,11,14,17,20,23,26,29,32,35,38,41,44\}$

6. Arithmetic graph of $(P_n;\ S_3)$

Theorem 5.1: *For all* ***positive*** *integer k ,d, and n, the graph $(P_n\ ;\ S_3)$is (k, d)*

$$\textit{arithmetic if}\quad \begin{cases} k < d; & \textit{for all } k \textit{ and } d \\ k > d; & k \textit{ is not a multiple of } d \end{cases}$$

Proof: Denote the vertices of the path P_n as $u_1, u_2,\ldots,u_n$. Let $v_1,v_2,\ldots,v_n$ and $w_1,w_2,\ldots,w_{2n}$ be vertices of the Star S_3.

Define a labeling f: V $((P_n;\ S_3)\ \rightarrow \{0,1,\ldots\}$ as follows

$$f(u_{2i-1}) = (5i - 2)\,d,\ \ 1 \le i \le \lceil \tfrac{n}{2} \rceil,$$
$$f(u_{2i}) = k + (5i - 4)\,d,\ \ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor,$$
$$f(v_{2i-1}) = k + 5\,(i - 1)\,d,\ \ 1 \le i < \lceil \tfrac{n}{2} \rceil,$$
$$f(v_{2i}) = (5i - 1)\,d,\ \ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor,$$
$$f\,(w_i) = \begin{cases} (i-j\,)d & if\ \ i \equiv 1,2\,,3 (mod\ 6\,)\ , \\ k + (i-1-j)d, & if\ \ i \equiv 0,4,5\ (mod\ 6)\ \ ,1\ \le i \le 3n \end{cases}$$

where $j = \lceil \frac{i}{6} \rceil$

Hence $(P_n;\ S_3)$ is (k, d) arithmetic graph.

For example, (4, 3) arithmetic labeling of $(P_4;\ S_3)$ is shown in Figure 6.

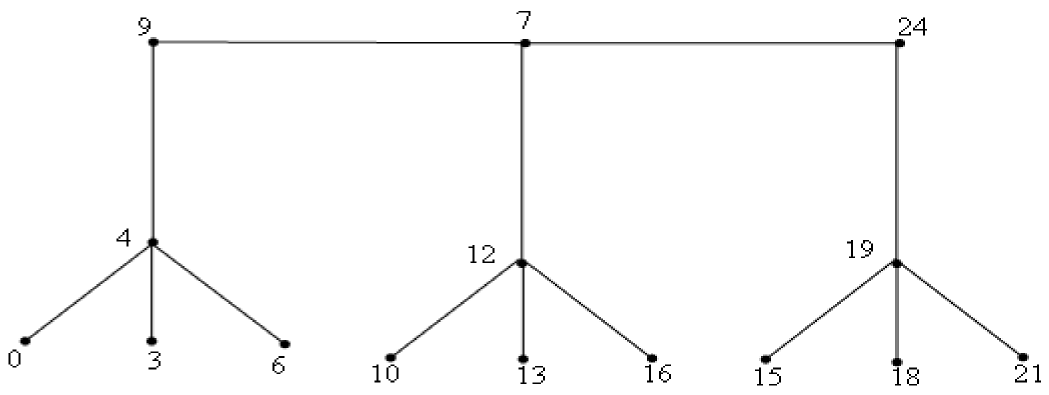

Figure 6.

$f^*(E(P_3,S_3) = \{4,7,10,13,16,19,22,25,28,31,34,37,40,43\}$

REFERENCES

[1] B.D Acharya and S.M.Hedge, *Arithemetic graphs*, J. Graph Theory, 14(3), (1990) 275-299.

[2] J. A. Gallian, *A dynamic survey of graph labeling*, The Electronic journal of Combinatorics (2010).

[3] F. Harary, *Graph theory*, Addison Wesley, Reading Mass (1972).

[4] S. M. Hedge, and Sudhakar Shetty, *On Arithemetic graphs*, J. Pure appl Math., 33(8), (2002) 1275-1283.

[5] M.I. Jinnah and G.S. Singh, A note on arithmetic numberings of graphs, Proc. Symposium on Graphs and combinatorics, Kerala, india (1991) 83-87.

Source of support: Nil, Conflict of interest: None Declared