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FULLY FUZZY LINEAR PROGRAMS WITH TRIANGULAR FUZZY NUMBERS

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ABSTRACT

In this paper we propose a new method for solving fully fuzzy linear programming problem whose parameters are triangular fuzzy number. We prove some of the important theorems for the solution of fuzzy linear programming problem which in turn helps us to solve the given fully fuzzy linear programming problem by applying the fuzzy version of Simplex algorithm without converting to classical problem. A numerical example is provided to illustrate the proposed method.

Keywords: *Triangular fuzzy numbers, Fuzzy Linear programming, Ranking, location index number, fuzziness index functions.*

AMS Subject Classification: *90C05, 03E72, 90C70.*

1. INTRODUCTION

Linear programming is a one of the most important operational research (OR) techniques applied to solve many decision making problems. To solve optimization models in operation research, the decision parameters of the model must be known precisely. But in real world systems, in many cases, the decision parameters involved in the problem are imprecise in nature due to many reasons and have to be interpreted as fuzzy numbers to reflect the real world situation. A crisp linear programming problem is a mathematical programming problem having a linear objective function and linear constraints whose coefficients are crisp real numbers. If the coefficients involved in the objective and constraint functions are imprecise in nature and is interpreted as fuzzy numbers, then the resulting mathematical problem is referred to as a fuzzy linear programming problem. Fuzzy linear programming problem occur in many fields such as Mathematical modeling, Control theory and Management sciences, etc.

The idea of fuzzy set was introduced by Zadeh [11] in 1965. The concept of a fuzzy decision making was first proposed by Bellman and Zadeh [3]. An application of fuzzy optimization techniques to linear programming problem with single and multi-objective functions has been proposed by Zimmermann [12].

In literature several authors have proposed different approaches for solving fuzzy linear programming problem such as Inuiguchi [5], Ganesan and Veeramani [4], Rommelfanger [9], Nasseri [8], Tanaka and Asai [10] and Maleki [6] etc. Most of the existing methods are based on the concept of comparison of fuzzy numbers by use of ranking function. Inuiguchi et.al [5] deals with the fuzzy linear program with continuous piecewise linear membership functions. They proposed a technique to solve the problem using a standard linear programming when membership functions are strictly quasiconcave and the minimum operator is adopted for aggregating fuzzy goals. Ganesan and Veeramani [4] introduced a new method based on primal simplex algorithm for solving linear programming problem with symmetric trapezoidal fuzzy numbers without converting them to crisp linear programming problems. Rommelfanger [8] proposed a new method for solving stochastic linear programming problems with fuzzy parameters. Nasseri et al. [8], have proposed a new method for solving fuzzy number linear programming problems, by use of linear ranking function. Tanaka and Asai [10] also proposed formulation of fuzzy linear programming with fuzzy constraints and proposed a method for its solution based on inequality relation between fuzzy numbers. Maleki [6] proposed a method to unify some of the existing approaches which are using different ranking functions for solving fuzzy programming problems.

In this paper we propose a new method for solving fully fuzzy linear programming problem whose coefficients all are represented by triangular fuzzy number. We propose a fuzzy version of Simplex method to solve the given fully fuzzy

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linear programming problem with triangular fuzzy number without converting the given problem to crisp equivalent problem.

The rest of the paper is organized as follows. In section 2, we recall the basic concepts and the results of triangular fuzzy numbers and their arithmetic operations. In section 3, we introduce the fully fuzzy linear programming problem with triangular fuzzy numbers and related results. In section 4, we prove some of the important theorems for the solution of fully fuzzy linear programming problem with triangular fuzzy numbers and apply fuzzy version of simplex algorithm and obtained the fuzzy optimal solution. A numerical example is also provided to illustrate the theory developed in this paper.

2. PRELIMINARIES

Definition 2.1: A fuzzy set $\tilde{a}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following characteristics:

(i). $\tilde{a}$ is convex, i.e. $\tilde{a}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\tilde{a}(x_1),\ \tilde{a}(x_2)\}$, for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$.

(ii). $\tilde{a}$ is normal i.e. there exists an $x \in R$ such that $\tilde{a}(x) = 1$

(iii). $\tilde{a}$ is Piecewise continuous.

Definition 2.2: A fuzzy number $\tilde{a}$ on R is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following characteristics:

$$\tilde{a}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \leq x \leq a_3 \\ 0, & \text{elsewhere} \end{cases}$$

We denote this triangular fuzzy number by $\tilde{a} = (a_1, a_2, a_3)$. We use $F(R)$ to denote the set of all triangular fuzzy numbers.

