

NOTES ON INTUITIONISTIC FUZZY SUBHEMIRINGS OF A HEMIRING

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of an intuitionistic fuzzy subhemirings of a hemiring.

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Keywords: Fuzzy set, fuzzy subhemiring, anti-fuzzy subhemiring, intuitionistic fuzzy set, intuitionistic fuzzy subhemiring, and pseudo intuitionistic fuzzy coset.

INTRODUCTION:

There are many concepts of universal algebras generalizing an associative ring $(R; +; \cdot)$. Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also half-rings) are algebras $(R; +; \cdot)$ share the same properties as a ring except that $(R; +)$ is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra $(R; +, \cdot)$ is said to be a semiring if $(R; +)$ and $(R; \cdot)$ are semigroups satisfying $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$ for all a, b and c in R . A semiring R is said to be additively commutative if $a + b = b + a$ for all a, b and c in R . A semiring R may have an identity 1 , defined by $a \cdot 1 = a = a \cdot 1$ and a zero 0 , defined by $0 + a = a = a + 0$ and $a \cdot 0 = 0 = 0 \cdot a$ for all a in R . A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A. Zadeh [15], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subsets (IFS) was introduced by K. T. Atanassov [4], as a generalization of the notion of fuzzy set. The notion of Fuzzy left h -ideals in hemirings with respect to a s -norm was introduced in [2]. In this paper, we introduce the some Theorems in intuitionistic fuzzy subhemiring of a hemiring.

1. PRELIMINARIES

1.1 Definition: Let X be a non-empty set. A fuzzy subset A of X is a function $A: X \rightarrow [0, 1]$.

1.2 Definition: Let R be a hemiring. A fuzzy subset A of R is said to be a fuzzy subhemiring (FSHR) of R if it satisfies the following conditions:

- (i) $A(x + y) \geq \min\{A(x), A(y)\}$,
- (ii) $A(xy) \geq \min\{A(x), A(y)\}$, for all x and y in R .

1.3 Definition: Let R be a hemiring. A fuzzy subset A of R is said to be an anti-fuzzy subhemiring (AFSHR) of R if it satisfies the following conditions:

- (i) $A(x + y) \leq \max\{A(x), A(y)\}$,
- (ii) $A(xy) \leq \max\{A(x), A(y)\}$, for all x and y in R .

1.4 Definition: An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$, where $\mu_A: X \rightarrow [0, 1]$ and $\nu_A: X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.5 Definition: Let R be a hemiring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic fuzzy subhemiring (IFSHR) of R if it satisfies the following conditions:

- (i) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
- (ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,
- (iii) $\nu_A(x + y) \leq \max\{\nu_A(x), \nu_A(y)\}$,
- (iv) $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R .

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1.6 Definition: Let $(R, +, \cdot)$ be a hemiring. An intuitionistic fuzzy subhemiring A of R is said to be an intuitionistic fuzzy normal subhemiring (IFNSHR) of R if it satisfies the following conditions:

- (i) $\mu_A(xy) = \mu_A(yx)$,
- (ii) $\nu_A(xy) = \nu_A(yx)$, for all x and y in R .

1.5 Definition: If $(R, +, \cdot)$ and $(R^1, +, \cdot)$ are any two hemirings, then the function $f: R \rightarrow R^1$ is called a **homomorphism** if $f(x+y) = f(x)+f(y)$ and $f(xy)=f(x)f(y)$, for all x and y in R .

1.6 Definition: If $(R, +, \cdot)$ and $(R^1, +, \cdot)$ are any two hemirings, then the function $f: R \rightarrow R^1$ is called an **anti-homomorphism** if $f(x+y) = f(y)+f(x)$ and $f(xy)=f(y)f(x)$, for all x and y in R .

1.7 Definition: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two hemirings. Then the function $f: R \rightarrow R^1$ be a hemiring homomorphism. If f is one-to-one and onto, then f is called a **hemiring isomorphism**.

1.8 Definition: Let $(R, +, \cdot)$ and $(R^1, +, \cdot)$ be any two hemirings. Then the function $f: R \rightarrow R^1$ be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a **hemiring anti-isomorphism**.

