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DUAL GRAPHS AND CELLULAR FOLDING OF 2-MANIFOLDS

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ABSTRACT

In this paper we have constructed dual graphs associated with the cellular folding in a natural way. Then we obtained the condition under which we may have a successive foldings of a CW-complex into itself. We also obtained the conditions for a cartesian product and a wedge sum of two cellular foldings to be a cellular folding. These conditions are obtained in terms of the dual graphs and by using the above results we also obtained some other results.

Keywords: *Cellular folding, Dual graph, Cartesian product, Wedge sum, Suspension.*

2001 Mathematics subject classification: *51H10. 57H10.*

1. INTRODUCTION

A cellular folding is a folding defined on regular *CW*-complexes first defined by, E. El-Kholy and H. Al-Khurasani, [1] and various properties of this type of folding are also studied by them.

By a cellular folding of regular *CW*-complexes, it is meant a cellular map $f : K \to L$ which maps *i*-cells of *K* to *i*-cells of *L* and such that $f \mid e^i$, for each *i*-cell *e*, is a homeomorphism onto its image.

The set of regular *CW*-complexes together with cellular foldings form a category denoted by $C(K, L)$. If $f \in C(K, L)$; then $x \in K$ is said to be a singularity of f iff f is not a local homeomorphism at *x*. The set of all singularities of f is denoted by $\sum f$. This set corresponds to the "folds" of the map. It is noticed that for a cellular folding *f*, the set $\sum f$ of singularities of f is a proper subset of the union of cells of dimension $\leq n-1$. Thus, when we consider any $f \in C(K, L)$, where *K* and *L* are connected regular *CW*-complexes of dimension 2, the set $\sum f$ will consists of 0-cells, and 1-cells, each 0-cell (vertex) has an even valency, [2, 3], of course, $\sum f$ need not to be connected. Thus in this case $\sum f$ has the structure of a locally finite graph Γ_f embedded in *K*, for which every vertex has an even valency. Note that if *K* is compact, then Γ_f is finite, also any compact connected 2-manifold without boundary (surface) *K* with a finite cell decomposition is a regular *CW*-complex, then the 0- and 1-cells of the decomposition *K* form a finite graph Γ_f without loops and f folds *K* along the edges or 1-cells of Γ_f.

Now a neat cellular folding $f : K \to L$ is a cellular folding such that $L^n - L^{n-1}$ consists of a single *n*-cell, $\operatorname{Int} L$, [2].

2. DEFINITIONS

Let $f \in C(K, L)$, then there is a cellular subdivision *S* on *K* by singularities of *f*. In the following we give the definition of the dual graph Γ_f^* associated to this stratification in a natural way.

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In fact the vertices of the graph Γ_f^* are just the *n*-cells of *S* and the edges are some $(n-1)$ cells. An edge $E \in S_{n-1}$ means that *E* lies in the frontiers of exactly two *n*-cells $\gamma, \gamma' \in S_n$ where $f(\gamma) = f(\gamma')$ such that $E, E' \in S_{n-1}$ are equivalent (the same) iff both *E* and E' lies in the frontier of γ and γ' and $\partial(\gamma) = \partial(\gamma')$ contains more than $(n-1)$-cells. We then say that *E* is an edge in Γ_f^* with end points γ, γ'. It is possible to realize Γ_f^* as a graph $\tilde{\Gamma}_f^*$ embedded in *K* as follows:

For each *n*-cell $\gamma \in S_n$, we choose any point $u \in \gamma$. If $\gamma, \gamma' \in S_n$ are end points of $E \in S_{n-1}$, then we can join *u* to *v* by an arc $\tilde{E}$ in *K* that runs from *u* through γ and γ' to *v* crossing *E* transversely at a single point. Trivially, the correspondence $\gamma \to u,\ E \to \tilde{E}$ is a graph isomorphism from Γ_f^* to $\tilde{\Gamma}_f^*$. It should be noted that the graph Γ_f^* has no multiple edges, no loops and generally disconnected. It will be connected in the case of neat cellular foldings, since all the *n*-cells will be send to the same *n*-cell. The graph Γ_f^* is the dual graph of Γ_f.