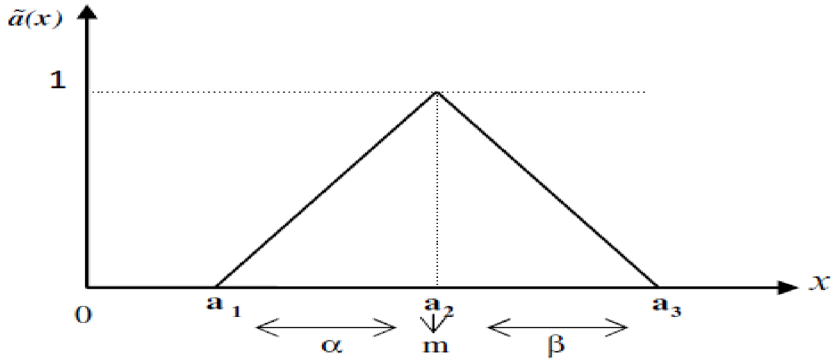

Fig 1. Triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

Also if $m = a_2$ represents the modal value or midpoint, $\alpha = (a_2 - a_1)$ represents the left spread and $\beta = (a_3 - a_2)$ represents the right spread of the triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$, then the triangular fuzzy number $\tilde{a}$ can be represented by the triplet $\tilde{a} = (\alpha, m, \beta)$. i.e. $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

Definition 2.3: A triangular fuzzy number $\tilde{a} \in F(R)$ can also be represented as a pair $\tilde{a} = (\underline{a}, \overline{a})$ of functions $\underline{a}(r)$ and $\overline{a}(r)$ for $0 \leq r \leq 1$ which satisfies the following requirements:

(i). $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.

(ii). $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.

(iii). $\underline{a}(r) \leq \overline{a}(r)$, $0 \leq r \leq 1$

Definition 2.4: For an arbitrary triangular fuzzy number $\tilde{a} = (\underline{a}, \overline{a})$, the number $a_0 = \left(\frac{\underline{a}(1)+\overline{a}(1)}{2}\right)$ is said to be a location index number of $\tilde{a}$. The two non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$, $a^* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function respectively. Hence every triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ can also be represented by $\tilde{a} = (a_0, a_*, a^*)$

2.1 Ranking of triangular Fuzzy Numbers

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari [1] Proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers. For an arbitrary triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ with parametric form $\tilde{a} = (\underline{a}(r), \overline{a}(r))$, we define the magnitude of the triangular fuzzy number $\tilde{a}$ by

$$\begin{aligned} Mag(\tilde{a}) &= \frac{1}{2}\left(\int_0^1 (\, \underline{a} + \overline{a} + a_0)\; f(r)\; d\; r\right) \\ &= \frac{1}{2}\left(\int_0^1 (\, a^* + 4a_0 - a_*)\; f(r)\; dr\right) \end{aligned}$$

where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0) = 0$, $f(1) = 1$ and $\int_0^1 f(r)\, dr = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r$.

Hence $Mag(\tilde{a}) = \left(\frac{a^* + 4a_0 - a_*}{4}\right) = \left(\frac{\underline{a} + \overline{a} + a_0}{4}\right)$.

The magnitude of a triangular fuzzy number $\tilde{a}$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. Mag ($\tilde{a}$) is used to rank fuzzy numbers. The larger Mag($\tilde{a}$), the larger fuzzy number.

For any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R), we define the ranking of $\tilde{a}$ and $\tilde{b}$ by comparing the Mag ($\tilde{a}$) and Mag ($\tilde{b}$) on R as follows:

(i). $\tilde{a} \succeq \tilde{b}$ if and only if Mag($\tilde{a}$) $\geq$ Mag($\tilde{b}$)

(ii). $\tilde{a} \preceq \tilde{b}$ if and only if Mag($\tilde{a}$) $\leq$ Mag($\tilde{b}$)

(iii). $\tilde{a} \approx \tilde{b}$ if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$)

Definition 2.5: A triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*)$ is said to be symmetric if and only if $a_* = a^*$.

Definition 2.6: A triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*)$ is said to be non-negative if and only if Mag($\tilde{a}$) $\geq$ 0 and is denoted by $\tilde{a} \succeq \tilde{0}$. Further if Mag($\tilde{a}$) > 0, then $\tilde{a} = (a_0, a_*, a^*)$ is said to be a positive fuzzy number and is denoted by $\tilde{a} \succ \tilde{0}$.

Definition 2.7: Two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R) are said to be equivalent if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$).That is $\tilde{a} \approx \tilde{b}$ if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$).Two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R) are said to be equal if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$. That is $\tilde{a} = \tilde{b}$ if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$.

2.2 Arithmetic operation on triangular Fuzzy Numbers

Ming Ma et al. [7] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for a, b∈L we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For arbitrary triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular fuzzy numbers are defined by $\tilde{a} * \tilde{b} = (a_0 * b_0,\ a_* \vee b_*,\ a^* \vee b^*)$.
In particular for any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$, we define

(i). Addition: $\tilde{a} + \tilde{b} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$

(ii). Subtraction: $\tilde{a} - \tilde{b} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$.

