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ANALYSIS OF SORET EFFECT ON THE ONSET OF NONLINEAR DDC IN A TWO COMPONENT BOUSSINESQ FLUID SATURATED ANISOTROPIC POROUS LAYER

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ABSTRACT

The double diffusive convection (DDC) in a horizontal anisotropic porous layer saturated with a Boussinesq fluid, which is heated and salted from below in the presence of Soret coefficient is studied analytically using nonlinear theory. The generalized Darcy model is employed for the momentum equation. The nonlinear theory provides the quantification of heat and mss transports and also explains the possibility of the finite amplitude motions. The effect of mechanical anisotropy parameter, thermal anisotropy parameter, Lewis number and Soret parameter on finite amplitude motion and heat and mass transfer are shown graphically.

Keywords: *Double diffusive convection, Soret parameter, Finite amplitude convection, Heat and Mass transfer.*

1. INTRODUCTION

The problem of convection induced by temperature and concentration gradients or by concentration gradients of two species, known as double diffusive convection, has attracted considerable interest in the last several decades. If gradients of two stratifying agencies having different diffusivities are simultaneously present in a fluid layer, a variety of interesting convective phenomena can occur that are not possible in single component fluids. The double diffusive convection in porous media has also become important in recent years because of its many applications in geophysics, particularly in saline geothermal fields where hot brines remain beneath less saline, cooler ground waters. A comprehensive review of the literature concerning double diffusive convection in a binary fluid saturated porous medium may be found in the book by Nield and Bejan [7]. Excellent review articles on double diffusive convection in porous media include those by Mojtabi and Charrier-Mojtabi ([3], [4]) and Mamou [12].

In a system where two diffusing properties are present, instabilities can occur only if one of the component is destabilizing. If the cross diffusion terms are included in the species transport equations, then the situation will be quite different. Due to the cross diffusion effect, each property gradient has a significant influence on the flux of the other property. A flux of salt caused by a spatial gradient of temperature is called the Soret effect.

There are many studies available on the onset of double diffusive convection in a porous medium with and without cross diffusion effects (see e.g. Nield and Bejan, [7]. Thermal convection in a binary fluid driven by the Soret and DuFour effects has been investigated by Knobloch [8]. He has shown that equations are identical to the thermosolutal problem except for a relation between the thermal and solute Rayleigh numbers. The double diffusive convection in a porous medium in the presence of Soret and DuFour coefficients has been analyzed by Rudraiah and Malashetty [15]. This work has been extended to weak nonlinear analysis by Rudraiah and Siddheshwar [16]. The effect of temperature dependent viscosity on double diffusive convection in an anisotropic porous medium in the presence of Soret coefficient has been studied by Patil and Subramanian [19]. Straughan and Hutter [5] have investigated the double diffusive convection with Soret effect in a porous layer using Darcy-Brinkman model. Bahloual et al. [1] have carried out an analytical and numerical study of the double diffusive convection in a shallow horizontal porous layer under the influence of Soret effect. Recently, Mansour et al. [2] have investigated the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and subject to horizontal solute gradient in the presence of Soret effect.

Most of the studies have usually been concerned with homogeneous isotropic porous structures. However during the last one decade, the effect of non-homogeneity and anisotropy of the porous medium have also been studied. The geological and pedagogical processes rarely form isotropic media as is usually assumed in transport studies. In geothermal system with a ground structure composed of many strata of different permeabilities, the overall horizontal

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permeability may be up to ten times as large as the vertical component. Process such as sedimentation, compaction, frost action, and reorientation of the solid matrix are responsible for the creation of anisotropic natural porous media. Anisotropy can also be a characteristic of artificial porous material like pelleting used in chemical engineering process, fiber materials used in insulating purposes.

There are many investigations available on the thermal convection in a single component fluid saturated anisotropic porous layer heated from below. A theoretical analysis of non-linear thermal convection in an anisotropic porous media is performed by Kvernvold and Tyvand [17]. Nilsen and Storesletten [22] have studied the problem of natural convection in both isotropic and anisotropic porous channels. Tyvand and Storesletten [18] investigated the problem concerning the onset of convection in an anisotropic porous layer in which the principal axes were obliquely oriented to the gravity vector. Natural thermal convection in horizontal anisotropic porous layers heated from below or in vertical cavities filled with an anisotropic porous layer subjected to a constant heat flux, as described in the work of Degan et al. [10]. Some other studies reported the anisotropy and heterogeneous character of porous media, and a summary of these can be found in the book of Nield and Bejan [7].

Recently many authors have studied the effect of anisotropy on the onset of convection in a porous layer (see e.g., Govinder [20], [21]: Malashetty and Swamy [13]; Malashetty and Heera [14]). Although some work on double diffusive convection in an isotropic porous medium is available (Malashetty and Heera [14], attention has not been given to the study of double diffusive convection in an anisotropic porous medium with Soret effect. The main objective of this study is therefore to investigate the effect of Soret coefficient, mechanical and thermal anisotropy on the double diffusive convection in a fluid saturated porous layer using nonlinear analysis.