If (M, N) is a *CW*-pair consisting of a cell complex *M* and a subcomplex *N*, then the quotient space M/N inherits a natural cell complex structure from *M*. The cells of M/N are the cells of $M-N$ plus one new 0-cell, the image of *N* in M/N, [4].

The suspension SM of a complex *M* is the union of all line segments joining points of *M* to two external vertices, [4].

2-1 Examples

(a) Consider a complex *K* such that $|K|$ is a torus with cellular subdivision consisting of eight 0-cells, sixteen 1-cells and eight 2-cells, let $f: K \to K$ be a cellular folding given by

$$f(e_1^0, ..., e_8^0) = (e_1^0, e_2^0, e_3^0, e_4^0, e_2^0, e_1^0, e_3^0, e_4^0)$$
$$f(e_1^2, ..., e_8^2) = (e_1^2, e_2^2, e_1^2, e_2^2, e_1^2, e_2^2, e_1^2, e_2^2)$$

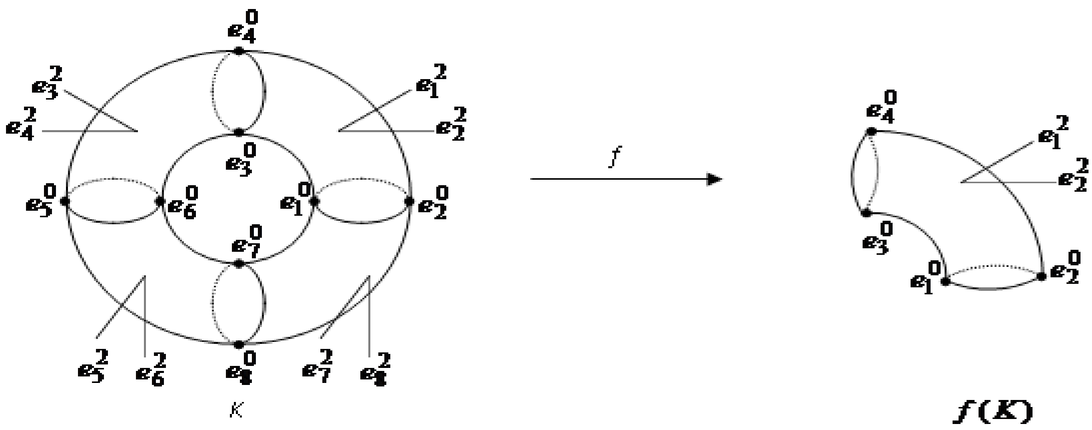

Fig. (1)

In this case the dual folding graph Γ_f^* is as shown in Fig. (2)

Fig. (2) : The dual folding graph Γ_f^*.

(b) Let K be a complex such that $|K|$ is a cylinder with cellular subdivision consisting of eight vertices, sixteen 1-cells and eight 2-cells, see Fig. (3). Let $f : K \to K$ be a cellular folding given by:

$$f(a, b, c, d, e, f, g, h) = (a, b, c, b, e, f, g, f)$$
$$f(\sigma_i) = \sigma_7, \qquad i = 1, \ldots, 7$$

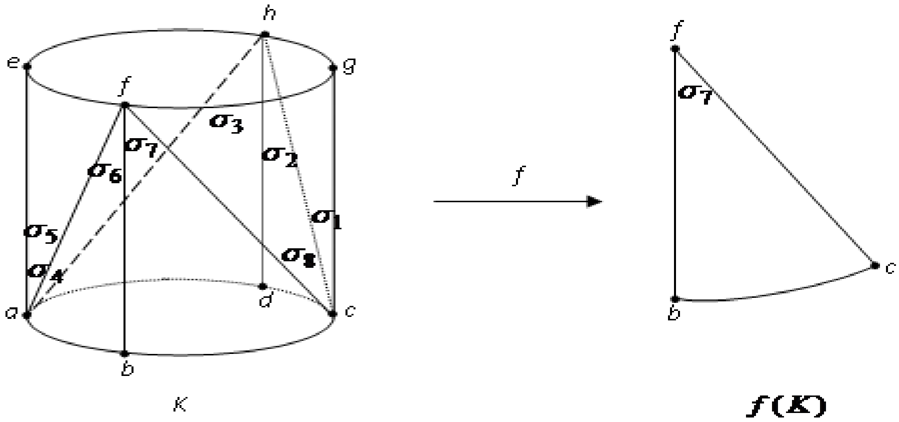

Fig. (3)

The dual folding graph Γ_f^* is as shown in Fig. (4), it is a connected graph.