1.9 Definition: Let R and R^1 be any two hemirings. Let $f: R \rightarrow R^1$ be any function and let A be an intuitionistic fuzzy subhemiring in R , V be an intuitionistic fuzzy subhemiring in $f(R) = R^1$, defined by $\mu_V(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) =$

$\inf_{x \in f^{-1}(y)} \nu_A(x)$, for all x in R and y in R^1 . Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.10 Definition: Let A be an intuitionistic fuzzy subhemiring of a hemiring $(R, +, \cdot)$ and a in R . Then the pseudo intuitionistic fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x) = p(a)\mu_A(x)$ and $((a\nu_A)^p)(x) = p(a)\nu_A(x)$, for every x in R and for some p in P .

2. INTUITIONISTIC FUZZY SUBHEMIRINGS OF A HEMIRING

2.1 Theorem: If A is an intuitionistic fuzzy subhemiring of a hemiring $(R, +, \cdot)$, then A is an intuitionistic fuzzy subhemiring of R .

Proof: Let A be an intuitionistic fuzzy subhemiring of a hemiring R . Consider $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \}$, for all x in R , we take $\square A = B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \}$, where $\mu_B(x) = \mu_A(x)$, $\nu_B(x) = 1 - \mu_A(x)$. Clearly, $\mu_B(x+y) \geq \min \{ \mu_B(x), \mu_B(y) \}$, for all x and y in R and $\mu_B(xy) \geq \min \{ \mu_B(x), \mu_B(y) \}$, for all x and y in R . Since A is an intuitionistic fuzzy subhemiring of R , we have $\mu_A(x+y) \geq \min \{ \mu_A(x), \mu_A(y) \}$, for all x and y in R , which implies that $1 - \nu_B(x+y) \geq \min \{ (1 - \nu_B(x)), (1 - \nu_B(y)) \}$, which implies that $\nu_B(x+y) \leq 1 - \min \{ (1 - \nu_B(x)), (1 - \nu_B(y)) \} = \max \{ \nu_B(x), \nu_B(y) \}$. Therefore, $\nu_B(x+y) \leq \max \{ \nu_B(x), \nu_B(y) \}$, for all x and y in R . And $\mu_A(xy) \geq \min \{ \mu_A(x), \mu_A(y) \}$, for all x and y in R , which implies that $1 - \nu_B(xy) \geq \min \{ (1 - \nu_B(x)), (1 - \nu_B(y)) \}$ which implies that $\nu_B(xy) \leq 1 - \min \{ (1 - \nu_B(x)), (1 - \nu_B(y)) \} = \max \{ \nu_B(x), \nu_B(y) \}$. Therefore, $\nu_B(xy) \leq \max \{ \nu_B(x), \nu_B(y) \}$, for all x and y in R . Hence $B = \square A$ is an intuitionistic fuzzy subhemiring of a hemiring R .

Remark: The converse of the above theorem is not true. It is shown by the following example:

Consider the hemiring $Z_5 = \{0, 1, 2, 3, 4\}$ with addition modulo 5 and multiplication modulo 5 operations. Then $A = \{ \langle 0, 0.7, 0.2 \rangle, \langle 1, 0.5, 0.1 \rangle, \langle 2, 0.5, 0.4 \rangle, \langle 3, 0.5, 0.1 \rangle, \langle 4, 0.5, 0.4 \rangle \}$ is not an intuitionistic fuzzy subhemiring of Z_5 , but $\square A = \{ \langle 0, 0.7, 0.3 \rangle, \langle 1, 0.5, 0.5 \rangle, \langle 2, 0.5, 0.5 \rangle, \langle 3, 0.5, 0.5 \rangle, \langle 4, 0.5, 0.5 \rangle \}$ is an intuitionistic fuzzy subhemiring of Z_5 .

2.2 Theorem: If A is an intuitionistic fuzzy subhemiring of a hemiring $(R, +, \cdot)$, then $\diamond A$ is an intuitionistic fuzzy subhemiring of R .

