

UNSTEADY OSCILLATORY MHD FLOW OF A VISCO-ELASTIC FLUID PAST
 A POROUS VERTICAL PLATE WITH PERIODIC SUCTION

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ABSTRACT

The two dimensional oscillatory MHD flow of a visco-elastic incompressible electrically conducting fluid past a porous vertical plate with periodic suction when the free stream velocity and temperature vary periodically with time has been investigated analytically. A magnetic field of uniform strength is transversely applied to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The multi-parameter perturbation scheme is used to solve the governing equations. The expression for dimensionless velocity, temperature and skin friction at the plate has been obtained and numerically worked out for different values of flow parameters involved in the solution. The dimensionless velocity has been presented graphically and skin friction at the plate has been presented numerically for various values of visco-elastic parameter with the combination of other flow parameters.

Key words and Phrases: Visco-elastic, electrically conducting fluid, MHD, periodic suction, viscous energy dissipation and magnetic dissipation.

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INTRODUCTION:

The theory of unsteady boundary layer flow draw the attention of many researchers as the boundary layer responses to imposed fluctuations in plate suction. Nanda and Sharma (1) has extended the above theory to the case of free convection boundary layers along a semi- infinite vertical plate. Watson (2) has generalized the Lighthill's (3) problem to the case of a flow against a flat plate normal to the stream and having an arbitrary transverse motion.

The effects of free convection currents on the unsteady motion past an infinite vertical plate in the presence of constant suction has been analyzed by Soundalgekar (4). Stuart (5) has derived some interesting results concerning skin friction and temperature for an oscillatory flow past an infinite plate with constant suction. The oscillations in the flow were induced by a time dependent free stream velocity. Messiha (6) has extended Stuart's work to the case of periodic plate suction. The study of an unsteady MHD free convection flow of viscous incompressible electrically conducting fluid past an infinite porous plate with constant suction in presence of heat sink has been studied by Sahoo et al. (7). The flow of an electrically conducting viscous incompressible fluid between two parallel flat porous plates in presence of a time varying pressure gradient has been investigated by Kishore et. al.(8). Ahmed et. al. (9) have studied the laminar boundary layers in oscillatory MHD flow past a porous vertical plate with periodic suction.

The aim of the present paper is to investigate the combined effects of viscous energy dissipation and magnetic dissipation in presence of oscillatory plate-suction and periodic free stream velocity on the flow and heat transfer of an electrically conducting visco-elastic incompressible fluid characterized by Walters liquid (Model B').

The constitutive equation for Walters liquid (Model B') is

$$\sigma^{ik} = -p g_{ik} + 2\eta_0 e^{ik} - 2k_0 e^{ik} \tag{1.1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i , v^i is the velocity vector, the contravariant form of e^{ik} is given by

$$e^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e_{,m}^{ik} - v_{,m}^k e^{im} - v_{,m}^i e^{mk} \tag{1.2}$$

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It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{,k}^i + v_{,i}^k \quad (1.3)$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau) d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau) d\tau, \quad (1.4)$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters (10, 11). This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau) d\tau, \quad n \geq 2$$

have been neglected. (1.5)

MATHEMATICAL FORMULATION:

We consider the flow of a visco-elastic incompressible electrically conducting fluid past an infinite porous vertical plate with the fluid region. The suction velocity is in the negative Y direction and varies periodically with time about a non-zero constant mean. The flow being incompressible, the density varies with the temperature only in the buoyancy term where Boussinesq approximation is used. All of the quantities, except possibly the pressure p , are independent of \bar{x} . Also, a magnetic field of uniform strength B_0 is applied transverse to the direction of the free stream flow. With the foregoing assumptions, the equations governing flow and heat transfer for the present problem become

$$\rho \left[\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right] = \rho \frac{\partial \bar{U}}{\partial \bar{t}} + \mu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\rho\beta(\bar{T} - \bar{T}_\infty) + \sigma B_0^2(\bar{U} - \bar{u}) - k_0 \left[\frac{\partial^3 \bar{u}}{\partial \bar{t} \partial \bar{y}^2} + \bar{v} \frac{\partial^3 \bar{u}}{\partial \bar{y}^3} \right] \quad (2.1)$$

