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CERTAIN NEW CONTINUED FRACTIONS FOR THE RATIO OF TWO ${}_3\psi_3$ SERIES

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ABSTRACT

In the present paper we have developed certain new continued fractions representations for the ratios of two basic bilateral hypergeometric series ${}_3\psi_3$.

Keywords: *Basic hypergeometric series, Basic bilateral hypergeometric series, Continued fraction.*

AMS Subject Classification: *33D15, 11A55.*

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1. INTRODUCTION

Now a days continued fractions have been become center of attraction for pure and applied mathematicians. The Indian mathematician Aryabhata(d. 550 AD) [4] used a continued fraction to solve a linear indeterminate equation. The development of continued fractions theory was stopped for more than a thousand years due to the specific applications. Major impacts of continued fractions in the different field of mathematics have been noticed from the beginning of 20th century. The Indian genius Ramanujan [16, 17] had shown new facet of continued fractions in the field of q-series. Ramanujan and his works on continued fractions had inspired a number of mathematicians working in the field of q-series to carry out researches in this direction. There are a number of mathematicians namely R. P. Agarwal [1, 2], G. E. Andrews and D. Bowman [3], B. C. Berndt [5], N. A. Bhagirathi [6,7], S. Bhargava, C. Adiga and D. D. Somashekara [8], R. Y. Denis [9, 10, 11], R. Y. Denis and S. N. Singh [12], Maheshwar Pathak and Pankaj Srivastava [14], P. Rai [15], S. N. Singh [18, 19, 20], Pankaj Srivastava [21], A. Verma, R. Y. Denis and K. Srinivasa Rao [22] etc. have established several results for basic hypergeometric series ${}_2\phi_1$, ${}_3\phi_2$ and basic bilateral hypergeometric series ${}_2\psi_2$, ${}_3\psi_3$ in terms of continued fractions. In the present paper certain new results for the quotients of two ${}_3\psi_3$ series in terms of continued fractions have been established following the techniques developed by Pankaj Srivastava [21] and also certain special cases have been deduced.

2. DEFINITIONS AND NOTATIONS

We shall use the following q-symbols:
For

$$|q|<1 \text{ and } |q^r|<1$$

$$(a;q)_n=\prod_{s=0}^{n-1}(1-aq^s), \quad n\geq 1$$

$$(a;q^r)_n=\prod_{s=0}^{n-1}(1-aq^{rs}), \quad n\geq 1$$

$$(a;q)_0=1, \quad (a;q^r)_0=1$$

$$(a;q^r)_\infty=\prod_{s=0}^{\infty}(1-aq^{rs})$$

$$(a)_n=(a;q)_n.$$

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A generalized basic hypergeometric series with base q is defined as:

$${}_A\phi_{A-1}\left[\begin{matrix} a_1, & a_2, & ..., & a_A \\ b_1, & b_2, & ..., & b_{A-1}\end{matrix} ;q;\quad z\right]=\sum_{n=0}^{\infty}\frac{(a_1;q)_n...(a_A;q)_n z^n}{(b_1;q)_n...(b_{A-1};q)_n(q;q)_n},$$

where $|z|<1,\ |q|<1$.

A generalized basic bilateral hypergeometric series with base q is defined as:

$${}_A\psi_A\left[\begin{matrix} a_1, & a_2, & ..., & a_A \\ b_1, & b_2, & ..., & b_A\end{matrix} ;q;\quad z\right]=\sum_{n=-\infty}^{\infty}\frac{(a_1;q)_n...(a_A;q)_n z^n}{(b_1;q)_n...(b_A;q)_n},$$

where $|b_1b_2...b_A/a_1a_2...a_A|<|z|<1,\ |q|<1$.

A finite continued fraction is an expression of the type:

$$\frac{a_1}{a_2+}\frac{a_3}{a_4+}\frac{a_5}{a_6+}\frac{a_7}{a_8+}\frac{a_9}{a_{10}+}\frac{a_{11}}{a_{12}+}{}_{...}\frac{a_{n-1}}{a_n}.$$

where $a_1, a_2, a_3, a_4, ...$ are real or complex numbers.