Proof: Let A be an intuitionistic fuzzy subhemiring of a hemiring R . That is $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \}$, for all x in R . Let $\diamond A = B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \}$, where $\mu_B(x) = 1 - \nu_A(x)$, $\nu_B(x) = \nu_A(x)$. Clearly, $\nu_B(x+y) \leq \max \{ \nu_B(x), \nu_B(y) \}$, for all x and y in R and $\nu_B(xy) \leq \max \{ \nu_B(x), \nu_B(y) \}$, for all x and y in R . Since A is an intuitionistic fuzzy subhemiring of R , we have $\nu_A(x+y) \leq \max \{ \nu_A(x), \nu_A(y) \}$, for all x and y in R , which implies that $1 - \mu_B(x+y) \leq \max \{ (1 - \mu_B(x)), (1 - \mu_B(y)) \}$ which implies that $\mu_B(x+y) \geq 1 - \max \{ (1 - \mu_B(x)), (1 - \mu_B(y)) \} = \min \{ \mu_B(x), \mu_B(y) \}$. Therefore, $\mu_B(x+y) \geq \min \{ \mu_B(x), \mu_B(y) \}$, for all x and y in R . And $\nu_A(xy) \leq \max \{ \nu_A(x), \nu_A(y) \}$, for all x and y in R , which implies that $1 - \mu_B(xy) \leq \max \{ (1 - \mu_B(x)), (1 - \mu_B(y)) \}$, which implies that $\mu_B(xy) \geq 1 - \max \{ (1 - \mu_B(x)), (1 - \mu_B(y)) \} = \min \{ \mu_B(x), \mu_B(y) \}$. Therefore, $\mu_B(xy) \geq \min \{ \mu_B(x), \mu_B(y) \}$, for all x and y in R . Hence $B = \diamond A$ is an intuitionistic fuzzy subhemiring of a hemiring R .

Remark: The converse of the above theorem is not true. It is shown by the following example:

Consider the hemiring $Z_5 = \{0, 1, 2, 3, 4\}$ with addition modulo 5 and multiplication modulo 5 operations. Then $A = \{\langle 0, 0.5, 0.1 \rangle, \langle 1, 0.6, 0.4 \rangle, \langle 2, 0.5, 0.4 \rangle, \langle 3, 0.6, 0.4 \rangle, \langle 4, 0.5, 0.4 \rangle\}$ is not an intuitionistic fuzzy subhemiring of Z_5 , but $\delta A = \{\langle 0, 0.9, 0.1 \rangle, \langle 1, 0.6, 0.4 \rangle, \langle 2, 0.6, 0.4 \rangle, \langle 3, 0.6, 0.4 \rangle, \langle 4, 0.6, 0.4 \rangle\}$ is an intuitionistic fuzzy subhemiring of Z_5 .

2.3 Theorem: Let $(R, +, \cdot)$ be a hemiring and A be a non empty subset of R . Then A is a subhemiring of R if and only if $B = \langle \chi_A, \overline{\chi_A} \rangle$ is an intuitionistic fuzzy subhemiring of R , where χ_A is the characteristic function.

Proof: Let $(R, +, \cdot)$ be a hemiring and A be a non empty subset of R . First let A be a subhemiring of R . Take x and y in R .

Case (i): If x and y in A , then $x+y, xy$ in A , since A is a subhemiring of R ,

$\chi_A(x) = \chi_A(y) = \chi_A(x+y) = \chi_A(xy) = 1$ and $\overline{\chi_A}(x) = \overline{\chi_A}(y) = \overline{\chi_A}(x+y) = \overline{\chi_A}(xy) = 0$. So, $\overline{\chi_A}(x+y) \geq \min \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R , $\chi_A(xy) \geq \min \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R . So, $\chi_A(x+y) \leq \max \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R , $\overline{\chi_A}(xy) \leq \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}$, for all x and y in R .

Case (ii): If x in A , y not in A (or x not in A , y in A), then $x+y, xy$ may or may not be in A , $\chi_A(x) = 1, \chi_A(y) = 0$ (or $\chi_A(x) = 0, \chi_A(y) = 1$), $\chi_A(x+y) = \chi_A(xy) = 1$ (or 0) and $\overline{\chi_A}(x) = 0, \overline{\chi_A}(y) = 1$ (or $\overline{\chi_A}(x) = 1, \overline{\chi_A}(y) = 0$), $\overline{\chi_A}(x+y) = \overline{\chi_A}(xy) = 0$ (or 1). Clearly $\overline{\chi_A}(x+y) \geq \min \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R , $\chi_A(xy) \geq \min \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R , and $\overline{\chi_A}(x+y) \leq \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}$, for all x and y in R , $\chi_A(xy) \leq \max \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R .