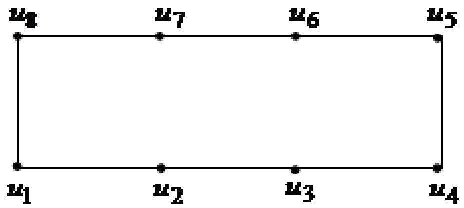

Fig. (4) : The dual folding graph Γ_f^*.

3. MAIN RESULTS

The following theorem gives the condition satisfied by the dual graphs in order to have a successive folding of *CW*-complex into itself.

Theorem 1: Let *K*, *L* and *M* be regular *CW*-complexes of the same dimension 2, such that $M \subset L \subset K$, let $f : K \to K,\ g : L \to L$ be cellular foldings such that $f(K) = L \neq K$ and $g(L) = M \neq L$ with dual folding graphs $\Gamma_f^* = (V_f, E_f)$ and $\Gamma_g^* = (V_g, E_g)$. Then $g \circ f$ is a cellular folding from *K* into *M* with dual folding graph $\Gamma_{g\circ f}^* = (V, E)$ such that $\Gamma_{g\circ f}^* = \Gamma_f^* \cup f^{-1}(\Gamma_g^*)$.

Proof: Let $f : K \to L,\ g : L \to M$ be cellular foldings, then *K* and *L* have stratifications S, S' respectively, such that $V_f = \{n - \text{cells of } S\}$ and $V_g = \{n - \text{cells of } S'\}$ where $S' \subset S$. But the composition map $g \circ f$ has the same stratification *S* on *K* i.e., $V_{g\circ f} = V_f$.

Also if $e \in E$, then $e \in S_{n-1}$ and lies in the frontier of exactly two *n*-cells $\gamma, \gamma' \in S_n$ such that $(g \circ f)(\gamma) = (g \circ f)(\gamma')$, thus $g(f(\gamma)) = g(f(\gamma'))$ where $f(\gamma)$ and $f(\gamma') \in S'_n$. Now there are three cases:

(1) $f(\gamma) = f(\gamma') = \sigma \in S'_n$, then there exists an edge belongs to E_f lies in the frontier of γ and γ'.

(2) $f(g) = \sigma \neq f(\gamma') = \sigma'$ and $g(\sigma) = g(\sigma') = \alpha$, then there exists an edge belongs two E_g lies in the frontier of σ and σ'.

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(3) $f(\gamma) = \sigma \neq f(\gamma') = \sigma'$ and $g(\sigma) = g(\sigma') = \sigma$ or $g(\sigma) = g(\sigma') = \sigma'$, then there exists an edge belongs to E' where $E' \cup E_g = f^{-1}(E_g)$, lies in the frontier of σ and σ'.

We conclude from the above possibilities that $E = E_f \cup f^{-1}(E_g)$. Thus we have $\Gamma^*_{g \circ f} = \Gamma^*_f \cup f^{-1}(\Gamma^*_g)$.

The above theorem can be generalized for a finite series of cellular foldings.

3-1 Example

Let K be a complex such that $|K|$ is a torus with cellular subdivision consisting of eight 0-cells, twenty 1-cells and sixteen 2-cells. Let $f : K \to K$ be a cellular folding given by:

$$f(e_1^0, ..., e_8^0) = (e_1^0, e_2^0, e_3^0, e_4^0, e_5^0, e_6^0, e_3^0, e_4^0)$$

$$f(e_1^2, ..., e_{16}^2) = (e_1^2, e_2^2, e_3^2, e_4^2, e_5^2, e_6^2, e_7^2, e_8^2, e_1^2, e_2^2, e_3^2, e_4^2, e_7^2, e_8^2, e_5^2, e_6^2)$$