(iii). Multiplication: $\tilde{a} \times \tilde{b} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$.

(iv). Division: $\tilde{a} \div \tilde{b} = (a_0, a_*, a^*) \div (b_0, b_*, b^*) = (a_0 \div b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$.

3. FUZZY LINEAR PROGRAMMING PROBLEM

Let F(R) be set of all triangular fuzzy numbers. A fuzzy linear programming problem with triangular fuzzy numbers is defined as follows:

$$\max \tilde{Z} \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$$

$$\text{subject to } \sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}_j \preceq \tilde{b}_i \quad \text{for all } i=1,2,.....,m_0$$

$$\sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}_j \succeq \tilde{b}_i \quad \text{for all } i = m_0+1,........m \tag{1}$$

$$\text{and } \tilde{x}_j \succeq \tilde{0} \quad \text{for all } j = 1, 2, ...n.$$

where $\tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i \in F(R)$, $i = 1, 2, 3, \ldots\ldots.m$ and $j = 1, 2, 3, \ldots\ldots, n$.

If $\tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j$ and $\tilde{b}_i$ are represented by location index number, left fuzziness index function and right fuzziness index function respectively, then the above problem can be rewritten as follows:

$$\max \tilde{Z} \approx \sum_{j=1}^{n} \left\langle (c_j)_0, (c_j)_*, (c_j)^* \right\rangle \times \left\langle (x_j)_0, (x_j)_*, (x_j)^* \right\rangle \tag{2}$$

$$\sum_{j=1}^{n} \left\langle (a_{ij})_0, (a_{ij})_*, (a_{ij})^* \right\rangle \times \left\langle (x_j)_0, (x_j)_*, (x_j)^* \right\rangle \preceq \left\langle (b_j)_0, (b_j)_*, (b_j)^* \right\rangle \quad \text{for all } i = 1, 2, ..., m_0$$

$$\sum_{j=1}^{n} \left\langle (a_{ij})_0, (a_{ij})_*, (a_{ij})^* \right\rangle \times \left\langle (x_j)_0, (x_j)_*, (x_j)^* \right\rangle \succeq \left\langle (b_j)_0, (b_j)_*, (b_j)^* \right\rangle \quad \text{for all } i = m_0 + 1, ...m$$

$$\left\langle (x_j)_0, (x_j)_*, (x_j)^* \right\rangle \succeq \tilde{0} \quad \text{for all } j = 1, 2, ...n.$$

Definition 3.1: Any vector $\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)^T \in (F(R))^n$

i.e. $\tilde{\mathbf{x}} = \left\{ \left\langle (x_1)_0, (x_1)_*, (x_1)^* \right\rangle, \left\langle (x_2)_0, (x_2)_*, (x_2)^* \right\rangle, ..., \left\langle (x_n)_0, (x_n)_*, (x_n)^* \right\rangle \right\} \in (F(R))^n$ is said to be a fuzzy feasible solution, if for each $\tilde{x}_j = \left\langle (x_j)_0, (x_j)_*, (x_j)^* \right\rangle \in (F(R))^n$ satisfies the constraints and non-negativity restrictions of (2).

Definition 3.2: A fuzzy feasible solution is said to be a fuzzy optimal solution if it optimizes the objective function of the fuzzy linear programming problem.

Definition 3.3: Suppose the constraints $\tilde{A}\tilde{\mathbf{x}} \approx \tilde{\mathbf{b}}$ where $\tilde{A} \in (F(R))^{m\times n}$, $\tilde{\mathbf{x}} \in (F(R))^n$ and $\tilde{\mathbf{b}} \in (F(R))^m$.
Let $\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)$ solves $\tilde{A}\tilde{\mathbf{x}} \approx \tilde{\mathbf{b}}$. If all $\tilde{x}_j = \left\langle (x_j)_0, (x_j)_*, (x_j)^* \right\rangle$ for some $(x_j)_0 \ge 0$, then $\tilde{\mathbf{x}}$ is said to be a fuzzy basic solution. If $\tilde{x}_j \ne \left\langle (x_j)_0, (x_j)_*, (x_j)^* \right\rangle$ for $(x_j)_0 \ge 0$, then $\tilde{\mathbf{x}}$ has some non-zero components, say, $\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_k$, $1 \le k \le n$.

