International Journal of Mathematical Archive-3(5), 2012, 1850-1853
IJMA Available online through www.ijma.info ISSN 2229 – 5046

NOTES ON INTUITIONISTIC FUZZY SUBHEMIRINGS OF A HEMIRING

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(Received on: 24-04-12; Accepted on: 14-05-12)

ABSTRACT

In this paper, we made an attempt to study the algebraic nature of an intuitionistic fuzzy subhemirings of a hemiring.

2000 AMS Subject classification: *03F55, 06D72, 08A72.*

Keywords: *Fuzzy set, fuzzy subhemiring, anti-fuzzy subhemiring, intuitionistic fuzzy set, intuitionistic fuzzy subhemiring, and pseudo intuitionistic fuzzy coset.*

INTRODUCTION:

There are many concepts of universal algebras generalizing an associative ring (*R; +; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (*R; +; .*) share the same properties as a ring except that (*R ; +*) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b + c) = a. b + a. c and (b + c) .a = b. a + c. a for all a, b and c in R. A semiring R is said to be additively commutative if a + b = b + a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0 + a = a = a + 0 and a. 0 = 0 = 0. a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A. Zadeh [15], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subsets (IFS) was introduced by K. T. Atanassov [4], as a generalization of the notion of fuzzy set. The notion of Fuzzy left *h*-ideals in hemirings with respect to a *s*-norm was introduced in **[2].** In this paper, we introduce the some Theorems in intuitionistic fuzzy subhemiring of a hemiring.

1. PRELIMINARIES

1.1 Definition: Let X be a non-empty set. A fuzzy subset A of X is a function A: X $\rightarrow$ [0, 1].

1.2 Definition: Let R be a hemiring. A fuzzy subset A of R is said to be a fuzzy subhemiring (FSHR) of R if it satisfies the following conditions:
(i) $A(x + y) \geq \min\{ A(x), A(y)\}$,
(ii) $A(xy) \geq \min\{ A(x), A(y)\}$, for all x and y in R.

1.3 Definition: Let R be a hemiring. A fuzzy subset A of R is said to be an anti-fuzzy subhemiring (AFSHR) of R if it satisfies the following conditions:
(i) $A(x + y) \leq \max\{ A(x), A(y) \}$,
(ii) $A(xy) \leq \max\{ A(x), A(y) \}$, for all x and y in R.

1.4 Definition: An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form $A = \{< x, \mu_A(x) , \nu_A(x) > / x\in X\}$, where $\mu_A:X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x\in X$ respectively and for every $x\in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.5 Definition: Let R be a hemiring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic fuzzy subhemiring (IFSHR) of R if it satisfies the following conditions:
(i) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(iii) $\nu_A(x + y) \leq \max\{\nu_A(x), \nu_A(y)\}$,
(iv) $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R.

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1.6 Definition: Let (R, +,.) be a hemiring. An intuitionistic fuzzy subhemiring A of R is said to be an intuitionistic fuzzy normal subhemiring (IFNSHR) of R if it satisfies the following conditions:
(i) $\mu_A(xy) = \mu_A(yx)$,
(ii) $\nu_A(xy) = \nu_A(yx)$, for all x and y in R.

1.5 Definition: If (R, +, **.**) and (R^l, +, **.**) are any two hemirings, then the function f: R → R^l is called a **homomorphism** if f(x+y) = f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R.

1.6 Definition: If (R, +, **.**) and (R^l, +, **.**) are any two hemirings, then the function f: R→ R^l is called an **anti-homomorphism** if f(x+y) = f(y)+f(x) and f(xy)=f(y)f(x), for all x and y in R.

1.7 Definition: Let (R, +,.) and (R^l, +, .) be any two hemirings. Then the function f: R → R^l be a hemiring homomorphism. If f is one-to-one and onto, then f is called a **hemiring isomorphism**.

1.8 Definition: Let (R, +,.) and (R^l, +, .) be any two hemirings. Then the function f: R → R^l be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a **hemiring anti-isomorphism**.