$$\rho C_p \left[\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} \right] = K \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 + \sigma B_0^2(\bar{U} - \bar{u})^2 - k_0 \left[\frac{\partial \bar{u}}{\partial \bar{y}} \frac{\partial^2 \bar{u}}{\partial \bar{t} \partial \bar{y}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right] \quad (2.2)$$

where $\bar{v} = -v_0(1 + \epsilon A e^{i\omega \bar{t}})$, ρ is the fluid density, g is the acceleration due to gravity, C_p is the specific heat at constant pressure, K is the thermal conductivity of the fluid medium, T is the fluid temperature, σ is the electrical conductivity of the fluid, β is the coefficient of volume expansion of the fluid, μ is the coefficient of viscosity of the fluid, ϵ is a small perturbation parameter, k_0 is the non-Newtonian parameter, A (suction parameter) is a positive real constant and other symbols have their usual meanings. The relative boundary conditions are as under:

$$\begin{aligned} \bar{y} = 0; \quad \bar{u} = 0, \quad \bar{T} = \bar{T}_0 \\ \bar{u} \rightarrow \infty; \quad \bar{u} \rightarrow \bar{U}, \quad \bar{T} \rightarrow \bar{T}_\infty \end{aligned} \quad (2.3)$$

We now introduce the following non-dimensional quantities:

$$y = \frac{\bar{y}v_0}{v}, \quad t = \frac{v_0^2 \bar{t}}{v}, \quad \omega = \frac{\bar{\omega}v}{v_0^2}, \quad \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_0 - \bar{T}_\infty}, \quad u = \frac{\bar{u}}{\bar{U}_0}, \quad U = \frac{\bar{U}}{\bar{U}_0},$$

$$P_r = \frac{\mu C_p}{K}, \quad G = \frac{v g \beta (\bar{T}_0 - \bar{T}_\infty)}{\bar{U}_0 v_0^2}, \quad E_c = \frac{\bar{U}_0^2}{C_p (\bar{T}_0 - \bar{T}_\infty)}, \quad M = \frac{\sigma B_0^2 v}{\rho v_0^2}$$

where \bar{U}_0 is the reference velocity and $v = \frac{\mu}{\rho}$ is the kinematic viscosity.

The non-dimensional forms of (2.1), (2.2) and (2.3) are as follows:

$$\frac{\partial u}{\partial t} - (1 + \epsilon A e^{i\omega t}) \frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} + M(U - u) + G\theta - k_1 \left[\frac{\partial^3 u}{\partial t \partial y^2} - (1 + \epsilon A e^{i\omega t}) \frac{\partial^3 u}{\partial y^3} \right] \quad (2.4)$$

$$\begin{aligned} P_r \left[\frac{\partial \theta}{\partial t} - (1 + \epsilon A e^{i\omega t}) \frac{\partial \theta}{\partial y} \right] \\ = \frac{\partial^2 \theta}{\partial y^2} + E_c P_r \left(\frac{\partial u}{\partial y} \right)^2 + M E P_r (U - u)^2 - E_c P_r k_1 \left[\frac{\partial u}{\partial y} \frac{\partial^3 u}{\partial t \partial y^2} - (1 + \epsilon A e^{i\omega t}) \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \right] \end{aligned} \quad (2.5)$$

where

$$k_1 = \frac{k_0 v_0^2}{\rho v^2}$$

The corresponding boundary conditions in non-dimensional forms are

$$y = 0, \quad u = 0, \quad \theta = 0$$

$$y \rightarrow \infty, \quad u \rightarrow 0, \quad \theta \rightarrow 0 \tag{2.6}$$

METHOD OF SOLUTION:

To solve equations (8.2.4) and (8.2.5) subject to the boundary conditions (2.6), the non-dimensional free stream velocity U , boundary layer velocity u and temperature θ are assumed, following Messiha (6) to be of the form

$$U = 1 + \varepsilon e^{i\omega t} \tag{3.1}$$

$$u = 1 - f_1(y) + \varepsilon e^{i\omega t} [1 - f_2(y)] \tag{3.2}$$

$$\theta = \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y) \tag{3.3}$$

Now, substituting (8.3.1) to (8.3.3) in equations (8.2.4) and (8.2.5), and equating the coefficients of ε^0 , ε^1 and neglecting those of ε^2 , the following equations for f_1 , f_2 and θ_0 , θ_1 are obtained:

