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INVENTORY SYSTEM WITH RANDOM ARRIVAL OF SHIPMENTS TO MAXIMISE THE REVENUE

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ABSTRACT

In single period inventory models, neither excess demand nor excess inventory can be carried forward to the next period. In stochastic multi period models, an order is placed at some point of time during the cycle. At the time an order is placed, the amount of inventory which will be on hand when replenishment arrives is unknown. In this paper, we propose a variation of these models to determine the optimal economic order quantity without using lost sales. Shipments arrive according to a Poisson process. The status of the inventory becomes known at the time of the arrival of the shipment, and the selection of order quantity is made at that time. However the arrival time of future shipments is uncertain.

Keywords: *Lost sales, Poisson processes Random replenishment.*

MSC Subject Classification No: *90B05.*

INTRODUCTION

The multi period inventory problem of stochastic demand and shortages during lead time has been well studied. The single period inventory problem with either instantaneous or constant demand, with and without backorders, has also been studied in the context of the classical newsboy problem and its extensions. Some of these models are detailed by Silver and Peterson [1], Elsayed and Boucher [2] and Tersine [3], Rose [4] considers the problem of deterministic demand and random replenishment in a reversal of the usual newsboy problem. In this paper, we consider a variation of these models.

In order to make assumptions that are relevant to practical situations, we consider multi period inventory problem. Shipments arrive at random according to a poisson process. From these shipments, orders are filled subject to the inventory on hand. The arrival time of the next shipment is unknown, and the demand during lead time is either constant or probabilistic. Consequently, the amount of inventory which will on hand when the next shipment arrives is unknown, it may in fact be negative, meaning that some stock is backordered. The inventory level becomes known upon arrival of shipments. Under these circumstances the following model has been developed to find the economic order quantity.

MODEL

A typical inventory situation is considered, Shipments arrive according to a Poisson process with inter arrival time distributed as follows:

$$f(\tau) = \lambda e^{-\lambda\tau}, \qquad \tau \geq 0,$$

where $\dfrac{1}{\lambda}$ is the expected interarrival time between consecutive shipments. Let S be the predetermined maximum inventory or capacity level. Every time a shipment arrives, an order of size Q is placed so that the stock on hand becomes S. The arriving shipments are assumed to be sufficient to fill the orders, therefore, immediate replenishment is necessary as soon as shipment arrives. Thus, Q is a random variable which depends on the demand between successive arrivals. The demand (D) per unit time is assumed to be constant throughout the planning horizon. If the demand between arrival of two consecutive shipments is less than S, there is a surplus of inventory at the end of the

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cycle. The cost of procurement is C_p per unit, and is assumed to be independent of Q. Let the items be sold at mC_p, so that the profit margin per unit sold, be (m-1) C_p. The cost of ordering (C_o) is constant and is ignored because it is incurred every time a shipment arrives. A cost of C_h per unit is incurred for every time an item is carried as inventory.

In traditional inventory models where there are no lost sales, demand is completely satisfied, and therefore, the minimization of total inventory-related costs becomes a valid objective However, in the present case, both the procurement costs and revenue incurred out of demand depend on the order level S, rather than the total demand. The order level S, in turn, depends on the fraction of demand actually fulfilled. The objective in this situation is the maximization of net revenue, rather than the minimization of total costs. The problem is to find the order level S that maximizes the total net revenue by satisfying a fraction of the demand between successive arrivals. M.A. Rahim [5] found the total net revenue by allowing backorders and lost sales. In his study, he concluded that when the backordering cost and lost sales cost is high, the total revenue may decreases. Hence, in this study to improve the net revenue, a special cost C_k may be included when the demand is greater than the stock in hand and considering without lost sales.

The total expected net revenue per cycle is comprised of the following:

C_1. Expected net revenue generated due to satisfied demand.

C_2. Expected cost of holding inventory.

C_3. Expected cost of backorders.

Hence the total expected net revenue per cycle is R= C_1 - C_2 - C_3, where $S \geq D\tau$, procurement (in next period) is $D\tau$, and $S \leq D\tau$, procurement is S + $\beta(D\tau - S)$.