Then $\tilde{A}\tilde{\mathbf{x}} \approx \tilde{\mathbf{b}}$ can be written as $\tilde{\boldsymbol{a}}_1\,\tilde{x}_1 + \tilde{\boldsymbol{a}}_2\,\tilde{x}_2 + \dots\dots\dots + \tilde{\boldsymbol{a}}_{\boldsymbol{k}}\,\tilde{x}_k + \tilde{\boldsymbol{a}}_{\boldsymbol{k}+1}\,\tilde{x}_{k+1} + \dots\dots\dots + \tilde{\boldsymbol{a}}_{\boldsymbol{n}}\,\tilde{x}_n \approx \tilde{\mathbf{b}}$. If the columns $\tilde{\boldsymbol{a}}_1, \tilde{\boldsymbol{a}}_2, \dots\dots\dots, \tilde{\boldsymbol{a}}_{\boldsymbol{k}}$ corresponding to these non-zero components $\tilde{x}_1, \tilde{x}_2, \dots\dots, \tilde{x}_k$ are linearly dependent, then $\tilde{\mathbf{x}}$ is said to be a fuzzy basic solution. Let $\tilde{B}$ be any $(m \times m)$ matrix formed by 'm' linearly independent column of $\tilde{A}$. Then $\tilde{B}\tilde{\mathbf{x}} \approx \tilde{\mathbf{b}}$. Now $\tilde{\mathbf{x}} = \left(\tilde{x}_1, \tilde{x}_2, \dots\dots, \tilde{x}_k, \tilde{0}, \tilde{0}, \dots\dots\dots, \tilde{0}\right)$ is a fuzzy basic solution. We represent the fuzzy basic solution $\tilde{\mathbf{x}}$ by $\tilde{\mathbf{x}}_{\mathbf{B}}$.

Definition 3.4: A fuzzy basic solution which satisfies the non-negativity constraints is called Fuzzy basic feasible solution**.**

Definition 3.5: A standard form of fuzzy linear programming problem is defined as follows:

$$\max \tilde{Z} \approx \sum_{j=1}^{n} \tilde{C}_j \tilde{x}_j \quad \text{i.e} \quad \max \tilde{Z} \approx \tilde{\mathbf{C}}\tilde{\mathbf{x}}$$

$$\text{subject to } \tilde{A}\tilde{\mathbf{x}} \approx \tilde{\mathbf{b}} \tag{3}$$

$$\text{and } \tilde{\mathbf{x}} \succeq \tilde{0} \text{ where } \tilde{A} \in \left(F(R)\right)^{m\times n},\ \tilde{\mathbf{C}}, \tilde{\mathbf{x}} \in \left(F(R)\right)^{n} \text{ and } \tilde{\mathbf{b}} \in \left(F(R)\right)^{m}$$

4. IMPROVING FUZZY BASIC FEASIBLE SOLUTION

Consider the standard form of fuzzy linear programming problem,
$\max \tilde{Z} \approx \tilde{\mathbf{C}}\tilde{\mathbf{x}}$ subject to $\tilde{A}\tilde{\mathbf{x}} \approx \tilde{\mathbf{b}}$ and $\tilde{\mathbf{x}} \succeq \tilde{0}$. Here we assume that rank of $\tilde{A} = m$.
Let the columns of $\tilde{A}$ be given by $\tilde{\mathbf{a}}_{\mathbf{j}}$ .i.e $\tilde{A} = \left[\tilde{\mathbf{a}}_1, \tilde{\mathbf{a}}_2, \dots\dots, \tilde{\mathbf{a}}_{\mathbf{n}}\right]$ .Let $\tilde{B}$ be a $(m \times m)$ non-singular matrix whose columns are linearly independent column of $\tilde{A}$ and $\tilde{B} = [\tilde{\mathbf{b}}_1, \tilde{\mathbf{b}}_2, \dots\dots, \tilde{\mathbf{b}}_{\mathbf{m}}]$. Let $\tilde{\mathbf{x}}_{\mathbf{B}}$ be a fuzzy basic solution. i.e. $\tilde{\mathbf{x}}_{\mathbf{B}} = \left[\tilde{x}_{B_1}, \tilde{x}_{B_2}, \dots\dots, \tilde{x}_{B_m}\right]$ where $\tilde{B}\tilde{\mathbf{x}}_{\mathbf{B}} = \tilde{\mathbf{b}}$ .Corresponding to any such $\tilde{\mathbf{x}}_{\mathbf{B}}$,we define $\tilde{\mathbf{C}}_{\mathbf{B}}$,called reduced fuzzy cost vector, containing the prices of the basic variables i.e. $\tilde{\mathbf{C}}_{\mathbf{B}} = \left[\tilde{c}_{B_1}, \tilde{c}_{B_2}, \dots\dots, \tilde{c}_{B_m}\right]$.The value of the objective function is given by $\tilde{Z} = \tilde{C}_{\mathbf{B}}^{T}\tilde{\mathbf{x}}_{\mathbf{B}}$. To improve the objective function $\tilde{Z}$, we have to find other fuzzy basic feasible solution by replacing one of the columns of basis matrix $\tilde{B}$.