$$f_1'' + f_1' - Mf_1 = G\theta_0 - k_1 f_1''' \tag{3.4}$$

$$f_2'' + f_2' - (i\omega + M)f_2 = -Af_1' + G\theta_1 - k_1[f_2''' + Af_1''' - i\omega f_2] \tag{3.5}$$

$$\theta_0'' + P_r \theta_0' = -E_c P_r f_1'^2 - ME_c P_r f_1'^2 - E_c P_r k_1 f_1 f_1'' \tag{3.6}$$

$$\theta_1'' + P_r \theta_1' - i\omega P_r \theta_1 = -AP_r \theta_0' - 2E_c P_r f_1' f_2' - 2ME_c P_r k_1 f_1 f_2 + E_c P_r k_1 [i\omega f_1' f_2' - f_1' f_2'' - f_1'' f_2' - Af_1' f_1''] \tag{3.7}$$

Here, the dashes denote differentiation with respect to y .

The boundary conditions in (8.2.6) give

$$y = 0; \quad f_1 = f_1 = 1, \quad \theta_0 = 1, \quad \theta_1 = 0$$

$$y \rightarrow \infty; \quad f_1 = f_1 = 0, \quad \theta_0 = 0, \quad \theta_1 = 0 \tag{3.8}$$

The equations (3.4) and (3.5) are still coupled for the functions f_0 , f_1 , θ_0 and θ_1 . To solve them we note that $E < 1$ for all incompressible fluids and hence we assume that

$$f_1(y) = f_{10}(y) + E_c f_{11}(y) + E_c^2 f_{12}(y) + \dots$$

$$f_2(y) = f_{20}(y) + E_c f_{21}(y) + E_c^2 f_{22}(y) + \dots$$

$$\theta_0(y) = \theta_{00}(y) + E_c \theta_{01}(y) + E_c^2 \theta_{02}(y) + \dots$$

$$\theta_1(y) = \theta_{10}(y) + E_c \theta_{11}(y) + E_c^2 \theta_{12}(y) + \dots \tag{3.9}$$

Substituting from (3.9) in equations (3.4) to (3.7) and by equating the coefficients of E^0 , E^1 in each equation and neglecting those of E^2 , we get

$$f_{10}'' + f_{10}' - Mf_{10} = G\theta_{00} - k_1 f_{10}''' \tag{3.10}$$

$$f_{11}'' + f_{11}' - Mf_{11} = G\theta_{01} - k_1 f_{11}''' \tag{3.11}$$

$$f_{20}'' + f_{20}' - (i\omega + M)f_{20} = -Af_{10}' + G\theta_{10} - k_1 [f_{20}''' + Af_{10}''' - i\omega f_{20}] \tag{3.12}$$

$$f_{21}'' + f_{21}' - (i\omega + M)f_{21} = -Af_{11}' + G\theta_{11} - k_1 [f_{21}''' + Af_{11}''' - i\omega f_{21}] \tag{3.13}$$

$$\theta''_{00} + P_r \theta'_{00} = 0 \quad (3.14)$$

$$\theta''_{01} + P_r \theta'_{01} - P_r f_{10}^2 - MP_r f_{10}^2 - P_r k_1 f_{10} \quad (3.15)$$

$$\theta''_{10} + P_r \theta'_{10} - i\omega P_r \theta_{10} = -AP_r \theta'_{00} \quad (3.16)$$

$$\begin{aligned} \theta''_{11} + P_r \theta'_{11} - i\omega P_r \theta_{11} \\ = -AP_r \theta'_{01} - 2P_r f'_{10} f'_{20} - 2MP_r k_1 f_{10} f_{20} + P_r k_1 [i\omega f'_{10} f'_{20} - f'_{10} f'_{20} - f''_{10} f'_{20} - Af'_{10} f''_{10}] \end{aligned} \quad (3.17)$$

Using (3.9), the boundary conditions (3.8) reduce to

$$\begin{aligned} y = 0; f_{10} = f_{20} = 1, f_{11} = f_{21} = 0, \theta_{00} = 1, \theta_{01} = \theta_{10} = \theta_{11} = 0 \\ y \rightarrow \infty; f_{10} = f_{11} = f_{20} = f_{21} = 0, \theta_{00} = \theta_{01} = \theta_{10} = \theta_{11} = 0 \end{aligned} \quad (3.18)$$