It is an infinite continued fraction when $n\to\infty$:

In order to establish main results, we shall make use of the following known results:

S. N. Singh [20] has established the following general transformation formula between basic and basic bilateral hypergeometric series:

$${}_{r+3}\psi_{r+3}\left[\begin{matrix} \frac{a}{b}, & cq, & dq, & \frac{b_1}{b}q^{m_1}, & ..., & \frac{b_r}{b}q^{m_r} \\ \frac{q}{b}, & c, & d, & \frac{b_1}{b}, & ..., & \frac{b_r}{b}\end{matrix} ;q;\quad z\right]=\frac{\left(q,\frac{bq}{az},\frac{az}{b},\frac{q}{a};q\right)_\infty}{\left(\frac{q}{b},\frac{q}{az},az,\frac{bq}{a};q\right)_\infty}$$
$$\times\frac{(1-bc)(1-bd)(b_1;q)_{m_1}...(b_r;q)_{m_r}}{(1-c)(1-d)(b_1/b;q)_{m_1}...(b_r/b;q)_{m_r}}\times{}_{r+3}\phi_{r+2}\left[\begin{matrix} a, & bcq, & bdq, & b_1q^{m_1}, & ..., & b_rq^{m_r} \\ & bc, & bd, & b_1, & ..., & b_r\end{matrix} ;q;\quad z\right] \quad (1)$$

Bhargava, Adiga and Somashekara [8] have established the following results for the quotients of two ${}_3\phi_2$ series, which are as follows:

$$\frac{{}_3\phi_2\left[\begin{matrix} a, & b, & cq \\ & dq, & e\end{matrix} ;q;\quad de/abc\right]}{{}_3\phi_2\left[\begin{matrix} a, & b, & cq \\ & d, & e\end{matrix} ;q;\quad de/abc\right]}=\frac{(1-d)}{(1-d)+}\frac{E_0}{(1-dq)(1-e/cq)+}$$
$$\frac{F_0}{(1-dq^2)+}\cdots\frac{E_n}{(1-dq^{2n+1})(1-e/cq)+}\frac{F_n}{(1-dq^{2n+2})+}\cdots \quad (2)$$

where

$$E_n=(deq^n/abc)(1-aq^n)(1-bq^n)(dq^n-c),\ \ n=0,1,2,...$$

and

$$F_n=(e/cq)[1-(dq^{n+1}/a)][1-(dq^{n+1}/b)](1-cq^{n+1}),\ \ n=0,1,2,...$$

$$\frac{{}_3\phi_2\left[\begin{matrix} a, & b, & c \\ dq, & e & \end{matrix} ;q;\quad deq/abc\right]}{{}_3\phi_2\left[\begin{matrix} a, & b, & c \\ d, & e & \end{matrix} ;q;\quad de/abc\right]} = \frac{(1-d)[1-(de/abc)]}{(1-d)[1-(de/abc)]+}\frac{D_0}{(1-dq)+}$$

$$\frac{C_0}{(1-dq^2)[1-(de/abc)]+}\cdots\frac{D_n}{(1-dq^{2n+1})+}\frac{C_n}{(1-dq^{2n+2})[1-(de/abc)]+}\cdots \tag{3}$$

where

$$C_n = -eq^n[1-(dq^{n+1}/a)][1-(dq^{n+1}/b)][1-(dq^{n+1}/c)],\; n=0,1,2,\ldots$$

and

$$D_n = (de/abc)(1-aq^n)(1-bq^n)(1-cq^n),\; n=0,1,2,\ldots$$

$$\frac{{}_3\phi_2\left[\begin{matrix} aq, & bq, & c \\ d, & eq & \end{matrix} ;q;\quad de/abcq\right]}{{}_3\phi_2\left[\begin{matrix} a, & b, & c \\ d, & e & \end{matrix} ;q;\quad de/abc\right]}$$

$$= \frac{(1-e)/[1-(de/abcq)]}{1+}\frac{c[(de/abcq)-(e/c)][1-(d/c)]}{(1-d)[1-(de/abcq)]+}\frac{(de/abcq)(1-aq)(1-bq)(1-c)}{(1-aq)+}$$

$$\frac{B_1}{[1-(de/abcq)]+}\frac{A_1}{(1-aq)+}\cdots\frac{B_n}{[1-(de/abcq)]+}\frac{A_n}{(1-aq)+}\cdots \tag{4}$$