Theorem 4.1: Let $\tilde{B}\tilde{\mathbf{x}}_{\mathbf{B}} = \tilde{\mathbf{b}}$ is a fuzzy basic solution of (3).If for any column $\tilde{\mathbf{a}}_{\mathbf{j}}$ in $\tilde{A}$ which is not in $\tilde{B}$, the condition $\left(\tilde{Z}_j - \tilde{C}_j\right) \prec \tilde{0}$ hold and $\tilde{y}_{ij} \succ \tilde{0}$ for some $i = 1, 2, \dots\dots, m$ and then it is possible to obtain a new fuzzy basic feasible solution by replacing one of the columns in $\tilde{B}$ by $\tilde{\mathbf{a}}_{\mathbf{j}}$.

Proof: Suppose that $\tilde{\mathbf{x}}_{\mathbf{B}} = \left[\tilde{x}_{B_1}, \tilde{x}_{B_2}, \dots\dots, \tilde{x}_{B_m}\right]$ be a fuzzy basic feasible solution with *k* positive components such that $\tilde{B}\tilde{\mathbf{x}}_{\mathbf{B}} = \tilde{\mathbf{b}}$

where $\tilde{x}_{B_i} = \left\langle (\tilde{x}_{B_i})_0, (\tilde{x}_{B_i})_*, (\tilde{x}_{B_i})^* \right\rangle$ $i \in \{1, 2, \dots\dots, m\}$ and $(\tilde{x}_{B_i})_0 > 0$ for $i = 1, 2, \dots\dots, k$ and $(\tilde{x}_{B_i})_0 = 0$ for $i = k+1, \dots\dots, m$.

Now the equation $\tilde{B}\tilde{\mathbf{x}}_{\mathbf{B}} = \tilde{\mathbf{b}}$ becomes

$$\sum_{\mathbf{i}=1}^{\mathbf{k}} \tilde{x}_{B_i}\tilde{\mathbf{b}}_{\mathbf{i}} + \left\langle 0, (\tilde{x}_{k+1})_*, (\tilde{x}_{k+1})^* \right\rangle \tilde{\mathbf{b}}_{\mathbf{k}+1} + \dots\dots\dots + \left\langle 0, (\tilde{x}_m)_*, (\tilde{x}_m)^* \right\rangle \tilde{\mathbf{b}}_{\mathbf{m}} \approx \tilde{\mathbf{b}} \qquad \sum_{\mathbf{i}=1}^{\mathbf{k}} \tilde{x}_{B_i}\tilde{\mathbf{b}}_{\mathbf{i}} + \sum_{\mathbf{i}=\mathbf{k}+1}^{\mathbf{m}} \left\langle 0, (\tilde{x}_i)_*, (\tilde{x}_i)^* \right\rangle \tilde{\mathbf{b}}_{\mathbf{i}} \approx \tilde{\mathbf{b}} \tag{4}$$

Then for any column $\tilde{\mathbf{a}}_{\mathbf{j}}$ in $\tilde{A}$ which is not in $\tilde{B}$, we write

$$\tilde{\mathbf{a}}_{\mathbf{j}} = \sum_{\mathbf{i}=1}^{\mathbf{m}} \tilde{y}_{ij}\tilde{\mathbf{b}}_{\mathbf{i}} = \tilde{y}_{1j}\tilde{\mathbf{b}}_1 + \tilde{y}_{2j}\tilde{\mathbf{b}}_2 + \dots\dots + \tilde{y}_{rj}\tilde{\mathbf{b}}_{\mathbf{r}} + \dots\dots + \tilde{y}_{mj}\tilde{\mathbf{b}}_{\mathbf{m}} \approx \tilde{y}_j\tilde{\mathbf{B}}$$

We know that if the basis vector $\tilde{\mathbf{b}}_{\mathbf{r}}$ for which $\tilde{y}_{rj} \neq \tilde{0}$ is replaced by $\tilde{\mathbf{a}}_{\mathbf{j}}$ in $\tilde{A}$, then the new set of vectors $\left(\tilde{\mathbf{b}}_1, \tilde{\mathbf{b}}_2, \dots\dots, \tilde{\mathbf{b}}_{\mathbf{r}-1}, \tilde{\mathbf{a}}_{\mathbf{j}}, \tilde{\mathbf{b}}_{\mathbf{r}+1}, \dots\dots, \tilde{\mathbf{b}}_{\mathbf{m}}\right)$ still form a basis.