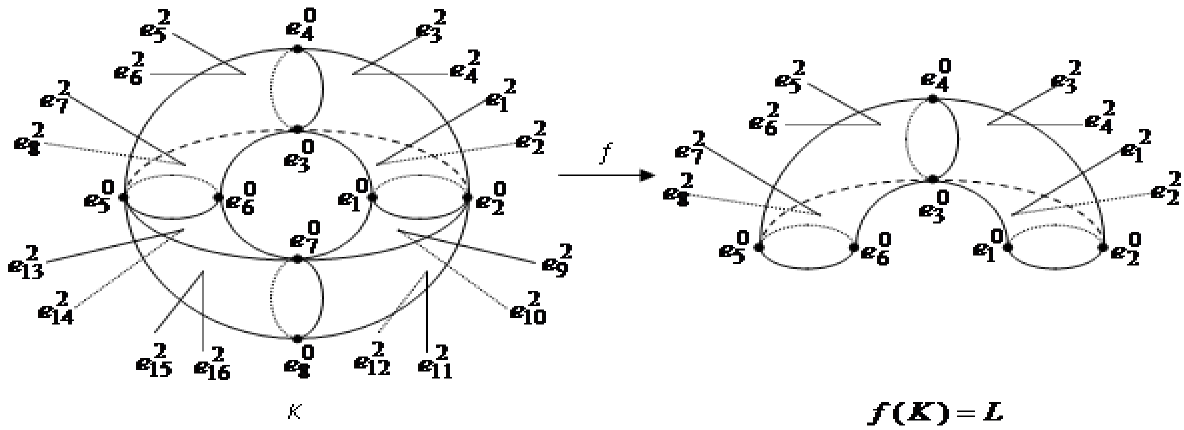

Fig. (5)

The dual folding graph Γ^*_f is as shown in Fig. (6)

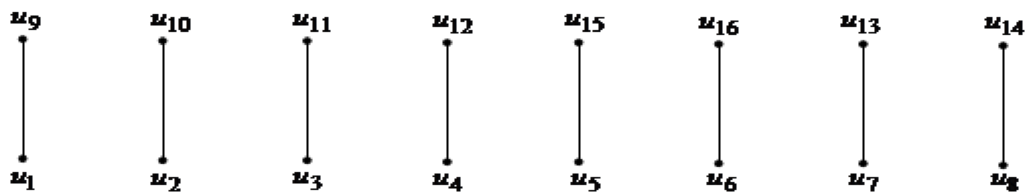

Fig. (6) : The dual folding graph Γ^*_f.

Now let $g : L \to L$ be a cellular folding where $L = f(K)$, defined by:

$$g(e_1^0, ..., e_6^0) = (e_1^0, e_2^0, e_3^0, e_4^0, e_2^0, e_1^0),$$

$$g(e_1^2, ..., e_8^2) = (e_1^2, e_2^2, e_3^2, e_4^2, e_3^2, e_4^2, e_1^2, e_2^2).$$

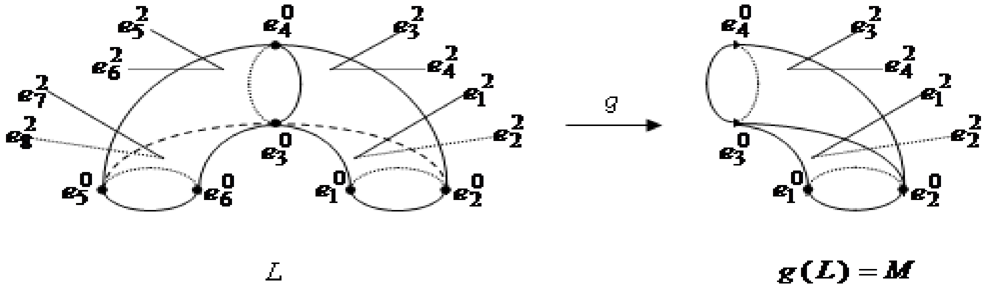

Fig. (7)

The dual folding graph Γ_g^* is as shown in Fig. (8)

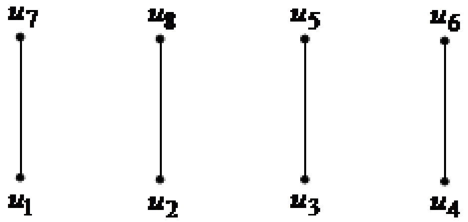

Fig. (8) : The dual folding graph Γ_g^*.