2 MATHEMATICAL FORMULATIONS

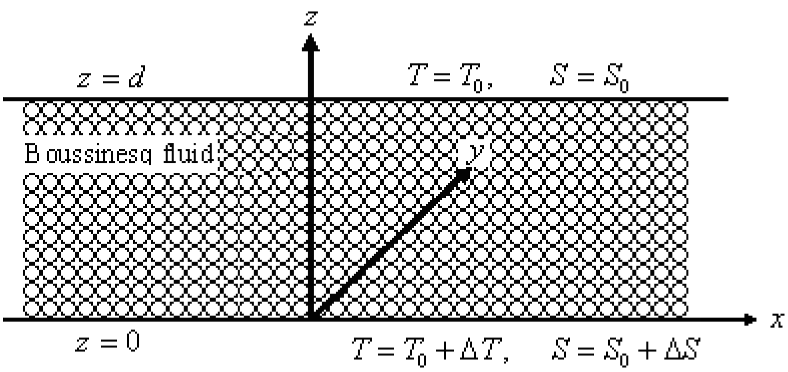

Fig. Physical configuration

The above figure shows the physical configuration of the problem. A horizontal porous layer held between two walls at $z = 0$ and $z = d$ saturated with a Boussinesq fluid, which is heated and salted from below, is considered. The porous medium is assumed to possess isotropy in horizontal plane in both thermal and mechanical properties. A constant gradient of temperature ΔT and salinity ΔS is maintained between the two walls. The generalized Darcy model has been employed for the momentum equation. With these assumptions the basic governing equations of motion are

$$\nabla.\mathbf{V} = 0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{V}}{\partial t} + \mu \mathbf{K}.\mathbf{V} = -\nabla p + \rho \mathbf{g}, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (\mathbf{V}.\nabla)T = D_1 \nabla^2 T, \tag{3}$$

$$\varepsilon \frac{\partial S}{\partial t} + (\mathbf{V}.\nabla)S = \varepsilon[D_2 S + D_3 T]\nabla^2, \tag{4}$$

$$\rho = \rho_0\left[1 - \alpha_T\left(T - T_0\right) + \alpha_S\left(S - S_0\right)\right], \tag{5}$$

where, $\mathbf{V}$ is the velocity vector (u, v, w), ρ is the density, t is time, p is pressure, μ is the dynamic viscosity, K is permeability tensor, $\mathbf{g}$ is gravitational acceleration, γ is specific heat ratio, T is temperature, ΔT is temperature difference between the walls, ΔS is salinity difference between the walls, D_1 is thermal diffusivity, ε is the porosity, S is solute concentration, D_2 is solute diffusivity, D_3 is cross diffusion due to T component, α_T is thermal expansion coefficient, α_S is solute expansion coefficient, T_b is the temperature of hot walls, T_0 is the temperature of cold walls.

2.1 Basic state

The basic state of the fluid is assumed to be quiescent and is given by,

$$\mathbf{V}_b = (0,0,0),\ p = p_b(z),\ T = T_b(z),\ S = S_b(z),\ \rho = \rho_b(z). \tag{6}$$

Using equation (6), equations (1) to (5) yield

$$\frac{dp_b}{dz} = -\rho_b g,\ \ \frac{d^2T_b}{dz^2} = 0,\ \ \frac{d^2S_b}{dz^2} = 0,\ \ \rho_b = \rho_0\left[1 - \alpha_T\left(T_b - T_0\right) + \alpha_S\left(S_b - S_0\right)\right]. \tag{7}$$

2.2 Perturbed state

On the basic state we superpose perturbations in the form

$$\begin{aligned} &\mathbf{V} = \mathbf{V}_b + \mathbf{V}'(x,y,z,t),\ T = T_b(z) + T'(x,y,z,t),\ S = S_b(z) + S'(x,y,z,t), \\ &p = p_b(z) + p'(x,y,z,t), \rho = \rho_b(z) + \rho'(x,y,z,t) \end{aligned} \tag{8}$$

where primes indicate perturbations.