where

$$A_n = (de/abcq)(1-bq^{n+1})(1-cq^n),\; n=1,2,3,\ldots$$

and

$$B_n = aq[1-(dq^{n-1}/a)][1-(eq^{n-1}/a)],\; n=1,2,3,\ldots$$

The identity due to Gasper and Rahman [13, III.9, pp. 241] is:

$${}_3\phi_2\left[\begin{matrix} a, & b, & c \\ d, & e & \end{matrix} ;q;\quad de/abc\right] = \frac{(e/a, de/bc;q)_\infty}{(e, de/abc;q)_\infty}\,{}_3\phi_2\left[\begin{matrix} a, & d/b, & d/c \\ d, & de/bc & \end{matrix} ;q;\quad e/a\right] \tag{5}$$

3. MAIN RESULTS

We have established the following results which are as follows:

$$\frac{{}_3\psi_3\left[\begin{matrix} a/b, & b_1q^{m_1}/b, & b_2q^{m_2+1}/b \\ q/b, & b_1/b, & b_2/b \end{matrix} ;q;\quad \frac{1}{aq^{m_1+m_2}}\right]}{{}_3\psi_3\left[\begin{matrix} a/b, & b_1q^{m_1}/b, & b_2q^{m_2}/b \\ q/b, & b_1/b, & b_2/b \end{matrix} ;q;\quad \frac{1}{aq^{m_1+m_2}}\right]} = \frac{(b_2q;q)_{m_2}(b_2/b;q)_{m_2}}{(b_2q/b;q)_{m_2}(b_2;q)_{m_2}}\times\left[\frac{(1-b_2)}{(1-b_2)+}\right.$$

$$\left.\frac{E_0}{(1-b_2q)(1-b_1/b_2q^{m_2+1})+}\frac{F_0}{(1-b_2q^2)+}\cdots\frac{E_n}{(1-b_2q^{2n+1})(1-b_1/b_2q^{m_2+1})+}\frac{F_n}{(1-b_2q^{2n+2})+}\cdots\right] \tag{6}$$

where

$$E_n = (q^n/aq^{m_1+m_2})(1-aq^n)(1-b_1q^{m_1+n})(b_2q^n - b_2q^{m_2}),\; n=0,1,2,\ldots$$

and

$$F_n = (b_1/b_2q^{m_2+1})(1-b_2q^{n+1}/a)(1-b_2q^{n+1}/b_1q^{m_1})(1-b_2q^{m_2+n+1}),\; n=0,1,2,\ldots$$

$$\frac{{}_3\psi_3\left[\begin{matrix} a/b, & b_1q^{m_1+1}/b, & b_2q^{m_2+1}/b \\ q/b, & b_1q/b, & b_2q/b \end{matrix} ;q; \frac{1}{aq^{m_1+m_2}}\right]}{{}_3\psi_3\left[\begin{matrix} a/b, & b_1q^{m_1}/b, & b_2q^{m_2}/b \\ q/b, & b_1/b, & b_2/b \end{matrix} ;q; \frac{1}{aq^{m_1+m_2}}\right]} = \frac{(b-b_1)(b-b_2)(1-b_1q^{m_1})}{(1-b_1)(1-b_2)(b-b_1q^{m_1})} \times \frac{(1-b_2q^{m_2})a(1-b_2)}{(b-b_2q^{m_2})(a-b_2)}$$

$$\left[\frac{(1-b_1)(a-b_2)}{(1-b_1)(a-b_2)+}\frac{D_0}{(1-b_1q)+}\frac{C_0}{(1-b_1q^2)(1-b_2/a)+}\cdots\frac{D_n}{(1-b_1q^{2n+1})+}\frac{C_n}{(1-b_1q^{2n+2})(1-b_2/a)+}\cdots\right] \tag{7}$$

where

$$C_n = -q^{n-(m_1+m_2)}(1-b_1q^{n+1}/a)(1-b_1q^{n+m_1+1})(1-b_2q^{n+m_2+1}),\ n = 0,1,2,\ldots$$

and

$$D_n = (b_2/a)(1-aq^n)(1-q^{n-m_1})(1-b_1q^{n-m_2}/b_2),\ n = 0,1,2,\ldots$$

$$\frac{{}_3\psi_3\left[\begin{matrix} aq/b, & b_1q^{m_1+1}/b, & b_2q^{m_2}/b \\ q/b, & b_1q/b, & b_2/b \end{matrix} ;q; \frac{1}{aq^{m_1+m_2+1}}\right]}{{}_3\psi_3\left[\begin{matrix} a/b, & b_1q^{m_1}/b, & b_2q^{m_2}/b \\ q/b, & b_1/b, & b_2/b \end{matrix} ;q; \frac{1}{aq^{m_1+m_2}}\right]}$$