Then by theorem (1) $g \circ f$ is a cellular folding and the dual folding graph of $g \circ f : K \to M$ is as shown in Fig. (9).

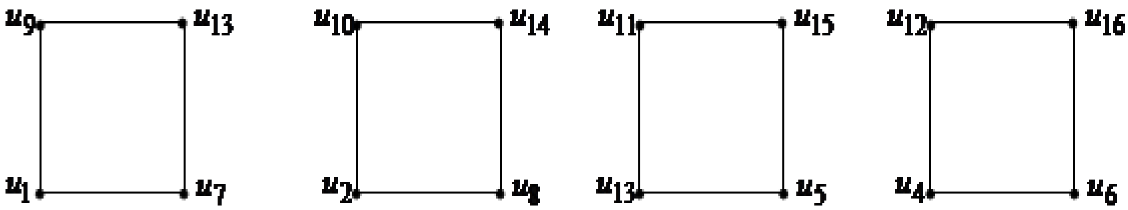

Fig. (9) : The dual folding graph $\Gamma_{g \circ f}^* = \Gamma_f^* \bigcup f^{-1}(\Gamma_g^*)$.

Again, let $h : M \to M$, $h(M) = N \neq M$ be a cellular folding defined by:

$$h(e_1^0, e_2^0, e_3^0, e_4^0) = (e_1^0, e_2^0, e_3^0, e_4^0), \ \ h(e_1^2, e_2^2, e_3^2, e_4^2) = (e_1^2, e_1^2, e_3^2, e_3^2).$$

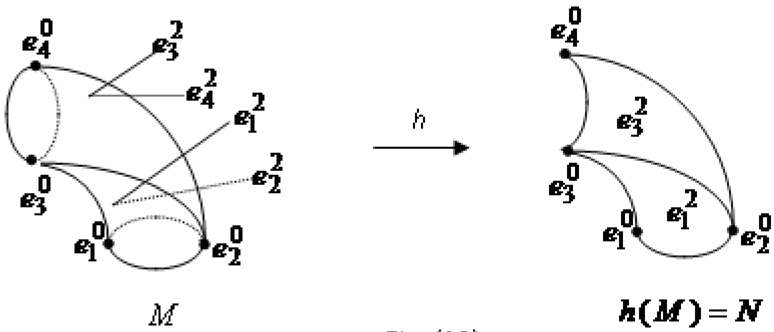

Fig. (10)

The dual folding graph Γ_h^* is as shown in Fig. (11)

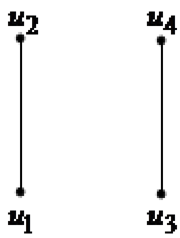

Fig. (11) : The dual folding graph Γ_h^*.

Then by theorem (1) $h \circ g \circ f$ is a cellular folding and the dual folding graph $\Gamma^*_{h \circ g \circ f}$ is as shown in Fig. (12).

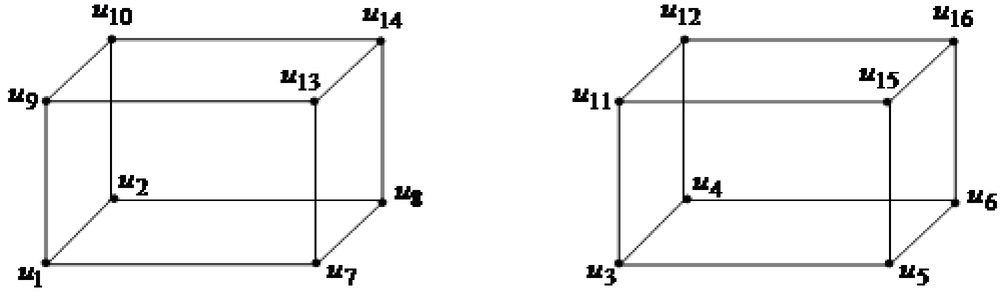

Fig. (12) : $\Gamma^*_{h \circ g \circ f} = \Gamma^*_f \cup f^{-1}(\Gamma^*_g) \cup (g \circ f)^{-1}(\Gamma^*_h)$.

