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GRAPH OF SIMPLICIAL COMPLEXS

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ABSTRACT

In this paper, we will define a graph of simplicial complex; folding and unfolding of this graph of simplicial complex into itself will be discussed. Theorems governing these types of transformations are achieved.

Keywords: *Graphs, Simplicial complex.*

2000 Mathematics Subject Classification: *51H10, 57N20.*

INTRODUCTION

It is a classical theory of Rado (1925) that any compact surface can be discussed into a finite number of triangular regions. The incidence structure of these regions determines a combinatorial object called a **Complex,** which is a higher dimensional analog of a graph. Whereas a graph has only zero- dimensional cells called vertices and one- dimensional cells called edges. A complex may include pieces of any dimension. In particular, a 2-complex also contains some two-dimensional cells. Often these cells correspond to triangular regions, but more generally they may be arbitrary polygons, with any number of sides.

Knowing that any closed surface can be represented by such a finite combinatorial object enables us classify the closed surfaces, that is, to construct a countable list that includes them all and contains no repetitions.

DEFINITIONS AND BACKGROUND:

We will give some definitions which we will need them in this paper:

(1) An" abstract graph" G is a diagram consisting of a finite non empty set of elements, called "vertices" denoted by $V(G)$ together with a set of unordered pairs of these elements, called "edges" denoted by $E(G)$.The set of vertices of the graph G is called "the vertex- set of G "and the list of the edges is called "the edge –list of G "[8, 9, 12].

(2) Let $\{v^0,\cdots,v^n\}$ be affinely independent set in R^n.The closed n-simplex S spanned by $v^0,\cdots,v^n$ (with vertices $v^0,\cdots,v^n$)is the set of all points affinely dependent on $v^0,\cdots,v^n$ and is denoted by $(v^0,\cdots,v^n)$, $S^{(n)}$ or S_n [10,11].

(3)A simplicial complex K in R^n is a collection of simplexes R^n such that:
(i) Every face of simplex of K is in K ,i.e if $s \in K$ and $t \prec s$ then $t \in K$.
(ii)If $s,t \in K$ then $s \cap t$ is either empty or a face of both s and t.

The dimension of K is the largest dimension of any simplex in K [13].

THE MAIN RESULTS

Aiming to our study we will introduce some definitions:

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Definition 1: The graph of simplicial complex is the set $\left(\overline{\overline{V}}, \overline{\overline{E}}\right)$ such that the vertices $\overline{\overline{V}}$ are lines and the edges $\overline{\overline{E}}$ are areas see Fig. (1) represent a graph of simplicial complex but in Fig. (2) not a graph of simplicial complex.

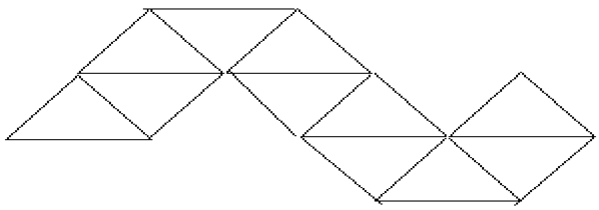


Fig.(1)

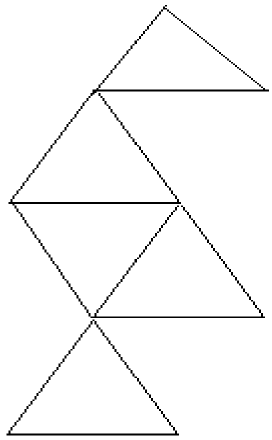


Fig.(2)

Remark 1:

(1) simplicial complex not necessary a graph.

(2) The condition of the simplicial complexes to represent a graph is the area adjacent another area in a line not a point.

Definition 2: A tree of simplicial complex is a graph of simplicial complex which is connected and has no loops as Fig. (3) but in Fig. (4) not a tree of simplicial complex.