Again, to solve equations (3.10) to (3.17), we consider very small values of k_1 and expand $f_{10}, f_{11}, f_{20}, f_{21}, \theta_{01}, \theta_{11}$ in terms of k_1 since $k_1 \ll 1$ as follows:

$$\begin{aligned} f_{10}(y) &= f_{100}(y) + k_1 f_{101}(y) + k_1^2 f_{102}(y) + \dots \\ f_{11}(y) &= f_{110}(y) + k_1 f_{111}(y) + k_1^2 f_{112}(y) + \dots \\ f_{20}(y) &= f_{200}(y) + k_1 f_{201}(y) + k_1^2 f_{202}(y) + \dots \\ f_{21}(y) &= f_{210}(y) + k_1 f_{211}(y) + k_1^2 f_{212}(y) + \dots \\ \theta_{01}(y) &= \theta_{010}(y) + k_1 \theta_{011}(y) + k_1^2 \theta_{012}(y) + \dots \\ \theta_{11}(y) &= \theta_{110}(y) + k_1 \theta_{111}(y) + k_1^2 \theta_{112}(y) + \dots \end{aligned} \quad (3.19)$$

On using (3.19) into the equations (3.10) to (3.13), (3.15) and (3.17) and by equating the coefficient of k_1^0, k_1^1 in each equation and neglecting those of k_1^2 , we get

$$f''_{100} + f'_{100} - Mf_{100} = G\theta_{00} \quad (3.20)$$

$$f''_{101} + f'_{101} - Mf_{101} = -f'''_{100} \quad (3.21)$$

$$f''_{110} + f'_{110} - Mf_{110} = G\theta_{010} \quad (3.22)$$

$$f''_{111} + f'_{111} - Mf_{111} = G\theta_{011} - f'''_{110} \quad (3.23)$$

$$f''_{200} + f'_{200} - (i\omega + M)f_{200} = -Af'_{100} + G\theta_{10} \quad (3.24)$$

$$f''_{201} + f'_{201} - (i\omega + M)f_{201} = -Af'_{101} - k_1 [f'''_{200} + Af'''_{100} - i\omega f_{200}] \quad (3.25)$$

$$f''_{210} + f'_{210} - (i\omega + M)f_{210} = -Af'_{110} + G\theta_{110} \quad (3.26)$$

$$f''_{211} + f'_{211} - (i\omega + M)f_{211} = -Af'_{111} + G\theta_{111} - k_1 [f'''_{210} + Af'''_{110} - i\omega f_{210}] \quad (3.27)$$

$$\theta''_{010} + P_r \theta'_{010} = -P_r f_{100}^2 - MP_r f_{100}^2 \quad (3.28)$$

$$\theta''_{011} + P_r \theta'_{011} = -P_r f'_{100} f'_{101} - MP_r f_{100} f_{101} - f_{100} f''_{100} \quad (3.29)$$

$$\theta''_{110} + P_r \theta'_{110} - i\omega P_r \theta_{110} = -AP_r \theta'_{010} - 2P_r f'_{100} f'_{200} - 2MP_r k_1 f_{100} f_{200} \quad (3.30)$$

$$\begin{aligned} \theta''_{111} + P_r \theta'_{111} - i\omega P_r \theta_{111} = -AP_r \theta'_{011} - 2P_r [f'_{100} f'_{201} + f'_{101} f_{200}] - 2MP_r [f_{100} f_{201} + f_{101} f_{200}] + P_r [i\omega f'_{100} f'_{200} - \\ f'_{100} f'_{200} - f'_{100} f'_{200} - Af'_{100} f'_{100}] \end{aligned} \quad (3.31)$$

Using (3.19), the corresponding boundary conditions reduce to

$$y = 0; f_{100} = f_{200} = 1, f_{101} = f_{201} = 0, f_{110} = f_{111} = f_{210} = f_{211} = 0, \theta_{010} = \theta_{011} = \theta_{110} = \theta_{111} = 0$$

$$y \rightarrow \infty; f_{100} = f_{101} = f_{200} = f_{201} = f_{110} = f_{111} = f_{210} = f_{211} = 0, \theta_{010} = \theta_{011} = \theta_{110} = \theta_{111} = 0 \quad (3.32)$$

Solutions of equations (3.14), (3.16) and (3.20) to (3.31) subject to the boundary conditions (3.18) and (3.32) are obtained, but not presented here for the sake of brevity.