$$= \frac{(a-1)(1-b_1q^{m_1})(b-b_1)}{(a-b)(b-b_1q^{m_1})(1-b_1)}$$

$$\left[\frac{(1-b_1)(1-1/aq^{m_1+m_2+1})}{1+}\frac{b_2q^{m_2}(1/aq^{m_1+m_2+1}-b_1q^{-m_2}/b_2)(1-q^{-m_2})}{(1-b_2)(1-1/aq^{m_1+m_2+1})+}\frac{(1/aq^{m_1+m_2+1})(1-aq)(1-b_1q^{m_1+1})(1-b_2q^{m_2})}{(1-aq)+}\right.$$

$$\left.\frac{B_1}{(1-1/aq^{m_1+m_2+1})+}\frac{A_1}{(1-aq)+}\cdots\frac{B_n}{(1-1/aq^{m_1+m_2+1})+}\frac{A_n}{(1-aq)+}\cdots\right],\ m_2 \neq 0 \tag{8}$$

where

$$A_n = (1/aq^{m_1+m_2+1})(1-b_1q^{m_1+n+1})(1-b_2q^{m_2+n}),\ n = 1,2,3,\ldots$$

and

$$B_n = aq(1-b_2q^{n-1}/a)(1-b_1q^{n-1}/a),\ n = 1,2,3,\ldots$$

4. PROOF OF MAIN RESULTS

The proof of main results 6, 7 and 8 are given below:

Proof of (6): Taking $r = 2, c = d = 0$ in transformation formula (1), we get

$${}_3\psi_3\left[\begin{matrix} a/b, & b_1q^{m_1}/b, & b_2q^{m_2}/b \\ q/b, & b_1/b, & b_2/b \end{matrix} ;q; z\right]$$
$$= \frac{\left(q, \frac{bq}{az}, \frac{az}{b}, \frac{q}{a}; q\right)_\infty}{\left(\frac{q}{b}, \frac{q}{az}, az, \frac{bq}{a}; q\right)_\infty} \times \frac{(b_1;q)_{m_1}(b_2;q)_{m_2}}{(b_1/b;q)_{m_1}(b_2/b;q)_{m_2}} \times {}_3\phi_2\left[\begin{matrix} a, & b_1q^{m_1}, & b_2q^{m_2} \\ & b_1, & b_2 \end{matrix} ;q; z\right] \tag{9}$$

replacing b_2 by b_2q and putting $z = 1/aq^{m_1+m_2}$ in (9), further taking $z = 1/aq^{m_1+m_2}$ in (9) and taking ratio of these two and making use of the result (2) and after simplification, we obtain the result (6).

Proof of (7): Taking $z = 1/aq^{m_1+m_2}$ in (9), we get

$$ {}_3\psi_3\left[\begin{matrix} a/b, & b_1q^{m_1}/b, & b_2q^{m_2}/b \\ q/b, & b_1/b, & b_2/b \end{matrix};q;\ \frac{1}{aq^{m_1+m_2}}\right] = \frac{\left(q, bq^{m_1+m_2+1}, 1/bq^{m_1+m_2}, q/a;q\right)_\infty}{\left(q/b, q^{m_1+m_2+1}, 1/q^{m_1+m_2}, bq/a;q\right)_\infty} $$

$$ \times\frac{(b_1;q)_{m_1}(b_2;q)_{m_2}}{(b_1/b;q)_{m_1}(b_2/b;q)_{m_2}}\times {}_3\phi_2\left[\begin{matrix} a, & b_1q^{m_1}, & b_2q^{m_2} \\ & b_1, & b_2 \end{matrix};q;\ \frac{1}{a\ q^{m_1+m_2}}\right] \tag{10} $$

making use of result (5) in (10) and then replace b_1, b_2 by b_1q, b_2q in that result. Again we use the result (5) in (10) and take the ratio of these two results, finally making use of the result (3) and after simplification; we get the result (7).