C_1. Expected net revenue per cycle

If we ignore the cost of ordering, the net revenue is considered to be the difference in the revenue obtained from items that are sold and the cost incurred on items that are procured. Thus the expected net revenue per inventory cycle depends on the order level, the stock on hand at the end of the cycle, and the profit margin per unit of satisfied demand. In addition, when the demand exceeds the stock in hand, a special holding cost C_k is included to maximize the revenue.

Average revenue per cycle is:

$$C_1 = (m-1)\, C_p \left[\int_0^{\frac{S}{D}} D\tau\lambda e^{-\lambda\tau} d\tau + \int_{\frac{S}{D}}^{\infty} (S + C_k(D\tau - S)\beta)\lambda e^{-\lambda\tau} d\tau \right]$$

$$= (m-1)\, C_p \left[D\lambda \int_0^{\frac{S}{D}} \tau e^{-\lambda\tau} d\tau + S\lambda \int_{\frac{S}{D}}^{\infty} e^{-\lambda\tau} d\tau + C_k\beta\lambda \int_{\frac{S}{D}}^{\infty} (D\tau - S) e^{-\lambda\tau} d\tau \right]$$

$$= (m-1)\, C_p \left[D\lambda \left(\frac{1}{\lambda^2} \left\{ 1 - (1 + \frac{\lambda S}{D}) e^{-\lambda \frac{S}{D}} \right\} \right) \right] + S e^{-\lambda \frac{S}{D}} + C_k \left[\frac{D}{\lambda} \beta e^{-\frac{\lambda S}{D}} + \beta C_k D \frac{S}{D} e^{-\frac{\lambda S}{D}} - C_k S \beta e^{-\frac{\lambda S}{D}} \right]$$

$$= (m-1)\, C_p \left[\frac{D}{\lambda} - \frac{D}{\lambda} e^{-\frac{\lambda S}{D}} (1 + \frac{\lambda S}{D}) + S e^{-\frac{\lambda S}{D}} + C_k D \beta \frac{e^{-\frac{\lambda S}{D}}}{\lambda} \right]$$

$$= (m-1)\, C_p \left[\frac{D}{\lambda} - \frac{D}{\lambda} e^{-\frac{\lambda S}{D}} + C_k \frac{D}{\lambda} \beta e^{-\frac{\lambda S}{D}} \right]$$

$$= (m-1)\, C_p \; \frac{D}{\lambda}\left[1 - e^{-\frac{\lambda S}{D}} (C_k \beta - 1) \right]$$

C_2. Expected inventory holding cost per cycle

Inventory holding cost is incurred only when the quantity on hand is positive and is computed on the average inventory level. The expected inventory holding cost per cycle is given by.

Average inventory level per cycle is,

$$C_2 = C_h \left[\int_0^{\frac{S}{D}} \left(S\tau - \frac{D\tau^2}{2} \right) \lambda e^{-\lambda\tau} d\tau + \int_{\frac{S}{D}}^{\infty} \left(\frac{S^2}{2D} \right) \lambda e^{-\lambda\tau} d\tau \right]$$

$$= C_h \left[S\lambda \int_0^{\frac{S}{D}} \tau e^{-\lambda\tau} d\tau - \frac{D\lambda}{2} \int_0^{\frac{S}{D}} \tau^2 e^{-\lambda\tau} d\tau \right] + C_h \left[\frac{S^2 \lambda}{2D} \int_{\frac{S}{D}}^{\infty} e^{-\lambda\tau} d\tau \right]$$

$$= C_h \left[S\lambda \left(-\frac{S}{D\lambda} e^{-\lambda\frac{S}{D}} + \frac{1}{\lambda} \int_0^{\frac{S}{D}} e^{-\lambda\tau} d\tau \right) \right] - C_h \frac{D\lambda}{2} \left[\tau^2 \frac{e^{-\lambda\tau}}{-\lambda} \right]_0^{\frac{S}{D}} - C_h \frac{D\lambda}{2} \int_0^{\frac{S}{D}} \frac{e^{-\lambda\tau}}{\lambda} 2\tau d\tau + C_h \frac{S^2\lambda}{2D} \left(\frac{1}{\lambda} e^{-\lambda\frac{S}{D}} \right)$$