We consider only two dimensional disturbances and define stream function ψ by

$$(u', w') = \left(\frac{\partial\psi}{\partial z},\ -\frac{\partial\psi}{\partial x}\right) \tag{9}$$

Introducing (8) in equations (1) - (5) and using basic state equations (7) and the transformations

$$\left(x^*,\ z^*\right) = \left(\frac{x}{d},\frac{z}{d}\right),\ \ t^* = \frac{t}{d^2/D_z},\ \ \psi^* = \frac{\psi}{D_z},\ \ T^* = \frac{T'}{\Delta T},\ S^* = \frac{S'}{\Delta S}, \tag{10}$$

where, D_z is the effective thermal diffusivity in vertical direction. To render the resulting equations dimensionless, we obtain (after dropping the asterisks and ε and γ are set equal to unity for simplicity).

$$\left[\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2 + \left(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\right]\psi = -R_T\frac{\partial T}{\partial x} + R_S\frac{\partial S}{\partial x}, \tag{11}$$

$$\left[\frac{\partial}{\partial t} - \left(\eta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\right]T = -\frac{\partial\psi}{\partial x} + \frac{\partial(\psi,T)}{\partial(x,z)}, \tag{12}$$

$$\left[\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right]S - Sr\frac{R_T}{R_S}\nabla^2 T = -\frac{\partial\psi}{\partial x} + \frac{\partial(\psi,S)}{\partial(x,z)}, \tag{13}$$

where , Pr_D is the Darcy Prandtl number $\left(\dfrac{vd^2\varepsilon}{K_z D_1}\right)$, ξ is the mechanical anisotropy parameter (K_x / K_z), R_T is thermal Rayleigh number $\left(\dfrac{\beta_T g\Delta T d K_z}{vD_1}\right)$, R_S is the solute Rayleigh number $\left(\dfrac{\beta_T g\Delta S d K_z}{vD_1}\right)$, η is thermal anisotropy parameter $\left(\dfrac{D_x}{D_z}\right)$, Le is the Lewis number $\left(\dfrac{D_z}{D_x}\right)$, Sr is the Soret parameter $\left(\dfrac{\alpha_S D_3}{\alpha_T D_1}\right)$, ψ is the stream function.

Equations (11) - (13) are solved for stress-free, isothermal, isohaline boundary conditions, namely,

$$\psi = T = S = 0 \quad \text{at} \quad z = 0,1. \tag{14}$$

We neglect the Jacobian in equations (11) - (13) and assume the solutions to be periodic waves of the form

$$\begin{pmatrix} \psi \\ T \\ S \end{pmatrix} = e^{\sigma t} \begin{pmatrix} \Psi \sin \pi\alpha x \\ \Theta \cos \pi\alpha x \\ \Phi \cos \pi\alpha x \end{pmatrix} \sin \pi z, \tag{15}$$

where ψ, Θ, Φ are the amplitudes of stream function, temperature and concentration field respectively. Substituting equations (15) in the linearized version of equations (11) – (13), we get

$$\left(\frac{\sigma}{Pr_D} a^2 + a_1^2 \right) \Psi = -R_T \pi\alpha\Theta + R_S \pi\alpha\Phi, \tag{16}$$

$$\left(\sigma + a_2^2 \right) \Theta = -\pi\alpha\Psi, \tag{17}$$

$$\left(\sigma + \frac{1}{Le} a^2 \right) \Phi + Sr \frac{R_T}{R_S} a^2 \Theta = -\pi\alpha\Psi, \tag{18}$$

where

$$a^2 = \pi^2 \left(\alpha^2 + 1 \right), \quad a_1^{\,2} = \pi^2 \left(\alpha^2 + \frac{1}{\xi} \right) \quad \text{and} \quad a_2^2 = \pi^2 \left(\eta\alpha^2 + 1 \right).$$

For non-trivial solution of Ψ, Θ and Φ, we require

$$\left(\frac{a^2}{Pr_D} \right) \sigma^3 + \left(a_1^2 + \frac{a^4}{Le\ Pr_D} + \frac{a^2 a_2^2}{Pr_D} \right) \sigma^2 + \left(\frac{a^2 a_1^2}{Le} + a_1^{\,2} a_2^2 + \frac{a^4 a_2^2}{Le\ Pr_D} + \pi^2 \alpha^2 \left(R_S - R_T \right) \right) \sigma + \frac{a^2 a_1^2 a_2^2}{Le} + a_2^2 \pi^2 \alpha^2 R_S - R_T \pi^2 \alpha^2 a^2 \left(Sr + \frac{1}{Le} \right) = 0. \tag{19}$$

3 NONLINEAR STABILITY ANALYSIS

In this section, we discuss the non-linear stability analysis, the effect of non-linear is to distort the temperature and concentration fields through the interaction of ψ, T and also ψ, S. The distortion of these fields will corresponds to a change in the horizontal mean, i.e. a component of the form $\sin(2\pi z)$ will be generated. Thus a minimal Fourier series which describes the finite amplitude free convection is given by,

$$\psi = A_1(t) \sin(\pi\alpha x) \sin(\pi z), \tag{20}$$

$$T = B_1(t) \cos(\pi\alpha x) \sin(\pi z) + B_2(t) \sin(2\pi z), \tag{21}$$

$$S = C_1(t) \cos(\pi\alpha x) \sin(\pi z) + C_2(t) \sin(2\pi z), \tag{22}$$

where the amplitudes $A_1(t)$, $B_1(t), B_2(t), C_1(t)$ and $C_2(t)$ are to be determined from the dynamics of the system.