Again, let $k : N \to N,\ k(N) = X \neq N$ be a cellular folding defined by:

$$k(e_1^0, e_2^0, e_3^0, e_4^0) = (e_1^0, e_2^0, e_3^0, e_1^0),$$

$$k(e_1^2, e_3^2) = (e_1^2, e_1^2).$$

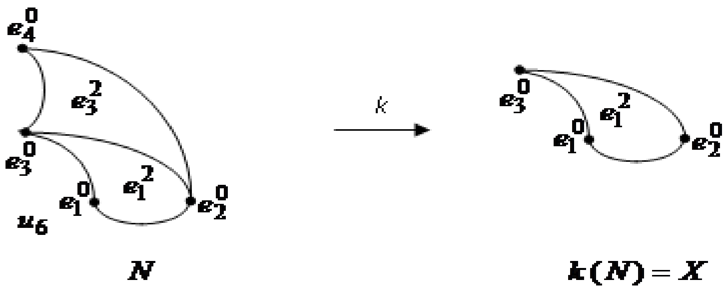

Fig. (13)

The dual folding graph Γ^*_k is as shown in Fig. (14).

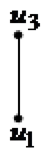

Fig. (14): The dual folding graph Γ^*_k

Again by theorem (1) $k \circ h \circ g \circ f$ is a cellular folding and the dual folding graph $\Gamma^*_{k \circ h \circ g \circ f}$ is as shown in Fig. (15).

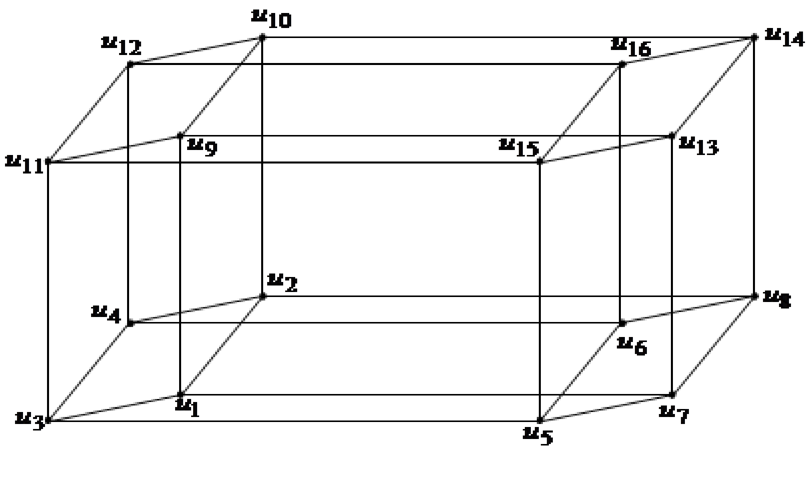

Fig. (15) : $\Gamma^*_{k \circ h \circ g \circ f} = \Gamma^*_f \cup f^{-1}(\Gamma^*_g) \cup (g \circ f)^{-1}(\Gamma^*_h) \cup (h \circ g \circ f)^{-1}(\Gamma^*_k)$.

Corollary 1: Let *K*, *L* and *M* be complexes of the same dimension 2 such that $M \subset L \subset K$, let $f : K \to L$ be a cellular folding with dual folding graph $\Gamma_f^* = (V_f, E_f)$, $g : L \to M$ be a cellular map and $h = g \circ f : K \to M$ be a cellular folding with dual folding graph $\Gamma_h^* = (V_h, E_h)$. Then $g : L \to M$ is a cellular folding with dual folding graph $\Gamma_g^* = (V_g, E_g)$ such that $\Gamma_g^* = f[\Gamma_h^* \setminus E_f]$.

3-2 Example

Consider a complex on $|K| = S^2$, with cellular subdivision consisting of six 0-cells, twelve 1-cells and eight 2-cells, let $f : K \to K$ be a cellular folding defined by:

$$f(e_1^0, ..., e_6^0) = (e_1^0, e_2^0, e_3^0, e_4^0, e_5^0, e_1^0),$$
$$f(e_1^1, ..., e_{12}^1) = (e_1^1, e_2^1, e_3^1, e_4^1, e_5^1, e_6^1, e_7^1, e_8^1, e_3^1, e_4^1, e_1^1, e_2^1),$$
$$f(e_1^2, ..., e_8^2) = (e_1^2, e_2^2, e_3^2, e_4^2, e_3^2, e_4^2, e_1^2, e_2^2).$$

In this case $f(K) = L$ is a complex with five 0-cells, eight 1-cells and four 2-cells, see Fig. (16).