1.9 Definition: Let R and R^l be any two hemirings. Let f: R → R^l be any function and let A be an intuitionistic fuzzy subhemiring in R, V be an intuitionistic fuzzy subhemiring in f (R) = R^l, defined by $\mu_V(y) = \sup_{x\in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x\in f^{-1}(y)} \nu_A(x)$, for all x in R and y in R^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.10 Definition: Let A be an intuitionistic fuzzy subhemiring of a hemiring (R, +,) and a in R. Then the pseudo intuitionistic fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x) = p(a)\mu_A(x)$ and $((a\nu_A)^p)(x) = p(a)\nu_A(x)$, for every x in R and for some p in P.

2. INTUITIONISTIC FUZZY SUBHEMIRINGS OF A HEMIRING

2.1 Theorem: If A is an intuitionistic fuzzy subhemiring of a hemiring (R, +,.), then A is an intuitionistic fuzzy subhemiring of R.

Proof: Let A be an intuitionistic fuzzy subhemiring of a hemiring R.Consider A = {⟨ x, $\mu_A(x)$, $\nu_A(x)$⟩}, for all x in R, we take □A = B ={⟨ x, $\mu_B(x)$, $\nu_B(x)$⟩}, where $\mu_B(x) = \mu_A(x)$, $\nu_B(x) = 1- \mu_A(x)$. Clearly, $\mu_B(x+y) \geq \min\{\mu_B(x), \mu_B(y)\}$, for all x and y in R and $\mu_B(xy) \geq \min\{\mu_B(x), \mu_B(y)\}$, for all x and y in R. Since A is an intuitionistic fuzzy subhemiring of R, we have $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R, which implies that $1- \nu_B(x+y) \geq \min\{(1- \nu_B(x)), (1- \nu_B(y))\}$,which implies that $\nu_B(x+y) \leq 1- \min\{(1- \nu_B(x)), (1- \nu_B(y))\} = \max\{\nu_B(x), \nu_B(y)\}$.Therefore, $\nu_B(x+y) \leq \max\{\nu_B(x), \nu_B(y)\}$, for all x and y in R. And $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R, which implies that $1-\nu_B(xy)\geq\min(1- \nu_B(x)), (-\nu_B(y))\}$which implies that $\nu_B(xy) \leq 1 -\min\{(1-\nu_B(x)),(1-\nu_B(y))\}=\max\{\nu_B(x),\nu_B(y)\}$. Therefore, $\nu_B(xy) \leq \max\{\nu_B(x), \nu_B(y)\}$, for all x and y in R. Hence B = □ A is an intuitionistic fuzzy subhemiring of a hemiring R.

Remark: The converse of the above theorem is not true. It is shown by the following example:

Consider the hemiring Z_5 = {0, 1, 2, 3, 4} with addition modulo 5 and multiplication modulo 5 operations. Then A = {⟨ 0, 0.7, 0.2⟩, ⟨1, 0.5, 0.1⟩, ⟨2, 0.5, 0.4⟩, ⟨3, 0.5, 0.1⟩, ⟨4, 0.5, 0.4⟩} is not an intuitionistic fuzzy subhemiring of Z_5, but □ A = {⟨0, 0.7, 0.3⟩, ⟨1, 0.5, 0.5⟩,⟨2, 0.5, 0.5⟩,⟨3, 0.5, 0.5⟩,⟨4, 0.5, 0.5⟩} is an intuitionistic fuzzy subhemiring of Z_5.

2.2 Theorem: If A is an intuitionistic fuzzy subhemiring of a hemiring (R, +, .), then ◊A is an intuitionistic fuzzy subhemiring of R.