Substituting equations (20) - (22) into equations (11) - (13) and equating the coefficients of like terms, we obtain the following non-linear autonomous system of differential equations

$$\dot{A}_1 = -Pr_D \frac{a_1^2}{a^2} A_1 - Pr_D \frac{\pi\alpha}{a^2} B_1 + R_S Pr_D \frac{\pi\alpha}{a^2} C_1, \tag{23}$$

$$\dot{B}_1 = -\pi\alpha A_1 - a_2^2 B_1 - \pi^2 \alpha A_1 B_2, \tag{24}$$

$$\dot{B}_2 = -4\pi^2 B_2 + \frac{\pi^2\alpha}{2} A_1 B_1, \quad (25)$$

$$\dot{C}_1 = -\pi\,\alpha\,A_1 - \frac{a^2}{Le} C_1 - \pi^2\alpha\,A_1 C_2 - a^2\,Sr\,\frac{R_T}{R_S} B_1\ , \quad (26)$$

$$\dot{C}_2 = -\frac{4\pi^2}{Le}\,C_2 + \frac{\pi^2\alpha}{2} A_1\,C_1 - 4\,\pi^2\,Sr\frac{R_T}{R_S} B_2\ , \quad (27)$$

where the over dot denotes the time derivative.

The non-linear system of autonomous differential equations is not suitable to analytical treatment for the general time-dependent variables and we have to solve it using numerical method. However, one can make qualitative predictions as discussed below. The system of equations (23) - (27) is uniformly bounded in time and possesses many properties of the full problem. Like the equations (23) - (27) must be dissipative. Thus volume in the phase space must contract. In order to prove the volume contraction, we must show that velocity field has a constant negative divergence. Indeed,

$$\frac{\partial \dot{A}_1}{\partial A_1} + \frac{\partial \dot{B}_1}{\partial B_1} + \frac{\partial \dot{B}_2}{\partial B_2} + \frac{\partial \dot{C}_1}{\partial C_1} + \frac{\partial \dot{C}_2}{\partial C_2} = -\left[Pr_D\,\frac{a_1^2}{a^2} + a_2^2 + 4\pi^2 + \frac{a^2}{Le} + \frac{4\pi^2}{Le} \right], \quad (28)$$

which is always negative and therefore the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase space; in particular, they may be attracted to a fixed point, a limit cycle or, perhaps, a strange attractor. From eq. (28) we conclude that if a set of initial points in phase space occupies a region $V_1(0)$ at time $t = 0,$ then after some time t, the end points of the corresponding trajectories will fill a volume

$$V_1(t) = V_1(0)\ \exp\left[-\left\{ Pr_D\,\frac{a_1^2}{a^2} + a_2^2 + 4\pi^2 + \frac{1}{Le}\left(a^2 + 4\pi^2\right) \right\} t \right]. \quad (29)$$

This expression indicates that the volume decreases exponentially with time. We can also infer that, the large Darcy Prandtl number and very small Lewis number $(Le < 1)$ tend to enhance dissipation. Finally, we note that the systems of equations (23) - (27) are invariant under the symmetry transformation $\left(A_1, B_1, B_2, C_1, C_2\right) \to \left(-A_1, -B_1, B_2, -C_1, -C_2\right).$

From qualitative predictions we look into the possibility of an analytical solution. In the case of steady motions, equations (23) - (27) can be solved in closed form. Setting the left hand side of equations (23) - (27) equal to zero, we get

$$\frac{a_1^2}{a^2}\,A_1 + R_T\,\frac{\pi\,\alpha}{a^2} B_1 - R_S\,\frac{\pi\,\alpha}{a^2} C_1 = 0\,, \quad (30)$$

$$\pi\,\alpha\,A_1 + a_2^{\,2}\,B_1 + \pi^2\alpha\,A_1 B_2 = 0\,, \quad (31)$$

$$8\pi^2 B_2 - \pi^2\alpha\,B_1 A_1 = 0\,, \quad (32)$$

$$\pi\,\alpha\,A_1 + \frac{a^2}{Le} C_1 + \pi^2\alpha\,A_1 C_2 + a^2\,Sr\frac{R_T}{R_S} B_1 = 0\,, \quad (33)$$

$$\frac{8\pi^2}{Le} C_2 - \pi^2\alpha\,A_1 C_1 + 8\pi^2\,Sr\frac{R_T}{R_S} B_2 = 0\,. \quad (34)$$

