

## ON SQUARING A NUMBER

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### ABSTRACT

*In this paper we give a method of finding the square of an integer and illustrate it by giving examples.*

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### INTRODUCTION AND STATEMENT OF RESULT

Square of an integer means multiple it by itself in our routine life squaring a number is an important problem. A square number can end only with digits 0, 1,4,6,9, or 25 in base 10, as follows: [1, 2]

1. If the last digit of a number is 0, its square ends in an even number of 0s (so at least 00) and the digits preceding the ending 0s must also form a square.
2. If the last digit of a number is 1 or 9, its square ends in 1 and the number formed by its preceding digits must be divisible by four.
3. If the last digit of a number is 2 or 8, its square ends in 4 and the preceding digit must be even.
4. If the last digit of a number is 3 or 7, its square ends in 9 and the number formed by its preceding digits must be divisible by four.
5. If the last digit of a number is 4 or 6, its square ends in 6 and the preceding digit must be odd.
6. If the last digit of a number is 5, its square ends in 25 and the preceding digits must be 0, 2, 06, or 56.

### IN THE PRESENT PAPER WE GIVE METHOD OF GETTING THE SQUARE OF ANY INTEGER.

Let  $m = (a_n a_{n-1} a_{n-2} \dots a_2 a_1)_{10}$  be a number in the decimal system (1)

Then for  $n = 1, ie(a_1)$

$$m^2 = a_1^2$$

For  $n = 2, ie(a_2 a_1)$

$$m^2 = (a_2)^2 (2a_1 a_2)^2 (a_1)^2$$

For  $n = 3, ie(a_3 a_2 a_1)$

$$m^2 = (a_3)^2 (2a_2 a_3)(2a_1 a_3 + a_2^2)(2a_1 a_2) a_1^2$$

For  $n = 4, ie(a_4 a_3 a_2 a_1)$

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$$m^2 = (a_4)^2 (2a_3a_4)(2a_2a_4 + a_3^2) [2(a_1a_4 + a_2a_3)] (2a_1a_2)(a_1)^2$$

In general the square of every integer "m" given by equation first can be expressed as

$$(a_n)^2 (2a_n a_{n-1}) \dots [2(a_1a_n + a_2a_{n-1})] [2a_1a_3 + a_2^2] (2a_1a_2)(a_1)^2$$

At each stage we have to carry on the digits to the next place if it is more than one digit number in calculation

#### FOR 2 DIGIT NUMBER

##### Examples

$$(21)^2$$

$$a_2 = 2, a_1 = 1$$

$$\begin{aligned} (m)^2 &= (a_2)^2 (2a_1a_2)(a_1)^2 \\ &= (2)^2 (2 \times 1 \times 2)(1)^2 \\ &= 441 \end{aligned}$$

$$(99)^2$$

$$a_2 = 9, a_1 = 9$$

$$\begin{aligned} (m)^2 &= (a_2)^2 (2a_1a_2)(a_1)^2 \\ &= (9)^2 (2 \times 9 \times 9)(9)^2 (81)(162)(81) \\ &= 9801 \end{aligned}$$

$$(76)^2$$

$$a_2 = 7, a_1 = 6$$

$$\begin{aligned} (m)^2 &= (a_2)^2 (2a_1a_2)(a_1)^2 \\ &= (7)^2 (2 \times 7 \times 6)(6)^2 \\ &= (49)(84)(36) \\ &= 5776 \end{aligned}$$