Proof: Let A be an intuitionistic fuzzy subhemiring of a hemiring R. That is A = {⟨ x, $\mu_A(x)$, $\nu_A(x)$⟩}, for all x in R. Let ◊A = B = {⟨x, $\mu_B(x)$, $\nu_B(x)$⟩}, where $\mu_B(x) = 1-\nu_A(x)$, $\nu_B(x) = \nu_A(x)$. Clearly, $\nu_B(x+y) \leq \max\{\nu_B(x), \nu_B(y)\}$, for all x and y in R and $\nu_B(xy) \leq \max\{\nu_B(x), \nu_B(y)\}$, for all x and y in R. Since A is an intuitionistic fuzzy subhemiring of R, we have $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R, which implies that $1-\mu_B(x+y) \leq \max\{(1- \mu_B(x)), (1- \mu_B(y))\}$which implies that $\mu_B(x+y) \geq 1- \max\{(1- \mu_B(x)), (1- \mu_B(y))\}= \min\{\mu_B(x), \mu_B(y)\}$.Therefore, $\mu_B(x+y) \geq \min\{\mu_B(x), \mu_B(y)\}$, for all x and y in R. And $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R,which implies that $1-\mu_B(xy) \leq \max\{(1-\mu_B(x)), (1-\mu_B(y))\}$,which implies that $\mu_B(xy) \geq 1 - \max\{(1-\mu_B(x)), (1-\mu_B(y))\}= \min\{\mu_B(x), \mu_B(y)\}$. Therefore, $\mu_B(xy) \geq \min\{\mu_B(x), \mu_B(y)\}$, for all x and y in R. Hence B = ◊A is an intuitionistic fuzzy subhemiring of a hemiring R.

Remark: The converse of the above theorem is not true. It is shown by the following example:

Consider the hemiring Z_5 = {0, 1, 2, 3, 4} with addition modulo 5 and multiplication modulo 5 operations. Then A = {⟨0, 0.5, 0.1⟩, ⟨1, 0.6, 0.4⟩, ⟨2, 0.5, 0.4⟩, ⟨3, 0.6, 0.4⟩, ⟨4, 0.5, 0.4⟩} is not an intuitionistic fuzzy subhemiring of Z_5, but ◊A = = {⟨0, 0.9, 0.1⟩,⟨1, 0.6, 0.4⟩,⟨2, 0.6, 0.4⟩,⟨3, 0.6, 0.4⟩,⟨4, 0.6, 0.4⟩} is an intuitionistic fuzzy subhemiring of Z_5.

2.3 Theorem: Let (R, +, .) be a hemiring and A be a non empty subset of R. Then A is a subhemiring of R if and only if B = $\langle \chi_A, \overline{\chi_A} \rangle$ is an intuitionistic fuzzy subhemiring of R, where χ_A is the characteristic function.

Proof: Let (R, +, .) be a hemiring and A be a non empty subset of R. First let A be a subhemiring of R. Take x and y in R.

Case (i): If x and y in A, then x+y, xy in A, since A is a subhemiring of R, χ_A(x) = χ_A(y) = χ_A(x+y) = χ_A(xy) =1 and $\overline{\chi_A}$(x) = $\overline{\chi_A}$(y) = $\overline{\chi_A}$(x+y) = $\overline{\chi_A}$(xy) = 0. So, χ_A(x+ y) ≥ min { χ_A(x), χ_A(y) }, for all x and y in R, χ_A(xy) ≥ min { χ_A(x), χ_A(y) }, for all x and y in R. So, $\overline{\chi_A}$(x+ y) ≤ max { $\overline{\chi_A}$(x), $\overline{\chi_A}$(y) }, for all x and y in R, $\overline{\chi_A}$(xy) ≤ max { $\overline{\chi_A}$(x), $\overline{\chi_A}$(y) } , for all x and y in R.

Case (ii): If x in A, y not in A (or x not in A, y in A), then x+y, xy may or may not be in A, χ_A(x) =1, χ_A(y) = 0 (or χ_A(x) =0, χ_A(y) =1), χ_A(x+y) = χ_A(xy) = 1 (or 0) and $\overline{\chi_A}$(x) = 0, $\overline{\chi_A}$(y) = 1(or $\overline{\chi_A}$(x)=1, $\overline{\chi_A}$(y) =0), $\overline{\chi_A}$(x+y) = $\overline{\chi_A}$(xy) = 0 (or 1). Clearly χ_A(x+ y) ≥ min { χ_A(x), χ_A(y) }, for all x and y in R, χ_A(xy) ≥ min { χ_A(x), χ_A(y)}, for all x and y in R,and $\overline{\chi_A}$(x + y) ≤ max { $\overline{\chi_A}$(x), $\overline{\chi_A}$(y) }, for all x and y in R $\overline{\chi_A}$(xy) ≤ max { $\overline{\chi_A}$(x), $\overline{\chi_A}$(y) }, for all x and y in R.