Case (iii): If x and y not in A , then $x+y, xy$ may or may not be in A , $\chi_A(x) = \chi_A(y) = 0, \chi_A(x+y) = \chi_A(xy) = 1$ or 0 and $\overline{\chi_A}(x) = \overline{\chi_A}(y) = 1, \overline{\chi_A}(x+y) = \overline{\chi_A}(xy) = 0$ or 1. Clearly $\overline{\chi_A}(x+y) \geq \min \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R , $\chi_A(xy) \geq \min \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R , and $\overline{\chi_A}(x+y) \leq \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \}$, for all x and y in R , $\chi_A(xy) \leq \max \{ \chi_A(x), \chi_A(y) \}$, for all x and y in R . So in all the three cases, we have B is an intuitionistic fuzzy subhemiring of a hemiring R .

Conversely, let x and y in A , since A is a non empty subset of R , so, $\chi_A(x) = \chi_A(y) = 1, \overline{\chi_A}(x) = \overline{\chi_A}(y) = 0$. Since $B = \langle \chi_A, \overline{\chi_A} \rangle$ is an intuitionistic fuzzy subhemiring of R , we have $\overline{\chi_A}(x+y) \geq \min \{ \chi_A(x), \chi_A(y) \} = \min \{ 1, 1 \} = 1, \chi_A(xy) \geq \min \{ \chi_A(x), \chi_A(y) \} = \min \{ 1, 1 \} = 1$. Therefore $\chi_A(x+y) = \chi_A(xy) = 1$. And, $\overline{\chi_A}(x+y) \leq \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \} = \max \{ 0, 0 \} = 0, \overline{\chi_A}(xy) \leq \max \{ \overline{\chi_A}(x), \overline{\chi_A}(y) \} = \max \{ 0, 0 \} = 0$. Therefore $\overline{\chi_A}(x+y) = \overline{\chi_A}(xy) = 0$. Hence $x+y$ and xy in A , so A is a subhemiring of R .

In the following Theorem is the composition operation of functions:

2.4 Theorem: Let A be an intuitionistic fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H . Then $A \circ f$ is an intuitionistic fuzzy subhemiring of R .

Proof: Let x and y in R and A be an intuitionistic fuzzy subhemiring of a hemiring H . Then we have, $(\mu_{A \circ f})(x+y) = \mu_A(f(x+y)) = \mu_A(f(x) + f(y))$, as f is an isomorphism $\geq \min \{ \mu_A(f(x)), \mu_A(f(y)) \} = \min \{ (\mu_{A \circ f})(x), (\mu_{A \circ f})(y) \}$, which implies that $(\mu_{A \circ f})(x+y) \geq \min \{ (\mu_{A \circ f})(x), (\mu_{A \circ f})(y) \}$. And, $(\mu_{A \circ f})(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y))$, as f is an isomorphism $\geq \min \{ \mu_A(f(x)), \mu_A(f(y)) \} = \min \{ (\mu_{A \circ f})(x), (\mu_{A \circ f})(y) \}$, which implies that $(\mu_{A \circ f})(xy) \geq \min \{ (\mu_{A \circ f})(x), (\mu_{A \circ f})(y) \}$. Then we have, $(\nu_{A \circ f})(x+y) = \nu_A(f(x+y)) = \nu_A(f(x) + f(y))$, as f is an isomorphism $\leq \max \{ \nu_A(f(x)), \nu_A(f(y)) \} = \max \{ (\nu_{A \circ f})(x), (\nu_{A \circ f})(y) \}$, which implies that $(\nu_{A \circ f})(x+y) \leq \max \{ (\nu_{A \circ f})(x), (\nu_{A \circ f})(y) \}$. And $(\nu_{A \circ f})(xy) = \nu_A(f(xy)) = \nu_A(f(x)f(y))$, as f is an isomorphism $\leq \max \{ \nu_A(f(x)), \nu_A(f(y)) \} = \max \{ (\nu_{A \circ f})(x), (\nu_{A \circ f})(y) \}$, which implies that $(\nu_{A \circ f})(xy) \leq \max \{ (\nu_{A \circ f})(x), (\nu_{A \circ f})(y) \}$. Therefore $(A \circ f)$ is an intuitionistic fuzzy subhemiring of a hemiring R .

2.5 Theorem: Let A be an intuitionistic fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H . Then $A \circ f$ is an intuitionistic fuzzy subhemiring of R .