#### FOR 3 DIGIT NUMBERS

$$(m)^2 = (a_3)^2 (2a_2a_3)(2a_1a_3 + a_2^2)(2a_1a_2)(a_1)^2$$

$$(126)^2$$

$$a_1 = 6, a_2 = 2, a_3 = 1$$

$$\begin{aligned} &= (1)^2 (2 \times 2 \times 1)(2 \times 6 \times 1 + 2^2)(2 \times 6 \times 2)(6)^2 \\ &= (1)(4)(16)(24)(36) \\ &= 15876 \end{aligned}$$

$$(269)^2$$

$$a_1 = 9, a_2 = 6, a_3 = 2$$

$$\begin{aligned} &= (2)^2 (2 \times 6 \times 2)[(2 \times 9 \times 2) + 6^2](2 \times 9 \times 6)(9)^2 \\ &= (4)(24)(72)(108)(81) \\ &= 72361 \end{aligned}$$

$$(987)^2$$

$$\begin{aligned} a_1 &= 7, a_2 = 8, a_3 = 9 \\ &= (9)^2 (2 \times 8 \times 9) [(2 \times 7 \times 9) + 8^2] (2 \times 7 \times 8) (7)^2 \\ &= (81)(144)(190)(112)(49) \\ &= 974169 \end{aligned}$$

**FOR FOUR DIGIT NUMBERS**

$$(m)^2 = (a_4)^2 (2a_3a_4) (2a_2a_4 + a_3^2) [2(a_1a_4 + a_2a_3)] (2a_1a_3 + a_2^2) (2a_1a_2) (a_1)^2$$

$$(1376)^2$$

$$\begin{aligned} a_1 &= 6, a_2 = 7, a_3 = 3, a_4 = 1 \\ &= (1)^2 (2 \times 3 \times 1) [(2 \times 7 \times 1) + 3^2] [2[(6 \times 1) + (7 \times 3)]] [(2 \times 6 \times 3) + 7^2] (2 \times 6 \times 7) (6)^2 \\ &= (1)^2 (2 \times 3 \times 1) [(2 \times 7 \times 1) + 3^2] [2[(6 \times 1) + (7 \times 3)]] [(2 \times 6 \times 3) + 7^2] (2 \times 6 \times 7) (6)^2 \\ &= 1893376 \end{aligned}$$

$$(9876)^2$$

$$\begin{aligned} a_1 &= 6, a_2 = 7, a_3 = 8, a_4 = 9 \\ &= (9)^2 (2 \times 8 \times 9) [(2 \times 7 \times 9) + 8^2] [2[(6 \times 9) + (7 \times 8)]] [(2 \times 6 \times 8) + 7^2] [(2 \times 6 \times 7) (6)^2] \\ &= (81)(144)(190)(220)(145)(84)(36) \\ &= 9735376 \end{aligned}$$

**FOR 5 DIGIT NUMBER**

$$(m)^2 = [2(a_2a_5 + a_3a_4)] [2(a_1a_5 + a_2a_4) + a_3^2] [2(a_1a_4 + a_2a_3)] (2a_1a_3 + a_2^2) (2a_1a_2) (a_1)^2$$

$$(a_5)^2 (2a_4a_5) (2a_3a_5 + a_4^2)$$

$$(13546)^2$$

$$\begin{aligned} a_1 &= 6, a_2 = 4, a_3 = 5, a_4 = 3, a_5 = 1 \\ &= (1)^2 (2 \times 3 \times 1) [(2 \times 5 \times 1) + 3^2] [2[(4 \times 1) + (5 \times 3)]] [2[(6 \times 1) + (4 \times 3) + 5^2] [2(6 \times 3 + 4 \times 5)]] \\ &= [2(6 \times 5) + 4^2] (2 \times 6 \times 4) (6)^2 \\ &= (1)(6)(19)(38)(61)(76)(76)(48)(36) \\ &= 183494116 \end{aligned}$$