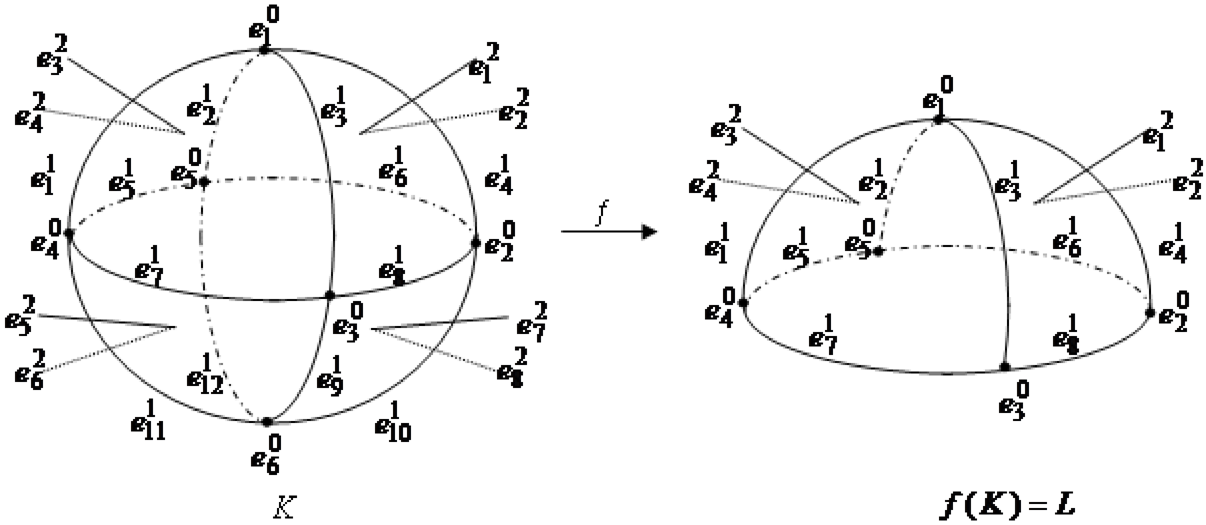

Fig. (16)

The dual folding graph Γ_f^* is as shown in Fig. (17).

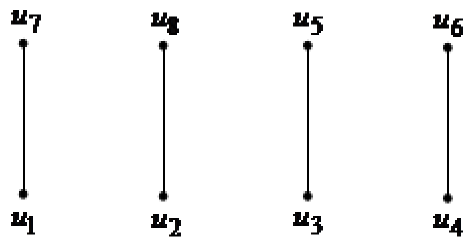

Fig. (17): The dual folding graph Γ_f^*.

Now, let $h : K \to K$ be a cellular folding defined by:

$$h(e_1^0, ..., e_6^0) = (e_1^0, e_2^0, e_3^0, e_2^0, e_3^0, e_1^0),$$
$$h(e_1^1, ..., e_{12}^1) = (e_4^1, e_3^1, e_3^1, e_4^1, e_8^1, e_8^1, e_8^1, e_8^1, e_3^1, e_4^1, e_4^1, e_3^1), \; h(e_i^2) = (e_1^2), \quad i = 1, ..., 8.$$