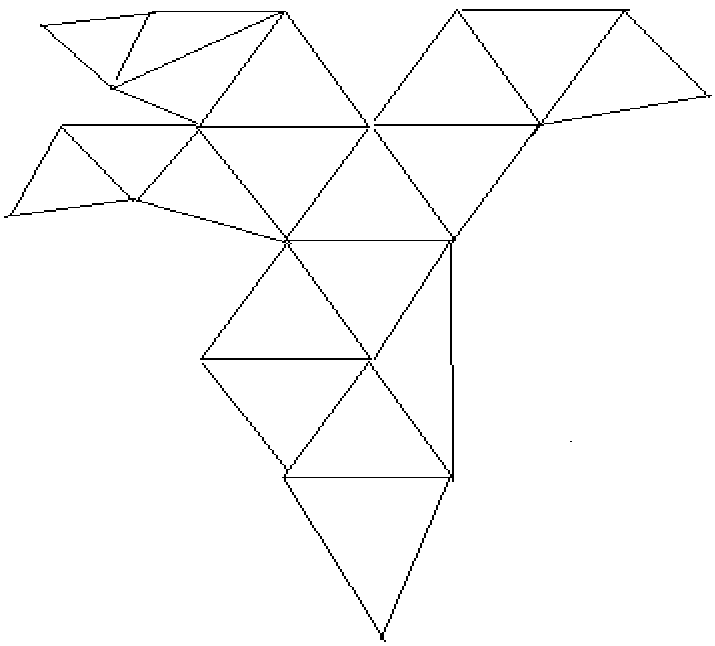


Fig.(3)

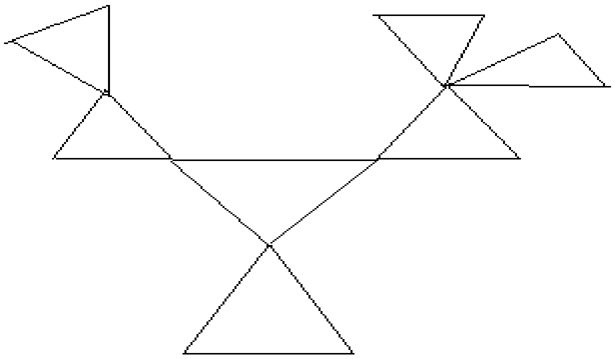
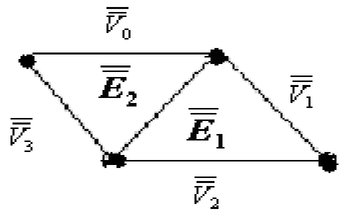


Fig.(4)

Definition 3: "Adjacency matrix of the graph of simplicial complex " consider the new graph with $n-$ vertices labeled $1,2,3,...,n$.The adjacency matrix $A\left(\overline{\overline{G}}\right)$ is the $n\times n$ matrix which the entry in row i and column j is the number of edges joining the vertices i and j.

Definition 4: "Incidence matrix of the graph of simplicial complex " let $\overline{\overline{G}}$ be the new graph with $n-$ vertices labeled $1,2,3,...,n$.And $m-$ edges labeled $1,2,3,...,m$ .The incidence matrix $I\left(\overline{\overline{G}}\right)$ is the $n\times m$ matrix which the entry in row i and column j is $\overline{\overline{1}}$ if vertex i is incident with edge j and $\overline{\overline{0}}$ otherwise .

Example: Consider the graph of simplicial complex: $\overline{\overline{V}}_3$



The adjacency matrix $A\left(\overline{\overline{G}}\right)$ is:

$$A\left(\overline{\overline{G}}\right)=\begin{array}{c} \\ \overline{\overline{V}}_0 \\ \overline{\overline{V}}_1 \\ \overline{\overline{V}}_2 \\ \overline{\overline{V}}_3 \end{array}\begin{array}{c} \begin{array}{cccc} \overline{\overline{V}}_0 & \overline{\overline{V}}_1 & \overline{\overline{V}}_2 & \overline{\overline{V}}_3 \end{array} \\ \begin{bmatrix} \overline{\overline{0}} & \overline{\overline{1}} & \overline{\overline{0}} & \overline{\overline{1}} \\ \overline{\overline{1}} & \overline{\overline{0}} & \overline{\overline{1}} & \overline{\overline{0}} \\ \overline{\overline{0}} & \overline{\overline{1}} & \overline{\overline{0}} & \overline{\overline{1}} \\ \overline{\overline{1}} & \overline{\overline{0}} & \overline{\overline{1}} & \overline{\overline{0}} \end{bmatrix} \end{array}$$