Proof of (8): Replacing a by aq and b_1 by b_1q in (10), we get

$$ {}_3\psi_3\left[\begin{matrix} aq/b, & b_1q^{m_1+1}/b, & b_2q^{m_2}/b \\ q/b, & b_1q/b, & b_2/b \end{matrix};q;\ \frac{1}{aq^{m_1+m_2+1}}\right] = \frac{\left(q, bq^{m_1+m_2+1}, 1/bq^{m_1+m_2}, 1/a;q\right)_\infty}{\left(q/b, q^{m_1+m_2+1}, 1/q^{m_1+m_2}, b/a;q\right)_\infty} $$

$$ \times\frac{(b_1q;q)_{m_1}(b_2;q)_{m_2}}{(b_1q/b;q)_{m_1}(b_2/b;q)_{m_2}}\times {}_3\phi_2\left[\begin{matrix} aq, & b_1q^{m_1+1}, & b_2q^{m_2} \\ & b_1q, & b_2 \end{matrix};q;\ \frac{1}{a\ q^{m_1+m_2+1}}\right] \tag{11} $$

taking the ratio of (11) and (10) and making use of result (4) and after simplification, we get the result (8).

5. SPECIAL CASES:

In this section, we shall consider certain applications of the main results obtained in the §3.

(1) Putting $m_1 = 0$ in result (6), we get

$$ \frac{{}_2\psi_2\left[\begin{matrix} a/b, & b_2q^{m_2+1}/b \\ q/b, & b_2/b \end{matrix};q;\ \frac{1}{aq^{m_2}}\right]}{{}_2\psi_2\left[\begin{matrix} a/b, & b_2q^{m_2}/b \\ q/b, & b_2/b \end{matrix};q;\ \frac{1}{aq^{m_2}}\right]} = \frac{\left(b_2q;q\right)_{m_2}\left(b_2/b;q\right)_{m_2}}{\left(b_2q/b;q\right)_{m_2}\left(b_2;q\right)_{m_2}}\times\left[\frac{(1-b_2)}{(1-b_2)+}\ \frac{E_o}{(1-b_2q)\left(1-b_1/b_2q^{m_2+1}\right)+} \right. $$

$$ \left. \frac{F_0}{\left(1-b_2q^2\right)+}\cdots\frac{E_n}{\left(1-b_2q^{2n+1}\right)\left(1-b_1/b_2q^{m_2+1}\right)+}\ \frac{F_n}{\left(1-b_2q^{2n+2}\right)+}\cdots\right] \tag{12} $$

where

$$ E_n = \left(q^n/aq^{m_2}\right)\left(1-aq^n\right)\left(1-b_1q^n\right)\left(b_2q^n - b_2q^{m_2}\right), \qquad n = 0,1,2,... $$

and

$$ F_n = \left(b_1/b_2q^{m_2+1}\right)\left(1-b_2q^{n+1}/a\right)\left(1-b_2q^{n+1}/b_1\right)\left(1-b_2q^{m_2+n+1}\right), \qquad n = 0,1,2,... $$

(2) Putting $m_2 = 0$ in result (11) and making use of the Ramanujan $_1\psi_1$ summation formula [13, (5.2.1), pp. 126], we get

$$
{}_2\psi_2\left[\begin{matrix} a/b, & b_2q/b \\ & & ;q;\ \dfrac{1}{a} \\ q/b, & b_2/b \end{matrix}\right]
$$

$$
=\frac{\left(q,q/a,1/b,qb;q\right)_\infty}{\left(q/b,qb/a,1/a,b^2;q\right)_\infty}\times\left[\frac{(1-b_2)}{(1-b_2)+}\quad\frac{G_0}{(1-b_2q)(1-b_1/b_2q)+}\right.
$$

$$
\left.\frac{H_o}{\left(1-b_2q^2\right)+}\cdots\cdots\frac{G_n}{\left(1-b_2q^{2n+1}\right)(1-b_1/b_2q)+}\ \frac{H_n}{\left(1-b_2q^{2n+2}\right)+}\cdots\right] \tag{14}
$$

where

$$
G_n=\left(q^n/a\right)\left(1-aq^n\right)\left(1-b_1q^n\right)\left(b_2q^n-b_2\right),\qquad n=0,1,2,\ldots
$$

and

$$
H_n=\left(b_1/b_2q\right)\left(1-b_2q^{n+1}/a\right)\left(1-b_2q^{n+1}/b\right)\left(1-b_2q^{n+1}\right),\qquad n=0,1,2,\ldots
$$