Writing B_1 , B_2, C_1 and C_2 in terms of A_1, using equations (30) - (34) and substituting these in equation (30), with $\frac{A_1^2}{8} = x$, we obtain

$$\mathrm{a}_1\,x^2 + \mathrm{b}_1\,x + \mathrm{c}_1 = 0\,, \quad (35)$$

where

$$\mathrm{a}_1 = a_1^{\,2}\,\pi^4\alpha^4\,Le^2,$$

$$\mathrm{b}_1 = a_1^{\,2}\,a_2^{\,2}\pi^4\alpha^2\,Le^2 + a^2\,a_1^{\,2}\pi^2\alpha^2 + R_T\,\pi^4\alpha^4 Le^2\left(Sr - 1\right) + R_S\,\pi^4\alpha^4 Le\,,$$

$$c_1 = a^2\, a_1^{\,2} a_2^{\,2} - R_T\, \pi^2\alpha^2 a^2 \left(1 + Sr\, Le\right) + \pi^4\alpha^2\, Le\, R_S\, a_2^{\,2}\,.$$

The required root of equation (35) is,

$$x = \frac{1}{2\,a_1}\left\{-\,b_1 + \left(b_1^2 - 4\,a_1\,c_1\right)^{1/2}\right\}. \tag{36}$$

3. 1 Finite amplitude motions

When we let the radical in the above equation to vanish, we obtain the expression for finite amplitude Rayleigh number R_T^f , which characterizes the onset of finite amplitude steady motions. The finite amplitude Rayleigh number can be obtained in the form

$$R_T^f = \frac{1}{2x_1}\left\{-x_2 + \left(x_2^2 - 4x_1x_3\right)^{1/2}\right\}, \tag{37}$$

where

$$x_1 = Le^4\pi^8\left(-1 + Sr\right)^2 \alpha^8\ ,$$
$$x_2 = 2\,Le^2\,\pi^6\alpha^6\left(a^2a_1^2\left(1 + Sr + 2LeSr\right) + Le\left(-1 + Sr\right)\left(a_1^2\,a_2^2\,Le + \pi^2 R_S\,\alpha^2\right)\right),$$
$$x_3 = \pi^4\alpha^4\left(a_1^2\left(a^2 - a_2^2\,Le\right)\left(a^2 + a_2^2\,Le\right) + Le\,\pi^2\,R_S\,\alpha^2\right)^2.$$

The expression for the finite amplitude Rayleigh number given by equation (37) is evaluated for different values of the parameters and the results are discussed in section 4.

3.2 Heat and mass transports

In the study of convection in fluids, the quantification of heat and mass transport is important. That is because the onset of convection, as Rayleigh number is increased, is more readily detected by its effect on the heat and mass transport. In the basic state, heat and mass transport is by conduction alone.

If H and J are the rate of heat and mass transport per unit area respectively, then

$$H = -\,D_1\left\langle\frac{\partial T_{total}}{\partial z}\right\rangle_{z=0}, \tag{38}$$

$$J = -D_2\left\langle\frac{\partial S_{total}}{\partial z}\right\rangle_{z=0} -\ D_3\left\langle\frac{\partial T_{total}}{\partial z}\right\rangle_{z=0}, \tag{39}$$

where the angular bracket $\langle....\rangle$ corresponds to a horizontal average with the definition of

$$T_{total} = T_0 - \Delta T\,\frac{z}{d} + T\left(x,z,t\right), \tag{40}$$

$$S_{total} = S_0 - \Delta S\,\frac{z}{d} + S\left(x,z,t\right). \tag{41}$$

Substituting equations (21) and (22) into equations (40) and (41) respectively and using the resultant equations in equations (38) and (39), we get

$$H = \frac{D_3\,\Delta T}{d}\left(1 - 2\,\pi\,B_2\right), \tag{42}$$

$$J = \frac{D_2\,\Delta S}{d}\left[\left(1 - 2\pi\,C_2\right) + Sr\,Le\frac{R_T}{R_S}\left(1 - 2\,\pi\,B_2\right)\right]. \tag{43}$$

The thermal Nusselt number is given by

$$Nu = \frac{H}{D_3\,\Delta T / d} = \left(1 - 2\,\pi\,B_2\right), \tag{44}$$

Similarly, solute Nusselt number is defined by

$$Sh=\frac{J}{D_3\ \Delta S/d}=\left[\left(1-2\pi C_2\right)+Sr\,Le\frac{R_T}{R_S}\left(1-2\pi B_2\right)\right]. \tag{45}$$

Writing B_2 and C_2 in terms of A_1, using equations (30) - (34) and substituting in equations (40) and (41) respectively,

we obtain

$$Nu=1+\frac{2\pi^2\alpha^2 x}{a_2^2+\pi^2\alpha^2 x}\quad , \tag{46}$$

$$Sh=1-2\pi\left[\frac{Le\,\pi R_T\,Sr\,\alpha^2 x}{R_S\left(a_2^2+\pi^2\alpha^2 x\right)}-\frac{Le\pi\ \alpha^2 x}{\left(\dfrac{a^2}{Le}+Le\pi^2\alpha^2 x\right)}\left(1-\frac{a^2 R_T\,Sr}{R_S\left(a_2^2+\pi^2\alpha^2 x\right)}+\frac{Le\,\pi^2 R_T\,Sr\,\alpha^2 x}{R_S\left(a_2^2+\pi^2\alpha^2 x\right)}\right)\right]+Sr\,Le\frac{R_T}{R_S}\left[1+\frac{2\pi^2\alpha^2 x}{a_2^2+\pi^2\alpha^2 x}\right]. \tag{47}$$

The second term on the right-hand side of equations (46) and (47) represent the convective contribution to heat and mass transport respectively.