And the incidence matrix $I\left(\overline{\overline{G}}\right)$ is:

$$I\left(\overline{\overline{G}}\right)=\begin{array}{c} \\ \overline{\overline{V}}_0 \\ \overline{\overline{V}}_1 \\ \overline{\overline{V}}_2 \\ \overline{\overline{V}}_3 \end{array}\begin{array}{c} \begin{array}{cc} \overline{\overline{E}}_1 & \overline{\overline{E}}_2 \end{array} \\ \begin{bmatrix} \overline{\overline{0}} & \overline{\overline{1}} \\ \overline{\overline{1}} & \overline{\overline{0}} \\ \overline{\overline{1}} & \overline{\overline{0}} \\ \overline{\overline{0}} & \overline{\overline{1}} \end{bmatrix} \end{array}$$

Definition (5): The folding of a graph of simplicial complexes into itself is a map $f:K\to K$ defined as Fig. (5) the first type $f_1:K\to K$ such that:

$$d(a,b)\le\theta d\left(f_1(a),f_1(b)\right)\qquad 0\prec\theta\prec 1\ .$$

Then $f_1(K)=K_1,\ f_{12}(f_1(K))=K_{12}$, $\Delta K_{12}\le\Delta K_1\ldots f_{1n}(f_{1n-1}\cdots(K))=K_{1n}$, $\Delta K_{1n}\le\ \Delta K_{1n-1}\ \le\cdots\le\ \Delta K_{12}\ \le\ \Delta K_1,\ \lim\limits_{n\to\infty} f_n(K)=$ (0-simplicial complex).

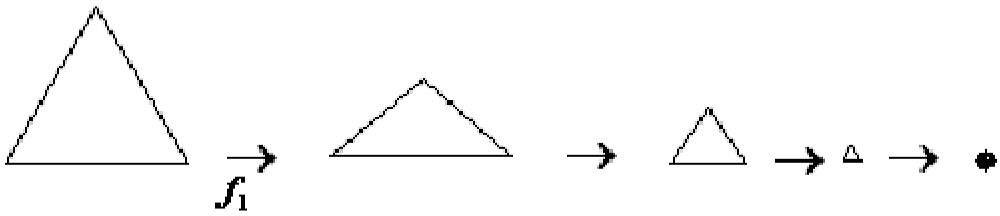


Fig.(5)

Consider another type of folding:

$f_2:K_2\to K_2,\ f_2(K_2)=K_2$

such that $f_2(L)=L$,see Fig.(6) $f_{21}(K_2)=K_{21}$,... $f_{2n}(f_{2n-1}\cdots(K_{21}))=K_{2n}$ and $\lim\limits_{n\to\infty} f_{2n}(\cdots(K_{2n}))=1-simplex=L$ see Fig.(6).

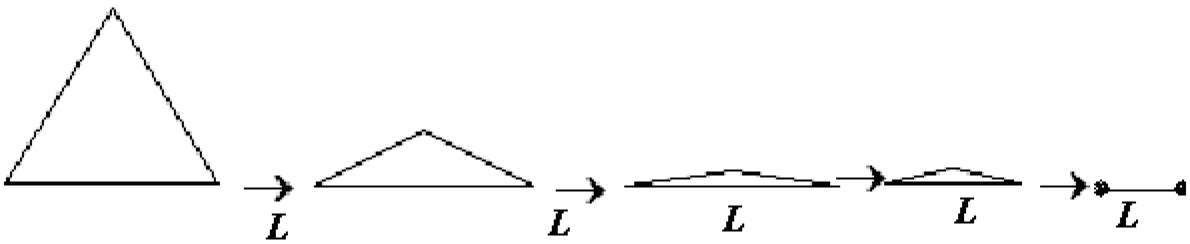


Fig. (6)

(1) The conditional folding f_1 is the folding which reduces the area of the simplex as Fig. (7).