(3) Putting $b_1=a$ and $b=bq$ in result (7), we get

$$
\frac{{}_2\psi_2\left[\begin{matrix} aq^{m_1}/b, & b_2q^{m_2}/b \\ & & ;q;\ \dfrac{1}{aq^{m_1+m_2}} \\ 1/b, & b_2/b \end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix} aq^{m_1-1}/b, & b_2q^{m_2-1}/b \\ & & ;q;\ \dfrac{1}{aq^{m_1+m_2}} \\ 1/b, & b_2/bq \end{matrix}\right]}=\frac{(bq-a)(bq-b_2)\left(1-aq^{m_1}\right)}{(1-a)(1-b_2)\left(bq-aq^{m_1}\right)}\times\frac{\left(\left(1-b_2q^{m_2}\right)a\left(1-b_2\right)\right)}{\left(b-qb_2q^{m_2}\right)\left(a-b_2\right)}
$$

$$
\left[\frac{(1-a)(a-b_2)}{(1-a)(a-b_2)+}\ \frac{D_0}{(1-aq)+}\ \frac{C_0}{\left(1-aq^2\right)(1-b_2/a)+}\cdots\frac{D_n}{\left(1-aq^{2n+1}\right)+}\ \frac{C_n}{\left(1-aq^{2n+2}\right)(1-b_2/a)+}\cdots\right] \tag{14}
$$

where

$$
C_n=-q^{n-(m_1+m_2)}\left(1-q^{n+1}\right)\left(1-aq^{n+m_1+1}\right)\left(1-b_2q^{n+m_2+1}\right),\qquad n=0,1,2,\ldots
$$

and

$$
D_n=\left(b_2/a\right)\left(1-aq^n\right)\left(1-q^{n-m_1}\right)\left(1-aq^{n-m_2}/b\right),\quad n=0,1,2,\ldots
$$

(4) Putting $b=1$ in result (6), we get

$$
\frac{{}_3\phi_2\left[\begin{matrix} a, & b_1q^{m_1}, & b_2q^{m_2+1} \\ & & & ;q;\ \dfrac{1}{aq^{m_1+m_2}} \\ & b_1, & b_2 \end{matrix}\right]}{{}_3\phi_2\left[\begin{matrix} a, & b_1q^{m_1}, & b_2q^{m_2} \\ & & & ;q;\ \dfrac{1}{aq^{m_1+m_2}} \\ & b_1, & b_2 \end{matrix}\right]}=\left[\frac{(1-b_2)}{(1-b_2)+}\ \frac{E_o}{(1-b_2q)\left(1-b_1/b_2q^{m_2+1}\right)+}\right.
$$

$$\frac{F_0}{\left(1-b_2q^2\right)+}\cdots\frac{E_n}{\left(1-b_2q^{2n+1}\right)\left(1-b_1/b_2q^{m_2+1}\right)+}\quad\frac{F_n}{\left(1-b_2q^{2n+2}\right)+}\cdots\Bigg] \tag{15}$$

where

$$E_n=\left(q^n/aq^{m_1+m_2}\right)\left(1-aq^n\right)\left(1-b_1q^{m_1+n}\right)\left(b_2q^n-b_2q^{m_2}\right),\qquad n=0,1,2,\ldots$$

and

$$F_n=\left(b_1/b_2q^{m_2+1}\right)\left(1-b_2q^{n+1}/a\right)\left(1-b_2q^{n+1}/b_1q^{m_1}\right)\left(1-b_2q^{m_2+n+1}\right),\qquad n=0,1,2,\ldots$$