4. RESULTS AND DISCUSSION

The double diffusive convection in a horizontal anisotropic porous layer saturated with Boussinesq fluid, which is heated and salted from below in the presence of Soret effect is studied analytically using nonlinear stability analyses. The effect of mechanical anisotropy parameter, thermal anisotropy parameter, Lewis number and Soret parameter on finite amplitude convection and heat and mass transfer are shown graphically and the results are discussed in this section.

Figures 1– 4 show the effect of various governing parameters on the finite amplitude convection. We find from Fig.1 that the effect of mechanical anisotropy parameter ξ is to destabilize the system in finite amplitude mode. From Fig. 2 we observe that the thermal anisotropy parameter η has stabilizing effect on the system for small values of the solute Rayleigh number while this trend reverses for large solute Rayleigh number. The effect of Lewis number Le on the finite amplitude motions is shown in Fig.3. We find that for small values of solute Rayleigh number the Lewis number has a stabilizing effect while for large solute Rayleigh number, it has a destabilizing effect. Fig.4 displays the effect of Soret parameter on the finite amplitude motions. We find from this figure that for small values of the solute Rayleigh number the negative Soret number has stabilizing effect and positive Soret number has destabilizing effect. However this trend reverses for large solute Rayleigh number.

The effect of mechanical anisotropy ξ on both the thermal Nusselt number and Sherwood number is shown in Fig. 5 when other parameters are fixed as, $\eta=0.5$, $R_S=100$, $S\text{r}=-0.01$ and $Le=1.0$. From Figs. 6 & 7 we observe that the heat transfer increases with increasing ξ. A similar effect is found for the thermal anisotropy parameter η and the Lewis number Le. Fig. 8 displays the effect of Soret parameter on both the thermal Nusselt number and Sherwood number. We find that a negative Soret parameter increases the heat transfer while positive Soret parameter decreases the heat transfer.

5. CONCLUSIONS

The onset of thermal convection in a two-component Boussinesq fluid saturated anisotropic porous layer which is heated and salted from below in the presence of Soret coefficient is investigated analytically using nonlinear theory. From this study the following conclusions are drawn:

- The value of critical Rayleigh number increases asymptotically with R_S to indicate the stabilizing effect of the solute Rayleigh number on finite amplitude mode.
- The effect of anisotropic properties is felt only for small values of R_S. In each mode the effect of the mechanical anisotropy parameter ξ is to destabilize the system while the effect of thermal anisotropy parameter η is to stabilize the system

- The effect of Le is destabilize the system in the finite amplitude mode.
- In case of finite amplitude convection, for small values of the solute Rayleigh number the negative Soret parameter has stabilizing effect while positive Soret number has destabilizing effect. However this trend reverses for large solute Rayleigh number.
- The anisotropy parameters and Lewis number increases the heat and mass transfer.
- The negative Soret parameter increases the heat transfer while positive Soret parameter decreases the heat transfer. A reverse trend is observed in case of mass transfer.

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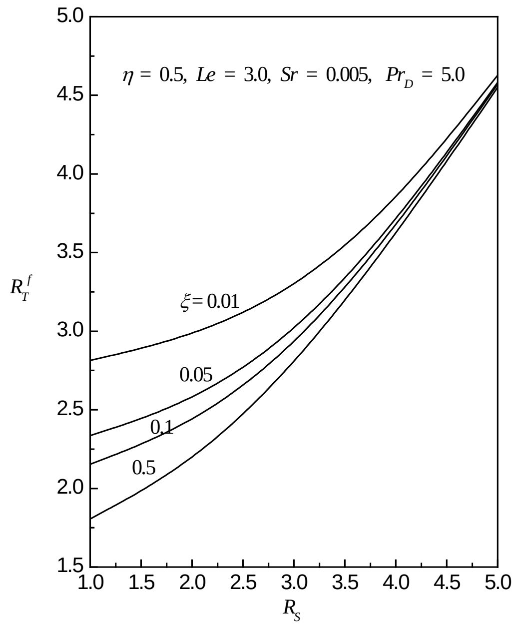

Fig. 1: Variation of finite amplitude Rayleigh number R_T^f with solute Rayleigh number R_S for different values of mechanical anisotropy parameter ξ.