Now for $\tilde{y}_{rj} \neq \tilde{0}$ and $r \le k$, we have, $\tilde{\mathbf{b}}_{\mathbf{r}} = \dfrac{\tilde{a}_j}{\tilde{y}_{rj}} - \sum\limits_{\substack{\mathbf{i=1}\\ \mathbf{i\neq r}}}^{\mathbf{m}} \dfrac{\tilde{y}_{ij}}{\tilde{y}_{rj}} \tilde{\mathbf{b}}_{\mathbf{i}} = \dfrac{\tilde{\mathbf{a}}_{\mathbf{j}}}{\tilde{y}_{rj}} - \sum\limits_{\substack{\mathbf{i=1}\\ \mathbf{i\neq r}}}^{\mathbf{k}} \dfrac{\tilde{y}_{ij}}{\tilde{y}_{rj}} \tilde{\mathbf{b}}_{\mathbf{i}} - \sum\limits_{\mathbf{i=k+1}}^{\mathbf{m}} \dfrac{\tilde{y}_{ij}}{\tilde{y}_{rj}} \tilde{\mathbf{b}}_{\mathbf{i}}$.

Then (4) becomes

$$\sum_{\substack{\mathbf{i=1}\\ \mathbf{i\neq r}}}^{\mathbf{k}} \tilde{x}_{B_i} \tilde{\mathbf{b}}_{\mathbf{i}} + \tilde{x}_{B_r} \left(\frac{\tilde{\mathbf{a}}_{\mathbf{j}}}{\tilde{y}_{rj}} - \sum_{\substack{\mathbf{i=1}\\ \mathbf{i\neq r}}}^{\mathbf{m}} \frac{\tilde{y}_{ij}}{\tilde{y}_{rj}} \tilde{\mathbf{b}}_{\mathbf{i}} = \frac{\tilde{\mathbf{a}}_{\mathbf{j}}}{\tilde{y}_{rj}} - \sum_{\substack{\mathbf{i=1}\\ \mathbf{i\neq r}}}^{\mathbf{k}} \frac{\tilde{y}_{ij}}{\tilde{y}_{rj}} \tilde{\mathbf{b}}_{\mathbf{i}} - \sum_{\mathbf{i=k+1}}^{\mathbf{m}} \frac{\tilde{y}_{ij}}{\tilde{y}_{rj}} \tilde{\mathbf{b}}_{\mathbf{i}} \right) + \sum_{\mathbf{i=k+1}}^{\mathbf{m}} \left\langle 0, (\tilde{x}_i)_*, (\tilde{x}_i)^* \right\rangle \tilde{\mathbf{b}}_{\mathbf{i}} \approx \tilde{\mathbf{b}}$$

$$\sum_{\substack{\mathbf{i=1}\\ \mathbf{i\neq r}}}^{\mathbf{k}} \left(\tilde{x}_{B_i} - \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}} \tilde{y}_{ij} \right) \tilde{\mathbf{b}}_{\mathbf{i}} + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}} \tilde{\mathbf{a}}_{\mathbf{j}} + \sum_{\mathbf{i=k+1}}^{\mathbf{m}} \left(\left\langle 0, (\tilde{x}_i)_*, (\tilde{x}_i)^* \right\rangle - \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}} \tilde{y}_{ij} \right) \tilde{\mathbf{b}}_{\mathbf{i}} \approx \tilde{\mathbf{b}}$$

$\sum\limits_{\substack{\mathbf{i=1}\\ \mathbf{i\neq r}}}^{\mathbf{k}} \left(\tilde{x}_{B_i} - \dfrac{\tilde{x}_{B_r}}{\tilde{y}_{rj}} \tilde{y}_{ij} \right) \tilde{\mathbf{b}}_{\mathbf{i}} + \dfrac{\tilde{x}_{B_r}}{\tilde{y}_{rj}} \tilde{\mathbf{a}}_{\mathbf{j}} \approx \tilde{\mathbf{b}}$ which gives a new fuzzy basic feasible solution to $\tilde{A}\tilde{x} \approx \tilde{b}$.

i.e. $\sum\limits_{\substack{\mathbf{i=1}\\ \mathbf{i\neq r}}}^{\mathbf{k}} \hat{\tilde{x}}_{B_i} \tilde{\mathbf{b}}_{\mathbf{i}} + \hat{\tilde{x}}_{B_r} \tilde{\mathbf{a}}_{\mathbf{j}} \approx \tilde{\mathbf{b}}$ where $\hat{\tilde{x}}_{B_i} = \tilde{x}_{B_i} - \dfrac{\tilde{x}_{B_r}}{\tilde{y}_{rj}} \tilde{y}_{ij}$ and $\hat{\tilde{x}}_{B_r} = \dfrac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}$.

Theorem 4.2: Let $\tilde{\mathbf{x}}_{\mathbf{B}}$ be a fuzzy basic feasible solution to a fuzzy linear programming problem with corresponding objective value $\hat{\tilde{Z}} = \tilde{C}_B^T \tilde{x}_B$. If $(\tilde{Z}_j - \tilde{C}_j) \succeq \tilde{0}$ for every column $\tilde{\mathbf{a}}_{\mathbf{j}}$ in $\tilde{A}$, then $\tilde{\mathbf{x}}_{\mathbf{B}}$ is optimal.