(5) Putting $a=\alpha q$ and $b=1$ in result (7), we get

$$\frac{{}_3\phi_2\left[\begin{matrix}\alpha q, & b_1q^{m_1+1}, & b_2q^{m_2+1} \\ & b_1q, & b_2q\end{matrix};q;\frac{1}{\alpha q^{m_1+m_2+1}}\right]}{{}_3\phi_2\left[\begin{matrix}\alpha q, & b_1q^{m_1}, & b_2q^{m_2} \\ & b_1, & b_2\end{matrix};q;\frac{1}{\alpha q^{m_1+m_2+1}}\right]}=\frac{\alpha q\left(1-b_2\right)}{\left(\alpha q-b_2\right)}\left[\frac{\left(1-b_1\right)\left(\alpha q-b_2\right)}{\left(1-b_1\right)\left(\alpha q-b_2\right)+}\right.$$

$$\left.\frac{D_o}{\left(1-b_1q\right)+}\frac{C_0}{\left(1-b_1q^2\right)\left(1-b_2/\alpha q\right)+}\cdots\frac{D_n}{\left(1-b_1q^{2n+1}\right)+}\frac{C_n}{\left(1-b_1q^{2n+2}\right)\left(1-b_2/\alpha q\right)+}\cdots\right] \tag{16}$$

where $|\alpha|>1$

$$C_n=-q^{n-(m_1+m_2)}\left(1-b_1q^n/\alpha\right)\left(1-b_1q^{n+m_1+1}\right)\left(1-b_2q^{n+m_2+1}\right),\qquad n=0,1,2,\ldots$$

and

$$D_n=\left(b_2/\alpha q\right)\left(1-\alpha q^{n+1}\right)\left(1-q^{n-m_1}\right)\left(1-b_1q^{n-m_2}/b_2\right),\qquad n=0,1,2,\ldots$$

(6) Putting $b_1=0$ in result (8), we get

$$\frac{{}_2\psi_2\left[\begin{matrix}aq/b, & b_2q^{m_2}/b \\ q/b, & b_2/b\end{matrix};q;\frac{1}{aq^{m_1+m_2+1}}\right]}{{}_2\psi_2\left[\begin{matrix}a/b, & b_2q^{m_2}/b \\ q/b, & b_2/b\end{matrix};q;\frac{1}{aq^{m_1+m_2}}\right]}=\frac{(a-1)}{(a-b)}\times\left[\frac{1/(1-1/aq^{m_1+m_2+1})}{1+}\ \frac{(b_2q^{-(m_1+1)}/a)(1-q^{-m_2})}{\left(1-b_2\right)(1-1/aq^{m_1+m_2+1})+}\right.$$

$$\left.\frac{(1-1/aq^{m_1+m_2+1})(1-aq)(1-b_2q^{m_2})}{(1-aq)+}\frac{B_1}{(1-1/aq^{m_1+m_2+1})+}\ \frac{A_1}{(1-aq)+}\cdots\frac{B_n}{(1-1/aq^{m_1+m_2+1})+}\ \frac{A_n}{(1-aq)+}\cdots\right],\ m_2\neq 0 \tag{17}$$

where

$$A_n=(1/aq^{m_1+m_2+1})(1-b_2q^{m_2+n}),\quad n=1,2,3,\ldots$$

and

$$B_n=aq(1-b_2q^{n-1}/a),\qquad n=1,2,3,\ldots$$

(7) Putting $b = 1$ and $b_1 = 0$ in result (8), we get

$$\frac{{}_2\phi_1\left[\begin{matrix} aq, & b_2q^{m_2} \\ & b_2 \end{matrix} ;q; \frac{1}{aq^{m_1+m_2+1}}\right]}{{}_2\phi_1\left[\begin{matrix} a, & b_2q^{m_2} \\ & b_2 \end{matrix} ;q; \frac{1}{aq^{m_1+m_2}}\right]} = \left[\frac{1/(1-1/aq^{m_1+m_2+1})}{1+}\,\frac{b_2q^{-(m_1+1)}/a(1-q^{-m_2})}{(1-b_2)(1-1/aq^{m_1+m_2+1})+}\,\frac{(1/aq^{m_1+m_2+1})(1-aq)(1-b_2q^{m_2})}{(1-aq)+}\right.$$

$$\left.\frac{B_1}{(1-1/aq^{m_1+m_2+1})+}\,\frac{A_1}{(1-aq)+}\cdots\frac{B_n}{(1-1/aq^{m_1+m_2+1})+}\,\frac{A_n}{(1-aq)+}\cdots\right],\ m_2 \neq 0 \tag{18}$$

where

$$A_n = (1/aq^{m_1+m_2+1})(1-b_2q^{m_2+n}), \qquad n = 1, 2, 3, \ldots$$

and

$$B_n = aq(1-b_2q^{n-1}/a), \qquad n = 1, 2, 3, \ldots$$

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