RESULTS AND DISCUSSIONS:

The non-dimensional skin friction at the plate $y=0$ is given by

$$\tau = H_0 + \varepsilon e^{i\omega t} H_1 \quad (4.1)$$

Where $H_0 = -f_1'(0) - k_1 f_1''(0)$, $H_1 = -f_2'(0)$

The dimensionless plate temperature $\theta(0)$ is given by

$$\theta(0) = \theta_0(0) + \varepsilon e^{i\omega t} \theta_1(0) \quad (4.2)$$

The purpose of this study is to bring out the effects of visco-elastic parameter on the unsteady MHD oscillatory flow past a porous vertical plate with periodic suction. The visco-elastic effect is exhibited through the non-dimensional parameter k_1 . The corresponding results for Newtonian fluid can be deduced from the above results by setting $k_1 = 0$ and it is worth mentioning here that these results coincide with that of Ahmed et. al.

Figures 1 to 5 depict the velocity profiles u against y for various values of the Prandtl number (P_r), Grashof number (G), Hartmann number (M), Eckert number (E) with $\omega t = \frac{\pi}{2}$, $\varepsilon = 0.01$ and $A = 4$ to observe the visco-elastic effects. From the figures it is observed that the velocity profile u diminishes due to the rise of the visco-elastic parameter in comparison with the Newtonian fluid ($k_1 = 0$). Also it is noticed that the velocity profile u decreases when Hartmann number increases (figures 1 & 2) & Prandtl number increases (figures 1 & 3) but the profiles of u increases when the Grashof number increases (figures 1 & 4) & Eckert number increases (figures 1 & 5) with the increasing values of the visco-elastic parameter $k_1 (= 0, 0.01, 0.02)$ in comparison with the Newtonian fluid.

Table 1 exhibits the variation of the shearing stress against different values of the flow parameters. It is seen from the table that the shearing stress τ diminishes when M enhances (cases I & II) and P increases (cases I & III) but τ increases when G increases (cases I & IV) and E increases with the increasing values of the visco-elastic parameter k_1 in comparison with the Newtonian fluid.

It is also observed that the temperature field is not significantly affected by the visco-elastic parameter k_1 .

CONCLUSION:

The unsteady MHD oscillatory flow of a visco-elastic fluid past a porous vertical plate with periodic suction has been studied in this chapter. In the analysis of the problem, the following conclusions are made.

- Increasing values of visco-elastic parameter lead to reduce the velocity profiles of non-Newtonian fluid in comparison with the Newtonian fluid.
- Growth of Magnetic parameter and Prandtl number decreases the velocity profile and shearing stress at the plate.
- Enhanced velocity profile and shearing stress at the plate are observed for increasing values of Grashof number and Eckert number.
- Plate temperature is not significantly affected by the visco-elastic parameter.

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TABLE AND GRAPHS:

Cases	M	P	G	E	Shearing Stress		
					$k_1=0$	$k_1=0.01$	$k_1=0.02$
I	3	5	7	0.01	1.5372	1.3297	1.1221
II	6	5	7	0.01	1.4987	1.2585	1.0184
III	3	8	7	0.01	0.8851	0.7173	0.5495
IV	3	5	10	0.01	2.2251	1.9296	1.6337
V	3	5	7	0.02	1.5521	1.3449	1.1376

Table 1: Shearing Stress with $\omega t = \frac{\pi}{2}$, $\varepsilon = 0.01$ and $A = 4$.