$$= C_h \left[-\frac{S^2}{D} e^{-\lambda\frac{S}{D}} - \frac{S}{\lambda} e^{-\frac{S}{D}} \right] + C_h \frac{D}{2} \frac{S^2}{D^2} e^{-\lambda\frac{S}{D}} + C_h \frac{S}{\lambda} e^{-\lambda\frac{S}{D}} + C_h \frac{1}{\lambda} \int_0^{\frac{S}{D}} e^{-\lambda\tau} d\tau + C_h \frac{S^2\lambda}{2D} \left(\frac{1}{\lambda} e^{-\lambda\frac{S}{D}} \right)$$

$$= C_h \left[-\frac{S^2}{D} e^{-\lambda\frac{S}{D}} - \frac{S}{\lambda} e^{-\lambda\frac{S}{D}} \right] + C_h \frac{D}{2} \frac{S^2}{D^2} e^{-\lambda\frac{S}{D}} + C_h \frac{S^2\lambda}{2D} \left(\frac{1}{\lambda} e^{-\lambda\frac{S}{D}} \right) + C_h \frac{S}{\lambda} e^{-\lambda\frac{S}{D}} + C_h \left(\frac{1}{\lambda^2} \left\{ e^{-\lambda\frac{S}{D}} - 1 \right\} \right)$$

$$= C_h \left[-\frac{S^2}{D} - \frac{S}{\lambda} + \frac{S^2}{2D} + \frac{S^2}{2D} + \frac{S}{\lambda} + \frac{1}{\lambda^2} \right] e^{-\lambda\frac{S}{D}} - C_h \frac{1}{\lambda^2}$$

$$C_2 = C_h \frac{D}{\lambda^2} \left[e^{-\lambda\frac{S}{D}} + \frac{\lambda S}{D} - 1 \right]$$

C_3. Expected backordering cost per cycle

The expected backordering cost per cycle is computed based on the average level of backorders and the period of time the customers had to wait. The expected cost per cycle is as follows:

Maximum level of backorders = $(D\tau - S)\beta$

Average backorders per cycle is:

$$C_3 = C_b \left[\int_{\frac{S}{D}}^{\infty} \frac{\beta (D\tau - S)^2}{2D} \lambda e^{-\lambda\tau} d\tau \right]$$

$$= C_b \frac{\beta\lambda}{2D} \int_{\frac{S}{D}}^{\infty} (D\tau - S)^2 e^{-\lambda\tau} d\tau$$

$$= C_b \frac{\beta\lambda}{2D} \left[\left\{ (D\tau - S)^2 e^{-\lambda\tau} / -\lambda \right\}_{\frac{S}{D}}^{\infty} + \int_{\frac{S}{D}}^{\infty} \frac{e^{-\lambda\tau}}{\lambda} 2D(D\tau - S) d\tau \right]$$

$$= C_b \frac{\beta}{1} \int_{\frac{S}{D}}^{\infty} (D\tau - S) e^{-\lambda\tau} d\tau$$

$$= C_b \frac{\beta D}{\lambda} \int_{\frac{S}{D}}^{\infty} e^{-\lambda\tau} d\tau$$

$$C_3 = C_b \frac{\beta D}{\lambda^2} e^{-\lambda \frac{S}{D}}$$

Thus the total expected net revenue per cycle is computed as follows:

$$R = C_1 - C_2 - C_3$$

$$R = (m-1) C_p \frac{D}{\lambda} \left[1 - e^{-\frac{\lambda S}{D}} (C_k \beta - 1) \right] + C_h \frac{D}{\lambda^2} \left[e^{-\lambda \frac{S}{D}} + \frac{\lambda S}{D} - 1 \right] + C_b \frac{\beta D}{\lambda^2} e^{-\lambda \frac{S}{D}}$$

$$= \frac{D}{\lambda} \left[(m-1) C_p + \frac{C_h}{\lambda} - \frac{C_h S}{D} - e^{-\frac{\lambda S}{D}} \left\{ (m-1) C_p (C_k \beta - 1) + \frac{C_h}{\lambda} + \frac{C_b \beta}{\lambda} \right\} \right]$$