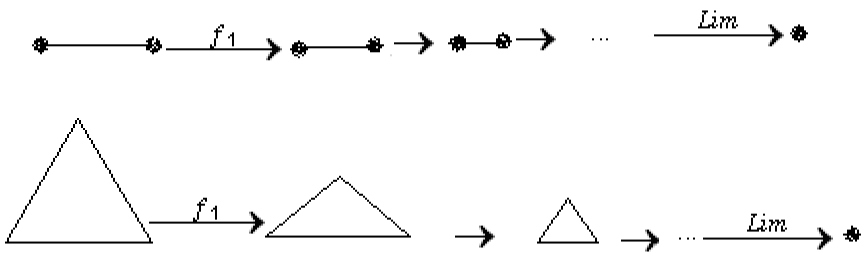
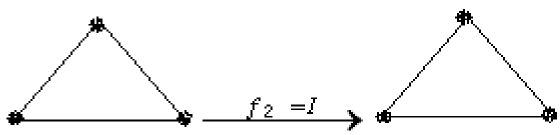


Fig.(7)

(2) $f_2:V_i\to V_j\quad ,E_i\to E_j\quad ,F_i\to F_j$ see Fig.(8) and Fig(9).



or:

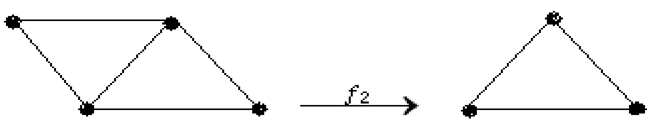


Fig.(8)

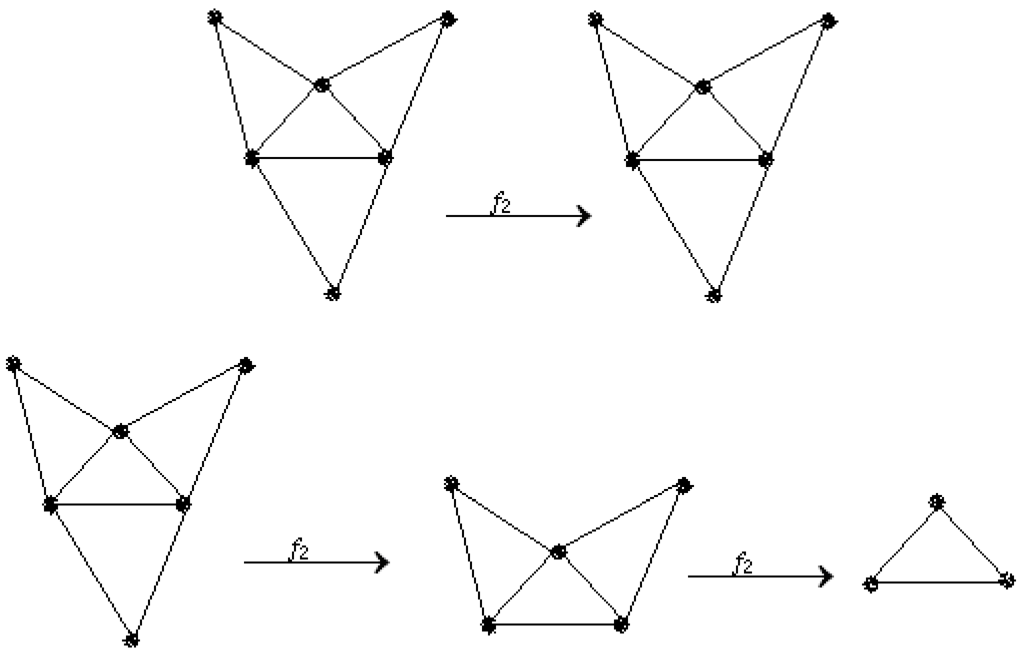


Fig.(9)

Theorem 1: The folding of the graph $\overline{G}$ into itself induces another folding of the incidence and adjacency matrices.