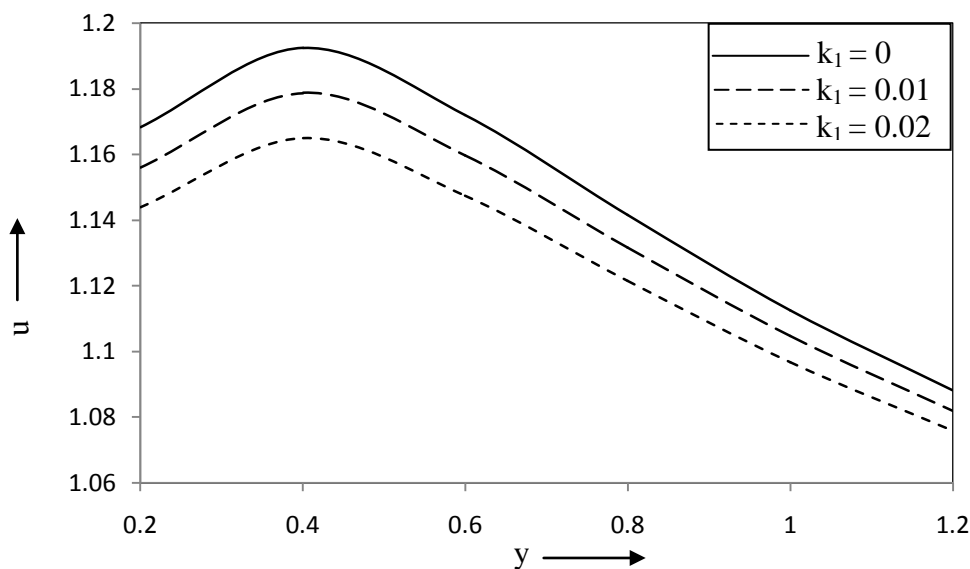


Figure 1: Velocity profile u against y for $M = 3$, $P = 5$, $G = 7$, $E = 0.01$.

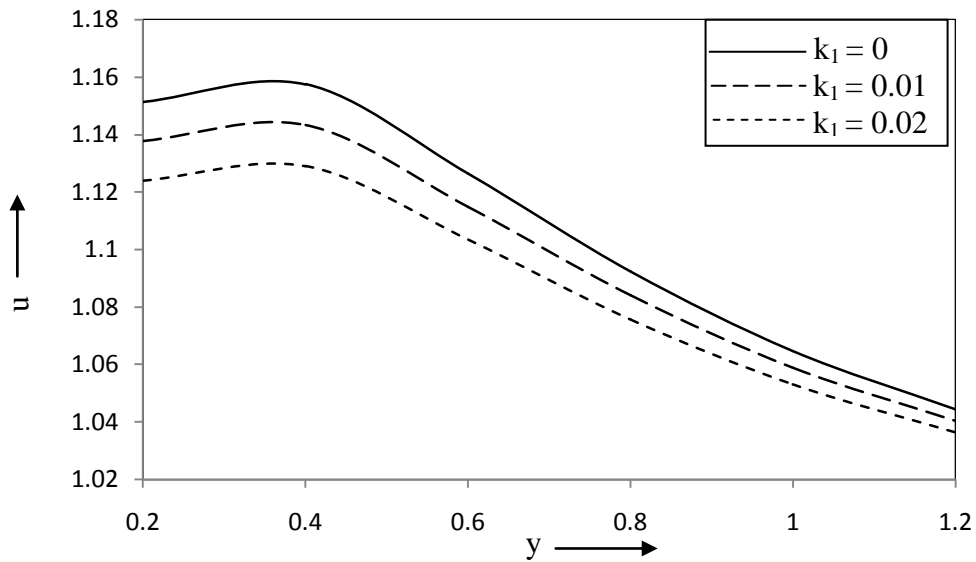


Figure 2: Velocity profile u against y for $M = 6, P = 5, G = 7, E = 0.01$.

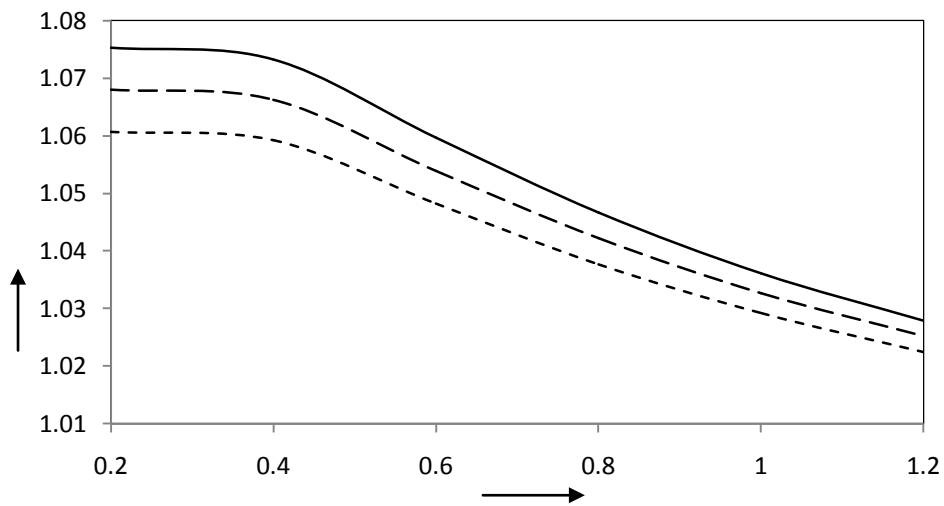


Figure 3: Velocity profile u against y for $M = 3, P = 8, G = 7, E = 0.01$.