Proof: For a given any feasible solution $\tilde{\mathbf{x}}$, we have $\left(\tilde{Z}_j - \tilde{C}_j\right) \succeq \tilde{0}$ for all $j = 1, 2, ..., n$ and by $\hat{\tilde{Z}} = \tilde{C}_B^T \tilde{x}_B$, we have $\tilde{Z} = \sum\limits_{j=1}^{n} \tilde{C}_j \tilde{x}_j \le \sum\limits_{j=1}^{n} \tilde{Z}_j \tilde{x}_j = \sum\limits_{j=1}^{n} \left(\tilde{C}_B^T \tilde{x}_B\right) \tilde{x}_j = \sum\limits_{j=1}^{n} \left(\sum\limits_{i=1}^{m} \tilde{C}_{B_i} \tilde{y}_{ij} \right) \tilde{x}_j = \sum\limits_{i=1}^{m} \left(\sum\limits_{j=1}^{n} \tilde{y}_{ij} \tilde{x}_j \right) \tilde{C}_{B_i}$

$\tilde{Z} \le \sum\limits_{i=1}^{m} \left(\sum\limits_{j=1}^{n} \tilde{y}_{ij} \tilde{x}_j \right) \tilde{C}_{B_i}$. That is $\tilde{x}_i = \sum\limits_{j=1}^{n} \tilde{y}_{ij} \tilde{x}_j = \tilde{x}_{B_i}$, $i \in \{1, 2, ..., m\}$

Since $\tilde{\mathbf{x}}$ is a feasible solution, $\tilde{A}\tilde{\mathbf{x}} \approx \tilde{\mathbf{b}}$ and by $\tilde{B}\tilde{y}_j = \tilde{a}_j$.

$\tilde{b} = \sum\limits_{j=1}^{n} \tilde{x}_j \tilde{a}_j = \sum\limits_{j=1}^{n} \tilde{x}_j \left(\tilde{B}\tilde{y}_j\right) = \sum\limits_{j=1}^{n} \left(\sum\limits_{i=1}^{m} \tilde{y}_{ij} \tilde{b}_i \right) \tilde{x}_j = \sum\limits_{i=1}^{m} \tilde{x}_i \tilde{b}_i = \tilde{B}\tilde{\mathbf{x}}$. Since $\tilde{B}$ is non-singular and $\tilde{B}\tilde{\mathbf{x}} \approx \tilde{\mathbf{b}}$. Therefore $\tilde{x} = \tilde{x}_B$. Hence $\tilde{Z} \le \sum\limits_{i=1}^{m} \tilde{x}_{B_i} \tilde{C}_{B_i} = \hat{\tilde{Z}}$ for all $\tilde{\mathbf{x}}$ in the feasible solution.

5. A NUMERICAL EXAMPLE

Consider an example discussed in Amit Kumar et al [2],

$$\begin{aligned} &\max\ (1,2,3)\tilde{x}_1 + (2,3,4)\tilde{x}_2 \\ &\text{such that } (0,1,2)\tilde{x}_1 + (1,2,3)\tilde{x}_2 \le (1,10,27) \\ &\qquad (1,2,3)\tilde{x}_1 + (0,1,2)\tilde{x}_2 \le (2,11,28) \\ &\qquad\qquad \tilde{x}_1, \tilde{x}_2 \ge \tilde{0} \end{aligned}$$

Representing the triangular fuzzy numbers in terms of left and right index function, we have

$$\begin{aligned} &\max\ (2,1-r,1-r)\tilde{x}_1 + (3,1-r,1-r)\tilde{x}_2 \text{ such that } (1,1-r,1-r)\tilde{x}_1 + (2,1-r,1-r)\tilde{x}_2 \le (10,9-9r,17-17r) \\ &\qquad (2,1-r,1-r)\tilde{x}_1 + (1,1-r,1-r)\tilde{x}_2 \le (11,9-9r,17-17r) \end{aligned}$$

Solution:
Initial iteration:

$\tilde{C}_B$	$\tilde{y}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	
$\tilde{0}$	$\tilde{s}_1$	$(10,9-9r,17-17r)$	$(1,1-r,1-r)$	$(2,1-r,1-r)$	$\tilde{1}$	$\tilde{0}$	$(5,9-9r,17-17r)$
$\tilde{0}$	$\tilde{s}_2$	$(11,9-9r,17-17r)$	$(2,1-r,1-r)$	$(1,1-r,1-r)$	$\tilde{0}$	$\tilde{1}$	$(11,9-9r,17-17r)$
$\tilde{Z}_j - \tilde{C}_j$		$\tilde{0}$	$(-2,1-r,1-r)$	$(-3,1-r,1-r)$	$\tilde{0}$	$\tilde{0}$	