Proof: Let x and y in R and A be an intuitionistic fuzzy subhemiring of a hemiring H . Then we have, $(\mu_A \circ f)(x+y) = \mu_A(f(x+y)) = \mu_A(f(y)+f(x))$, as f is an anti-isomorphism $\geq \min \{ \mu_A(f(x)), \mu_A(f(y)) \} = \min \{ (\mu_A \circ f)(x), (\mu_A \circ f)(y) \}$, which implies that $(\mu_A \circ f)(x+y) \geq \min \{ (\mu_A \circ f)(x), (\mu_A \circ f)(y) \}$. And, $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x))$, as f is an anti-isomorphism $\geq \min \{ \mu_A(f(x)), \mu_A(f(y)) \} = \min \{ (\mu_A \circ f)(x), (\mu_A \circ f)(y) \}$, which implies that $(\mu_A \circ f)(xy) \geq \min \{ (\mu_A \circ f)(x), (\mu_A \circ f)(y) \}$. Then we have, $(\nu_A \circ f)(x+y) = \nu_A(f(x+y)) = \nu_A(f(y)+f(x))$, as f is an anti-isomorphism $\leq \max \{ \nu_A(f(x)), \nu_A(f(y)) \} = \max \{ (\nu_A \circ f)(x), (\nu_A \circ f)(y) \}$, which implies that $(\nu_A \circ f)(x+y) \leq \max \{ (\nu_A \circ f)(x), (\nu_A \circ f)(y) \}$. And, $(\nu_A \circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(y)f(x))$, as f is an anti-isomorphism $\leq \max \{ \nu_A(f(x)), \nu_A(f(y)) \} = \max \{ (\nu_A \circ f)(x), (\nu_A \circ f)(y) \}$, which implies that $(\nu_A \circ f)(xy) \leq \max \{ (\nu_A \circ f)(x), (\nu_A \circ f)(y) \}$. Therefore $A \circ f$ is an intuitionistic fuzzy subhemiring of the hemiring R .

2.6 Theorem: Let A be an intuitionistic fuzzy subhemiring of a hemiring $(R, +, \cdot)$, then the pseudo intuitionistic fuzzy coset $(aA)^p$ is an intuitionistic fuzzy subhemiring of a hemiring R , for every a in R .

Proof: Let A be an intuitionistic fuzzy subhemiring of a hemiring R . For every x and y in R , we have, $((a\mu_A)^p)(x+y) = p(a)\mu_A(x+y) \geq p(a) \min \{ \mu_A(x), \mu_A(y) \} = \min \{ p(a)\mu_A(x), p(a)\mu_A(y) \} = \min \{ ((a\mu_A)^p)(x), ((a\mu_A)^p)(y) \}$. Therefore, $((a\mu_A)^p)(x+y) \geq \min \{ ((a\mu_A)^p)(x), ((a\mu_A)^p)(y) \}$. Now, $((a\mu_A)^p)(xy) = p(a)\mu_A(xy) \geq p(a) \min \{ \mu_A(x), \mu_A(y) \} = \min \{ p(a)\mu_A(x), p(a)\mu_A(y) \} = \min \{ ((a\mu_A)^p)(x), ((a\mu_A)^p)(y) \}$. Therefore, $((a\mu_A)^p)(xy) \geq \min \{ ((a\mu_A)^p)(x), ((a\mu_A)^p)(y) \}$. For every x and y in R , we have, $((a\nu_A)^p)(x+y) = p(a)\nu_A(x+y) \leq p(a) \max \{ \nu_A(x), \nu_A(y) \} = \max \{ p(a)\nu_A(x), p(a)\nu_A(y) \} = \max \{ ((a\nu_A)^p)(x), ((a\nu_A)^p)(y) \}$. Therefore, $((a\nu_A)^p)(x+y) \leq \max \{ ((a\nu_A)^p)(x), ((a\nu_A)^p)(y) \}$. Now, $((a\nu_A)^p)(xy) = p(a)\nu_A(xy) \leq p(a) \max \{ \nu_A(x), \nu_A(y) \} = \max \{ p(a)\nu_A(x), p(a)\nu_A(y) \} = \max \{ ((a\nu_A)^p)(x), ((a\nu_A)^p)(y) \}$. Therefore, $((a\nu_A)^p)(xy) \leq \max \{ ((a\nu_A)^p)(x), ((a\nu_A)^p)(y) \}$. Hence $(aA)^p$ is an intuitionistic fuzzy subhemiring of a hemiring R .

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