Proof: Let $\overline{G}$ be a graph of simplicial complex of n-vertices and m-edges then

$$A\left(\overline{\overline{G}}\right)=\begin{array}{c} \\ \overline{\overline{V}}_0 \\ \overline{\overline{V}}_1 \\ \vdots \\ \overline{\overline{V}}_n \end{array}\begin{array}{c} \begin{array}{cccc} \overline{\overline{V}}_0 & \overline{\overline{V}}_1 & \dots & \overline{\overline{V}}_n \end{array} \\ \begin{bmatrix} \overline{\overline{0}} & \overline{\overline{1}} & \cdots & \overline{\overline{1}} \\ \overline{\overline{1}} & \overline{\overline{0}} & \cdots & \overline{\overline{0}} \\ \vdots & \vdots & \vdots & \vdots \\ \overline{\overline{1}} & \overline{\overline{0}} & \cdots & \overline{\overline{0}} \end{bmatrix} \end{array} \quad \text{and} \quad I\left(\overline{\overline{G}}\right)=\begin{array}{c} \\ \overline{\overline{V}}_0 \\ \overline{\overline{V}}_1 \\ \vdots \\ \overline{\overline{V}}_n \end{array}\begin{array}{c} \begin{array}{cccc} \overline{\overline{E}}_0 & \vec{\overline{E}}_1 & \dots & \overline{\overline{E}}_m \end{array} \\ \begin{bmatrix} \overline{\overline{1}} & \overline{\overline{0}} & \cdots & \overline{\overline{1}} \\ \overline{\overline{0}} & \overline{\overline{1}} & \cdots & \overline{\overline{0}} \\ \vdots & \vdots & \vdots & \vdots \\ \overline{\overline{1}} & \overline{\overline{0}} & \cdots & \overline{\overline{0}} \end{bmatrix} \end{array}$$

If $f:\overline{\overline{G}}\to\overline{\overline{G}}$ is a folding of the new graph into itself, then $\bar{f}:A\left(\overline{\overline{G}}\right)\to A\left(\overline{\overline{G}}\right)$ such that:

$$\bar{f}\left(A\left(\overline{\overline{G}}\right)\right)=\begin{array}{c} \\ \overline{\overline{V}}_0 \\ \overline{\overline{V}}_1 \\ \vdots \\ \overline{\overline{V}}_s \end{array}\begin{array}{c} \begin{array}{cccc} \overline{\overline{V}}_0 & \overline{\overline{V}}_1 & \dots & \overline{\overline{V}}_s \end{array} \\ \begin{bmatrix} \overline{\overline{1}} & \overline{\overline{0}} & \cdots & \overline{\overline{1}} \\ \overline{\overline{1}} & \overline{\overline{0}} & \cdots & \overline{\overline{0}} \\ \vdots & \vdots & \vdots & \vdots \\ \overline{\overline{1}} & \overline{\overline{0}} & \cdots & \overline{\overline{1}} \end{bmatrix} \end{array}, \ s\le n \quad \text{and} \quad \bar{\bar{f}}:I\left(\overline{\overline{G}}\right)\to I\left(\overline{\overline{G}}\right) \text{ such that:}$$

$$\bar{\bar{f}}\left(I\left(\overline{\overline{G}}\right)\right)=\begin{array}{c} \\ \overline{\overline{V}}_0 \\ \overline{\overline{V}}_1 \\ \vdots \\ \overline{\overline{V}}_s \end{array}\begin{array}{c} \begin{array}{cccc} \overline{\overline{E}}_0 & \vec{\overline{E}}_1 & \dots & \overline{\overline{E}}_r \end{array} \\ \begin{bmatrix} \overline{\overline{0}} & \overline{\overline{1}} & \cdots & \overline{\overline{1}} \\ \overline{\overline{1}} & \overline{\overline{0}} & \cdots & \overline{\overline{0}} \\ \vdots & \vdots & \vdots & \vdots \\ \overline{\overline{1}} & \overline{\overline{0}} & \cdots & \overline{\overline{0}} \end{bmatrix} \end{array}$$

We find that $\bar{f}$ is a sub matrix of $A\left(\overline{\overline{G}}\right)$ and $\bar{\bar{f}}$ is a sub matrix of $I\left(\overline{\overline{G}}\right)$

Theorem 2: Any sequence of foldings of $\overline{\overline{G}}$ into itself induce two sequences of the matrices of $\overline{\overline{G}}$, $\lim_{i\to\infty} f_i\left(\overline{\overline{G}}\right)$ induces two limits : $\lim_{i\to\infty} \bar{f}_i\left(A\left(\overline{\overline{G}}\right)\right)$ and $\lim_{i\to\infty} \bar{\bar{f}}_i\left(I\left(\overline{\overline{G}}\right)\right)$