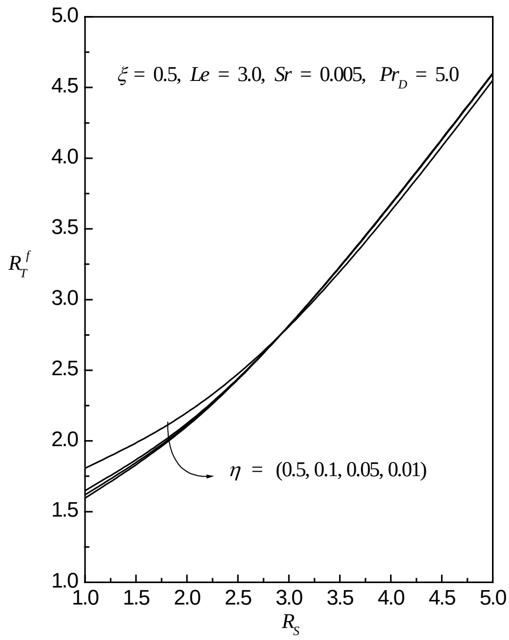

Fig.2: Variation of finite amplitude Rayleigh number R_T^f with solute Rayleigh number R_S for different values of thermal anisotropy parameter η.

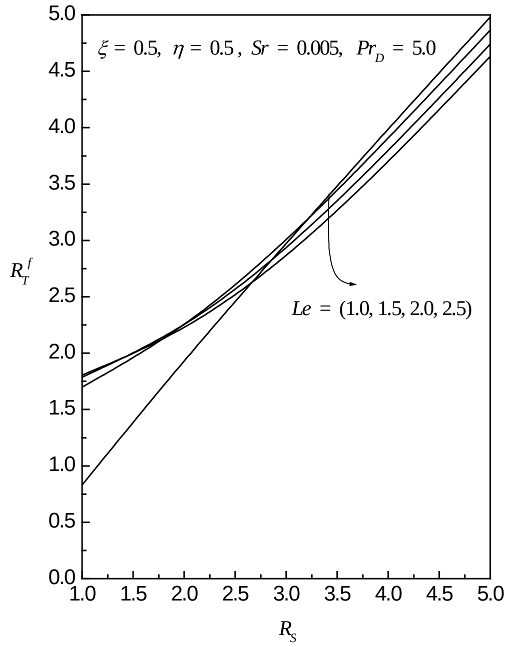

Fig.3: Variation of finite amplitude Rayleigh number R_T^f with solute Rayleigh number R_S for different values of Lewis number Le.

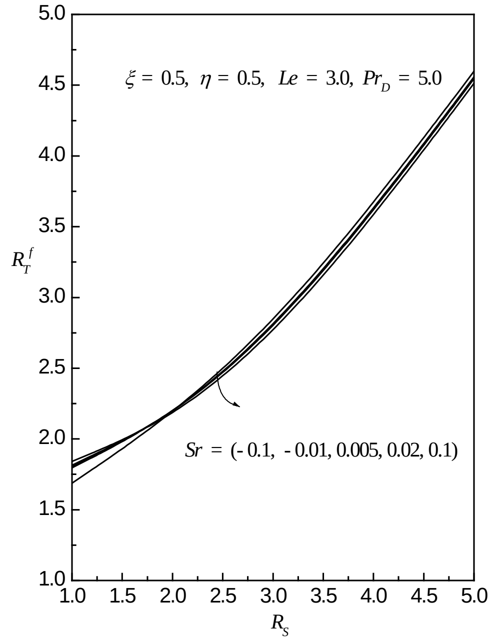

Fig.4: Variation of finite amplitude Rayleigh number R_T^f with solute Rayleigh number R_S for different values of Soret parameter Sr.

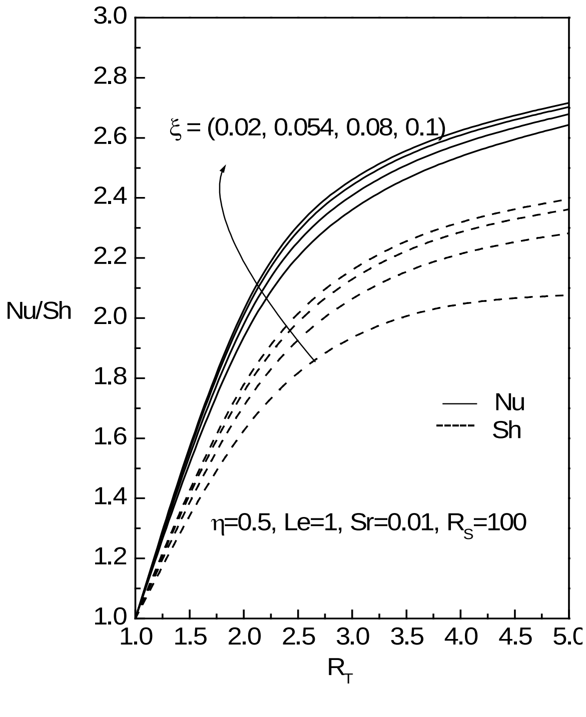

Fig. 5: Variation of thermal Nusselt number Nu and Sherwood number Sh with Rayleigh number R_T for different values of mechanical anisotropy parameter ξ.