Case (iii): If x and y not in A, then x+y, xy may or may not be in A, χ_A(x) = χ_A(y) = 0, χ_A(x+y) = χ_A(xy) =1 or 0 and $\overline{\chi_A}$(x) = $\overline{\chi_A}$(y) =1, $\overline{\chi_A}$(x+y) = $\overline{\chi_A}$(xy)= 0 or 1. Clearly χ_A(x+ y) ≥ min { χ_A(x), χ_A(y) }, for all x and y in R χ_A(xy) ≥ min { χ_A(x), χ_A(y) }, for all x and y in R, and $\overline{\chi_A}$(x + y) ≤ max { $\overline{\chi_A}$(x), $\overline{\chi_A}$(y) }, for all x and y in R. $\overline{\chi_A}$(xy) ≤ max { $\overline{\chi_A}$(x), $\overline{\chi_A}$(y) }, for all x and y in R.So in all the three cases, we have B is an intuitionistic fuzzy subhemiring of a hemiring R.

Conversely, let x and y in A, since A is a non empty subset of R, so, χ_A(x) = χ_A(y) = 1, $\overline{\chi_A}$(x) = $\overline{\chi_A}$(y) = 0.Since B = < χ_A, $\overline{\chi_A}$ > is an intuitionistic fuzzy subhemiring of R, we have χ_A(x + y) ≥ min{ χ_A(x), χ_A(y) }= min {1, 1} = 1, χ_A(xy) ≥ min { χ_A(x), χ_A(y) }= min {1,1 } = 1. Therefore χ_A(x + y) = χ_A(xy) =1.And, $\overline{\chi_A}$(x + y) ≤ max{ $\overline{\chi_A}$(x), $\overline{\chi_A}$(y) }= max{0, 0 } = 0, $\overline{\chi_A}$(xy) ≤ max { $\overline{\chi_A}$(x), $\overline{\chi_A}$(y) } = max { 0, 0 } = 0.Therefore $\overline{\chi_A}$(x + y) = $\overline{\chi_A}$(xy) = 0. Hence x + y and xy in A, so A is a subhemiring of R.

In the following Theorem ◦ is the composition operation of functions:

2.4 Theorem: Let A be an intuitionistic fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then A◦f is an intuitionistic fuzzy subhemiring of R.

Proof: Let x and y in R and A be an intuitionistic fuzzy subhemiring of a hemiring H. Then we have, $(\mu_A\circ f)(x+y)$ = μ_A(f(x+y))= μ_A(f(x)+ f(y)) , as f is an isomorphism ≥ min { μ_A(f(x)), μ_A(f(y))}, = min {$(\mu_A\circ f)(x)$, $(\mu_A\circ f)(y)$},which implies that $(\mu_A\circ f)(x+y)$ ≥ min { $(\mu_A\circ f)(x)$, $(\mu_A\circ f)(y)$}.And, $(\mu_A\circ f)(xy)$ = μ_A(f(xy))= μ_A(f(x)f(y)), as f is an isomorphism ≥ min {μ_A(f(x)), μ_A(f(y))} = min {$(\mu_A\circ f)(x)$, $(\mu_A\circ f)(y)$}, which implies that $(\mu_A\circ f)(xy)$ ≥ min{$(\mu_A\circ f)(x)$, $(\mu_A\circ f)(y)$}. Then we have, $(\nu_A\circ f)(x+y)$ = ν_A(f(x+y))= ν_A(f(x)+ f(y)) , as f is an isomorphism ≤ max {ν_A(f(x)), ν_A(f(y))}= max {$(\nu_A\circ f)(x)$, $(\nu_A\circ f)(y)$}, which implies that $(\nu_A\circ f)(x+y)$ ≤ max {$(\nu_A\circ f)(x)$, $(\nu_A\circ f)(y)$}.And $(\nu_A\circ f)(xy)$ = ν_A(f(xy)) = ν_A(f(x)f(y)), as f is an isomorphism≤ max { ν_A(f(x)), ν_A(f(y))}= max {$(\nu_A\circ f)(x)$, $(\nu_A\circ f)(y)$},which implies that $(\nu_A\circ f)(xy)$ ≤ max {$(\nu_A\circ f)(x)$, $(\nu_A\circ f)(y)$}.Therefore (A◦f) is an intuitionistic fuzzy subhemiring of a hemiring R.