The maximum total expected net revenue per cycle occurs for some S where the first derivate

$$\frac{\partial R}{\partial S} = 0,$$

ie., $$\frac{\partial R}{\partial S} = 0 = \frac{D}{\lambda} \left[\frac{-C_h}{D} + \left\{ (m-1) C_p (C_k \beta - 1) + \frac{C_b}{\lambda} + \frac{C_b \beta}{\lambda} \right\} e^{-\frac{\lambda S}{D}} \frac{\lambda}{D} \right]$$

OR $$\frac{C_h}{\lambda} = \left\{ (m-1)(1 - C_k \beta) C_p + \frac{C_b}{\lambda} + \frac{C_b \beta}{\lambda} \right\} e^{-\frac{\lambda S}{D}}$$

0r $$\left\{ \lambda (m-1)(1 - C_k \beta) C_p + C_h + C_b \beta \right\} e^{-\frac{\lambda S}{D}} = C_h$$

or $$\frac{\lambda S^*}{D} = \log_e \left[\lambda (m-1)(1 - C_k \beta) C_p / C_h + 1 + \frac{C_b}{C_h} \beta \right]$$

or $$S^* = \frac{D}{\lambda} \log_e \left[\lambda (m-1)(1 - C_k \beta) C_p / C_h + 1 + \frac{C_b}{C_h} \beta \right]$$

at this value of S, the second derivate,

$\frac{\partial^2 R}{\partial S^2}$ = - $e^{-\frac{\lambda S}{D}} \frac{\lambda}{D}\left[(m-1)(1-C_k\beta)C_p + \frac{C_h}{\lambda} + \frac{C_b}{\lambda}\beta\right]$, is negative. Hence, the expected net revenue per cycle is a maximum, and its value is as follows:

$$R_{\max} = \frac{D}{\lambda}\left[(m-1)C_p + \frac{C_h}{\lambda} - \frac{C_h S}{D} - e^{-\frac{\lambda S}{D}}\left\{(1-C_k\beta)(m-1)C_p + (C_h + \beta C_b)/\lambda\right\}\right]$$

$$R_{\max} = \frac{D}{\lambda}\left[(m-1)C_p + \frac{C_h}{\lambda} - \frac{C_h S^*}{D} - \frac{C_h}{\lambda}\right]$$

$$R_{\max} = (m-1)C_p\frac{D}{\lambda} - \frac{C_h S^*}{\lambda}$$

The maximum total expected net revenue over the time is given by

$$T.R_{\max} = \lambda R_{\max} = (m-1)C_p D - SC_h$$

CONCLUSION:

In the above result, we have obtained the maximum total expected net revenue, comprised of inventory holding cost, backordering cost without lost sales cost. In the original work, the author found that, when the value of backordering cost increases, the revenue decreases. But in this study, it is concluded that when the backordering cost increases, the maximum revenue will not affect mostly. The case of limited capacity and the case of not ordering every time can be incorporated in a future study.

REFERENCES

[1] Silver EA, Peterson R, Decision systems for inventory management and production planning, 2nd edn. (1985), Wiley, New York.

[2] Elsayed EA, Boucher TO, Analysis and control of production systems, Prentice-Hall, (1985), New York.

[3] Tersine RJ, Principles of inventory and materials management, 3rd etn. North-Holland, (1988), Amsterdam

[4] Rose JH, The newsboy with known demand and uncertain replenishment: applications to quality control and container fill. Operations Res Lett (1992), 11:111-117.

[5] M. A. Rahim Inventory systems with random arrival of shipments, Int J Adv Manuf. Technol. (2006), 29:197-201.

[6] Babu Krishnaraj. R, Ramasamy. K, Inventory system with random arrival of shipments without shortages, Archimedes Journal of Mathematics (2012), Vol. 2, No.1, Paper 3 (to be appeared)

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