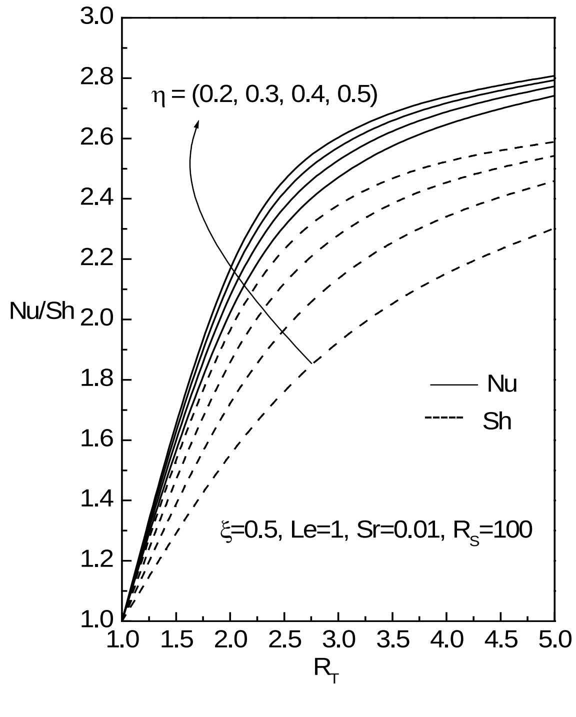

Fig.6: Variation of thermal Nusselt number Nu and Sherwood number Sh with Rayleigh number R_T for different values of thermal anisotropy parameter η.

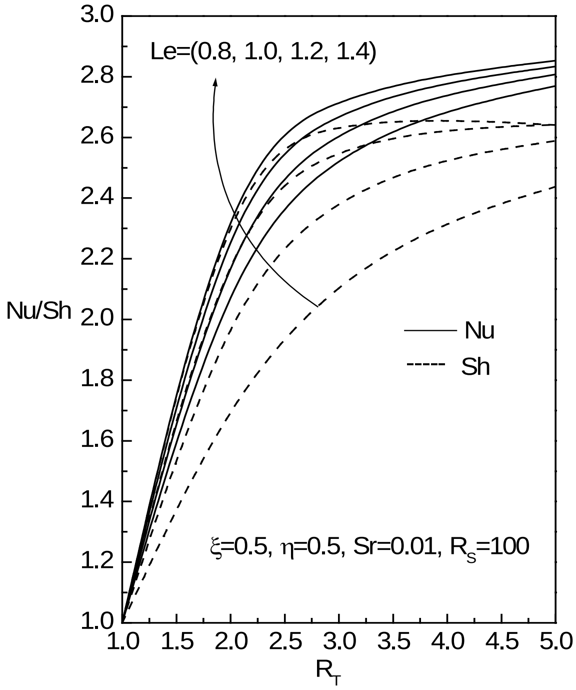

Fig.7: Variation of thermal Nusselt number Nu and Sherwood number Sh with Rayleigh number R_T for different values of Lewis number Le.

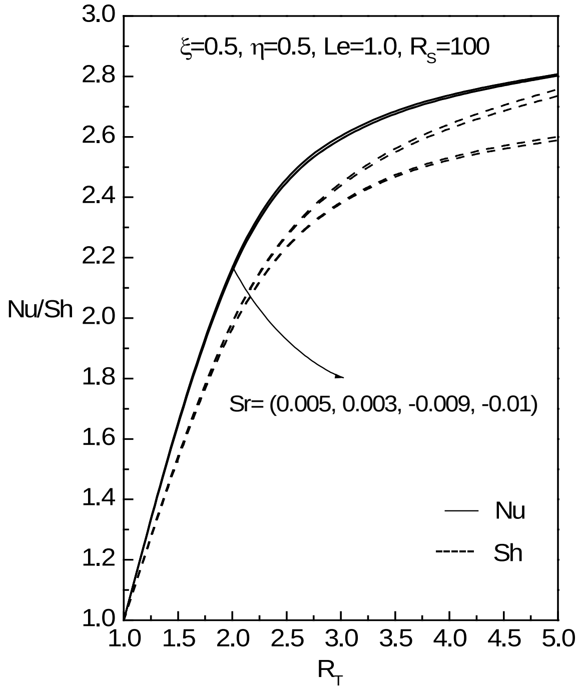

Fig. 8: Variation of thermal Nusselt number Nu and Sherwood number Sh with Rayleigh number R_T for different values of Soret parameter Sr.

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