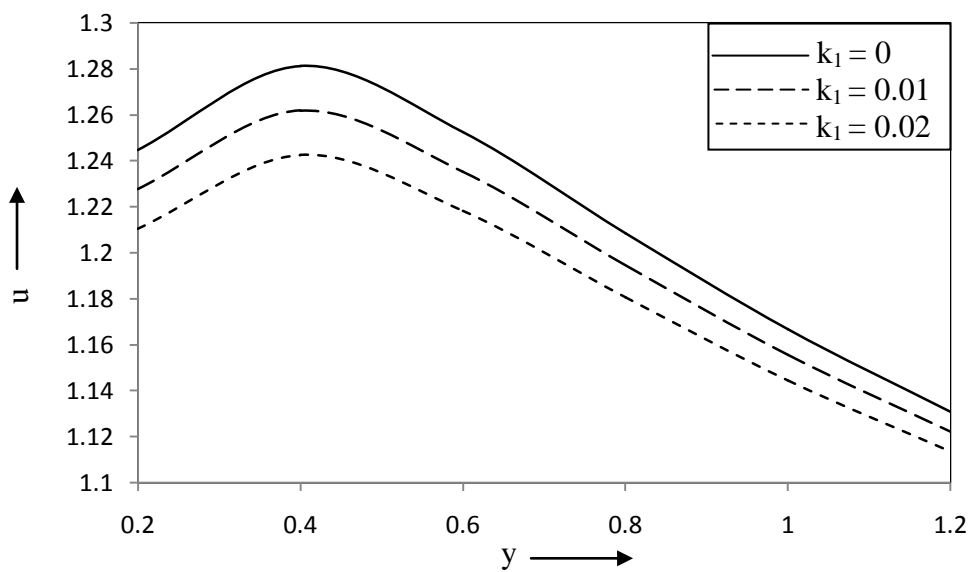


Figure 4: Velocity profile u against y for $M = 3, P = 5, G = 10, E = 0.01$.

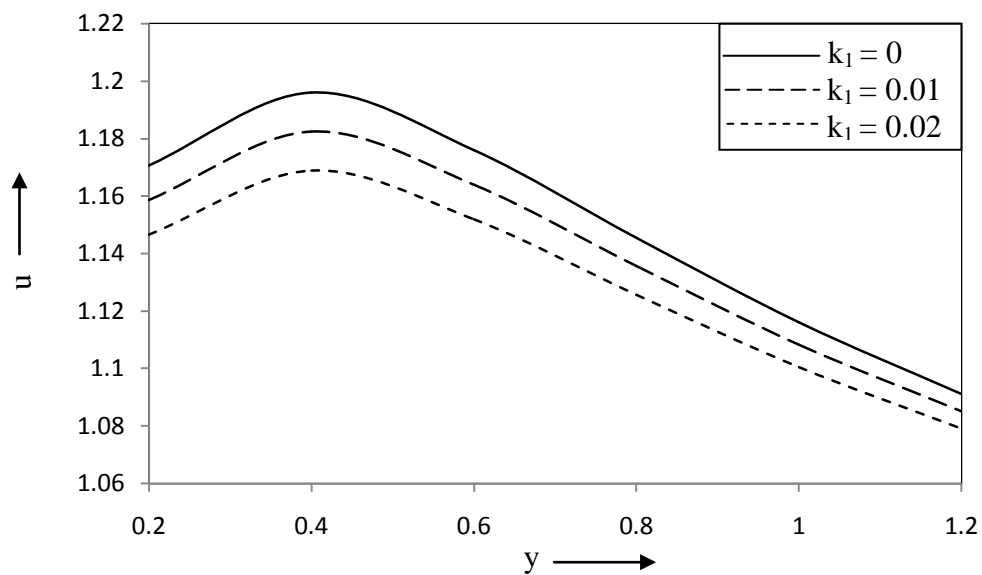


Figure 5: Velocity profile u against y for $M = 3$, $P = 5$, $G = 7$, $E = 0.02$.

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