2.5 Theorem: Let A be an intuitionistic fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then A◦f is an intuitionistic fuzzy subhemiring of R.

Proof: Let x and y in R and A be an intuitionistic fuzzy subhemiring of a hemiring H. Then we have,$(\mu_A \circ f)(x+y) = \mu_A(f(x+y)) = \mu_A(f(y)+f(x))$, as f is an anti-isomorphism $\geq \min\{\mu_A(f(x)), \mu_A(f(y))\} = \min\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$,which implies that $(\mu_A \circ f)(x+y) \geq \min\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$.And, $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x))$, as f is an anti-isomorphism $\geq \min\{\mu_A(f(x)), \mu_A(f(y))\}$, $= \min\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$, which implies that $(\mu_A \circ f)(xy) \geq \min\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$. Then we have, $(\nu_A \circ f)(x+y) = \nu_A(f(x+y)) = \nu_A(f(y)+f(x))$,as f is an anti-isomorphism $\leq \max\{\nu_A(f(x)), \nu_A(f(y))\} = \max\{(\nu_A \circ f)(x), (\nu_A \circ f)(y)\}$,which implies that $(\nu_A \circ f)(x+y) \leq \max\{(\nu_A \circ f)(x), (\nu_A \circ f)(y)\}$.And, $(\nu_A \circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(y)f(x))$, as f is an anti-isomorphism $\leq \max\{\nu_A(f(x)), \nu_A(f(y))\} = \max\{(\nu_A \circ f)(x), (\nu_A \circ f)(y)\}$, which implies that $(\nu_A \circ f)(xy) \leq \max\{(\nu_A \circ f)(x), (\nu_A \circ f)(y)\}$.Therefore $A \circ f$ is an intuitionistic fuzzy subhemiring of the hemiring R.

2.6 Theorem: Let A be an intuitionistic fuzzy subhemiring of a hemiring (R, +, .), then the pseudo intuitionistic fuzzy coset $(aA)^p$ is an intuitionistic fuzzy subhemiring of a hemiring R, for every a in R.

Proof: Let A be an intuitionistic fuzzy subhemiring of a hemiring R.For every x and y in R, we have, $((a\mu_A)^p)(x+y) = p(a)\mu_A(x+y) \geq p(a)\min\{\mu_A(x), \mu_A(y)\} = \min\{p(a)\mu_A(x), p(a)\mu_A(y)\} = \min\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$.Therefore, $((a\mu_A)^p)(x+y) \geq \min\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$. Now, $((a\mu_A)^p)(xy) = p(a)\mu_A(xy) \geq p(a)\min\{\mu_A(x), \mu_A(y)\} = \min\{p(a)\mu_A(x), p(a)\mu_A(y)\} = \min\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$.Therefore, $((a\mu_A)^p)(xy) \geq \min\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$. For every x and y in R, we have, $((a\nu_A)^p)(x+y) = p(a)\nu_A(x+y) \leq p(a)\max\{\nu_A(x), \nu_A(y)\} = \max\{p(a)\nu_A(x), p(a)\nu_A(y)\} = \max\{((a\nu_A)^p)(x), ((a\nu_A)^p)(y)\}$. Therefore, $((a\nu_A)^p)(x+y) \leq \max\{((a\nu_A)^p)(x), ((a\nu_A)^p)(y)\}$. Now, $((a\nu_A)^p)(xy) = p(a)\nu_A(xy) \leq p(a)\max\{\nu_A(x), \nu_A(y)\} = \max\{p(a)\nu_A(x), p(a)\nu_A(y)\} = \max\{((a\nu_A)^p)(x), ((a\nu_A)^p)(y)\}$. Therefore,$((a\nu_A)^p)(xy) \leq \max\{((a\nu_A)^p)(x), ((a\nu_A)^p)(y)\}$. Hence $(aA)^p$ is an intuitionistic fuzzy subhemiring of a hemiring R.

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