**FOR SIX DIGIT NUMBER**

$$(m)^2 = [2(a_1a_5 + a_2a_4) + a_3^2] [2(a_1a_4 + a_2a_3)] [2(a_1a_3 + a_2^2) (2a_1a_2) (a_1)^2]$$

$$[2a_3a_6 + a_4a_5] [2(a_2a_6 + a_4a_5) + a_4^2] [2(a_1a_6 + a_2a_5 + a_3a_4)]$$

$$(a_6)^2 (2a_5a_6) (2a_4a_6 + a_5^2)$$

$$(376475)^2$$

$$\begin{aligned} a_1 &= 5, a_2 = 7, a_3 = 4, a_4 = 6, a_5 = 7, a_6 = 3 \\ &= [2(5 \times 7) + (7 \times 6) + 4^2] [2(5 \times 6) + (7 \times 4)] [2(5 \times 4) + 7^2] [2(5 \times 7)] (5)^2 \\ &\quad [2(6 \times 3) + 7^2] [2((4 \times 3) + 6(7))] [2((7 \times 3) + (4 \times 7) + 6^2)] [2((5 \times 3) + (7 \times 7) + (4 \times 6))] \\ &\quad (3)^2 [2(7 \times 3)] \\ &= (9)(42)(85)(108)(134)(176)(170)(116)(89)(70)(25) \\ &= 141733425625 \end{aligned}$$

**FOR SEVEN DIGIT NUMBER**

$$\begin{aligned} (m)^2 &= [2(a_1 a_5 + a_2 a_4) + a_3^2] [2(a_1 a_4 + a_2 a_3)] (2a_1 a_3 + a_2^2) (2a_1 a_2) (a_1)^2 \\ &\quad [2(a_2 a_7 + a_3 a_6 + a_4 a_5)] [2(a_1 a_7 + a_2 a_6 + a_3 a_5) + a_4^2] [2(a_1 a_6 + a_2 a_5 + a_3 a_4)] \\ &\quad (a_7)^2 2(a_6 a_7) (2a_5 a_7 + a_6^2) [2(a_4 a_2 + a_5 a_6)] [2(a_3 a_7 + a_4 a_6) + a_5^2] \end{aligned}$$

**Examples**

$$(1111111)^2$$

$$\begin{aligned} a_1 &= 1, a_2 = 1, a_3 = 1, a_4 = 1, a_5 = 1, a_6 = 1, a_7 = 1 \\ &= [2((1 \times 1) + (1 \times 1)) + 1^2] [2((1 \times 1) + (1 \times 1))] [2(1 \times 1) + 1^2] (2(1 \times 1)) (1)^2 \\ &= [2((1 \times 1) + (1 \times 1) + (1 \times 1))] (2((1 \times 1) + (1 \times 1) + (1 \times 1)) + 1^2) (2(1 \times 1) + (1 \times 1) + (1 \times 1)) \\ &= (2((1 \times 1) + 1^2)) (2((1 \times 1) + (1 \times 1) + 1^2)) (1)^2 (2(1 \times 1)) \\ &= 1234567654321 \end{aligned}$$

$$(2132143)^2$$

$$\begin{aligned} a_1 &= 3, a_2 = 4, a_3 = 1, a_4 = 2, a_5 = 3, a_6 = 1, a_7 = 2 \\ &= [2((3 \times 3) + (4 \times 2)) + 1^2] [2((3 \times 2) + (4 \times 1))] [2(3 \times 1) + 4^2] (2(3 \times 4)) (3)^2 \\ &\quad [2((4 \times 2) + (1 \times 1) + (2 \times 3))] (2((3 \times 2) + (4 \times 1) + (1 \times 3)) + 2^2) (2(3 \times 1) + (4 \times 3) + (1 \times 2)) \\ &\quad (2)^2 (2((1 \times 2))) (2((3 \times 2) + 1^2)) (2(2 \times 2) + (3 \times 1)) (2((1 \times 2) + (2 \times 1)) + 3^2) \\ &= (4)(4)(13)(14)(17)(30)(30)(34)(35)(20)(22)(24)(9) \\ &= 4546033772449 \end{aligned}$$

**CONCLUSION**

From the above examples we conclude that the square of every integer can be expressed as

$$(a_n)^2 (2a_n a_{n-1}) \dots [2(a_1 a_4 + a_2 a_3)] [2a_1 a_3 + a_2^2] (2a_1 a_2) (a_1)^2$$

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