First iteration:

$\tilde{C}_B$	$\tilde{y}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	
$(3,1-r,1-r)$	$\tilde{x}_2$	$(5,9-9r,17-17r)$	$(\frac{1}{2},1-r,1-r)$	$(1,1-r,1-r)$	$(1,1-r,1-r)$	$(0,1-r,1-r)$	$(10,9-9r,17-17r)$
$\tilde{0}$	$\tilde{s}_2$	$(6,9-9r,17-17r)$	$(\frac{3}{2},1-r,1-r)$	$(0,1-r,1-r)$	$(-1,1-r,1-r)$	$(1,1-r,1-r)$	$(4,9-9r,17-17r)$
$\tilde{Z}_j-\tilde{C}_j$		$(15,1-r,1-r)$	$(-\frac{1}{2},1-r,1-r)$	$(0,1-r,1-r)$	$(3,1-r,1-r)$	$(0,1-r,1-r)$	

Second iteration:

$\tilde{C}_B$	$\tilde{y}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$
$(3,1-r,1-r)$	$\tilde{x}_2$	$(3,9-9r,17-17r)$	$(0,1-r,1-r)$	$(1,1-r,1-r)$	$(\frac{2}{3},1-r,1-r)$	$(-\frac{1}{3},1-r,1-r)$
$(2,1-r,1-r)$	$\tilde{x}_1$	$(4,9-9r,17-17r)$	$(1,1-r,1-r)$	$(0,1-r,1-r)$	$(-\frac{2}{3},1-r,1-r)$	$(\frac{2}{3},1-r,1-r)$
$\tilde{Z}_j-\tilde{C}_j$		$(17,9-9r,17-17r)$	$(0,1-r,1-r)$	$(0,1-r,1-r)$	$(\frac{8}{3},1-r,1-r)$	$(\frac{1}{3},1-r,1-r)$

Since all $(\tilde{Z}_j-\tilde{C}_j)\succeq\tilde{0}$, the current fuzzy basic feasible solution is fuzzy optimal. The fuzzy optimal solution is $\tilde{x}_1=(4,9-9r,17-17r)$, $\tilde{x}_2=(3,9-9r,17-17r)$ with $\tilde{Z}=(17,9-9r,17-17r)$.

For $r=0$, we have the fuzzy optimal solution in terms of location index and fuzziness index as $\tilde{x}_1=(4,9,17)$; $\tilde{x}_2=(3,9,17)$; $\tilde{Z}=(17,9,17)$. Hence the fuzzy optimal solution in the general form, in terms of $\tilde{a}=(a_1, a_2, a_3)$ for $r=0$ is $\tilde{x}_1=(-5,4,21)$, $\tilde{x}_2=(-6,3,20)$ with $\tilde{Z}=(8,17,34)$.

Table 5.1: Comparison of Fuzzy Optimal Solution obtained by our method and by Amit kumar et al method.

	Our Method	**Amit Kumar et al Method**
Fuzzy Optimal Solution	$\max\ \tilde{Z}=(1,2,3)\tilde{x}_1+(2,3,4)\tilde{x}_2 =(17,9-9r,17-17r)$	$\max\ \tilde{Z}=(1,2,3)\tilde{x}_1+(2,3,4)\tilde{x}_2$
For $r=0$	$\max\ \tilde{Z}=(8,17,34)$	$\max\ \tilde{Z}=(4,17,38)$
For $r=0.25$	$\max\ \tilde{Z}=(10.25,17,29.75)$	-------------------------
For $r=0.5$	$\max\ \tilde{Z}=(12.5,17,25.5)$	--------------------------
For $r=0.75$	$\max\ \tilde{Z}=(14.75,17,21.25)$	--------------------------
For $r=1$	$\max\ \tilde{Z}=17$	----------------------------

It is seen that the Decision Maker have the flexibility of choosing $r\in[0,1]$ depending upon the situation and his wish by applying the proposed method in this paper whereas it is not possible by applying Amit kumar et al method.

6. CONCLUSION

In this paper, we proposed a new method for solving fully fuzzy linear programming problem with triangular fuzzy number. The triangular fuzzy numbers are represented in terms of location index number, left fuzziness index function and right fuzziness index function respectively. We applied fuzzy version of simplex algorithm for the fuzzy optimal solution of the fully fuzzy linear programming problem. A numerical example discussed by Amit kumar et al [2] is solved using the proposed method without converting the given problem to crisp equivalent problem. It is to be noted that the Decision Maker have the flexibility of choosing $r \in [0,1]$ depending upon the situation and his wish by applying the proposed method.

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