Proof: Let $f_i : \overline{\overline{G}} \to \overline{\overline{G}}$ be a sequence of foldings of the new graph into itself. We find $\lim_{i\to\infty} f_i\left(\overline{\overline{G}}\right)$=one edge see Fig. (10)

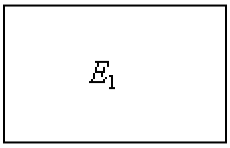


Fig. (10)

There is two induced chains of foldings of the incidence and adjacency matrices i.e.

$$\overline{\overline{G}} \xrightarrow{f_1} \overline{\overline{G}}_1 \xrightarrow{f_2} \overline{\overline{G}}_2 \xrightarrow{f_3} \cdots \xrightarrow{\lim_{i\to\infty} f_i} E_1$$

Induces

$$A\left(\overline{\overline{G}}\right) \xrightarrow{\bar{f}_1} A\left(\overline{\overline{G}}_1\right) \xrightarrow{\bar{f}_2} A\left(\overline{\overline{G}}_2\right) \xrightarrow{\bar{f}_3} \cdots \xrightarrow{\lim_{i\to\infty} \bar{f}_i} A(E_1)$$

And:

$$I\left(\overline{\overline{G}}\right) \xrightarrow{\bar{f}_1} I\left(\overline{\overline{G}}_1\right) \xrightarrow{\bar{f}_2} I\left(\overline{\overline{G}}_2\right) \xrightarrow{\bar{f}_3} \cdots \xrightarrow{\lim_{i\to\infty} \bar{f}_i} I(E_1)$$

REMARK

If f_1 preserves the number of vertices and edges then:

$$f_1(incidence) = incidence,\ f_1(adjacency) = adjacency$$

REFERENCES

[1] M. El-Ghoul: *Folding of fuzzy graphs and fuzzy spheres*, Fuzzy Sets and Systems, Germany, 58, 355-363, (1993).

[2] M.El-Ghoul; A.El-Ahmady; Homda: *Retraction of simplicial complexes*, International Journal of Applied Mathmatics & Statistics,Vol.4(J06), 54-67,India, 2006

[3] M. El-Ghoul: *Unfolding of graph and uncertain graph*, The Australian Mathematics Journal, Sandy Bay 7006, Tasmania,Australia, (in press).

[4] M. El-Ghoul; A.El-Ahmady; H.Rafat: *Folding-retraction of chaotic dynamical manifold and the VAC of vacuum fluctuation.* Chaos, Solitons and Fractals, UK, 20, 209-217, (2004).

[5] A.El-Ahmady: *The deformation retract of a manifold admitting a simple transitive group of motions and its topological folding.* Bull. Cal.Math.Soc. 96, 4,279-284, 2004.

[6] A.El-Ahmady; H.M.Shamara: *Fuzzy deformation retract of fuzzy horosphere*. India J. Pure Appl.Math. 32, 10, 1501-1506, 2001.

[7] M.El-Ghoul, H.I.Attiya: *The dynamical fuzzy topological space and its folding: The Journal of fuzzy mathematics Vol. 12 No. 3, 2004 Los Angeles U.S.A.*

[8] A.Gibbons: *Algorithmic graph theory. Cambridge University Press, Cambridge, UK, 1985*

[9] P.J.Giblin: *Graphs, surfaces and homology, an introduction to algebraic topology.* Chapman and Hall Ltd, London 1977.

[10] J.L.Groos; T.W.Tucker: *Topological graph theory.* Jon Wiley& Sons, Inc, Canada 1987.

[11] A.T.White: *Graphs, groups and surfaces.* Amsterdam, North-Holland Publishing Company 1973.

[12] R.J. Wilson: *Introduction to graph theory*. Oliver & Boyed, Edinburgh 1972.

[13] R.J. Wilson; J.J. Watkins: *Graphs, an introductory approach, a first course in discrete mathematics.* Jon Wiley& Sons Inc, Canada 1990.

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