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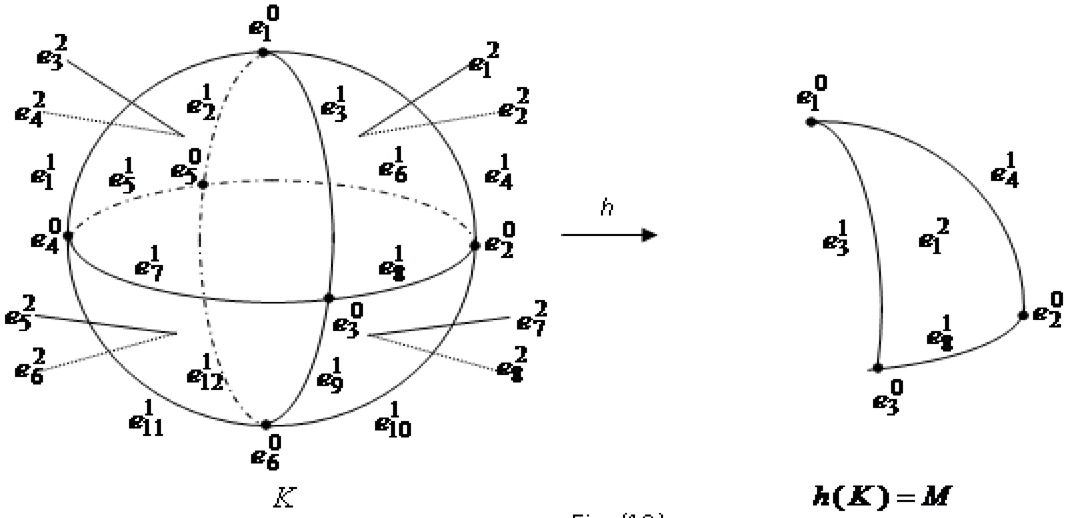

Fig. (18)

The dual folding graph Γ_h^* is as shown in Fig. (19).

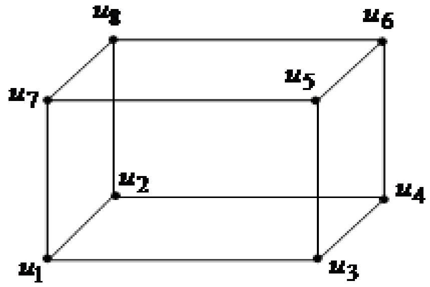

Fig. (19): The dual folding graph $\Gamma_h^* = \Gamma_{g \circ f}^*$.

Then by corollary (1) $g : L \to M$ is a cellular folding with dual folding graph is as shown in Fig. (20).

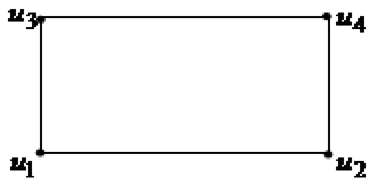

Fig. (20): $\Gamma_g^* = f[\Gamma_h^* \setminus E_f]$

The following theorem gives the condition satisfied by dual graphs in order to have the cartesian product of two cellular foldings is a cellular folding.

Theorem 2: Let *K*, *L*, *M*, and *N* be regular *CW*-complexes of the same dimension 2, let $f : K \to M$ and $g : L \to N$ be cellular foldings with dual folding graphs $\Gamma_f^* = (V_f, E_f)$ and $\Gamma_g^* = (V_g, E_g)$ respectively. Then $f \times g : K \times L \to M \times N$ is a cellular folding with dual folding graph $\Gamma_{f\times g}^* = (V, E)$ such that $\Gamma_{f\times g}^* = \Gamma_f^* \times \Gamma_g^*$. i.e., $V = V_f \times V_g$, $E = (V_f \times E_g) \bigcup (V_g \times E_f)$.

Proof: Let $f : K \to M$, $g : L \to N$ be cellular foldings, then *K* and *L* have stratifications *S*, S' respectively such that $V_f = \{n - \text{cells of } S\}$ and $V_g = \{n - \text{cells of } S'\}$. The map $f \times g : K \times L \to M \times N$ is a cellular folding iff there exists another stratification S'' on $K \times L$ such that $S'' = S_n \times S'_n$, thus $V_{f\times g} = V_f \times V_g$.

Now let $e \in E$, then $e \in S''_{n-1}$ and lies in the frontier of exactly two *n*-cells $\gamma, \gamma' \in S''_n$, $\gamma = (u, v)$, $\gamma' = (u', v')$ such that $(f \times g)(\gamma) = (f \times g)(\gamma')$ or $(f \times g)(u, v) = (f \times g)(u', v')$. Then there are two cases:

(1) $f(u) = f(u')$, $u, u' \in S_n$, then there exists an edge belongs to E_f lies in the frontier of u and u'.

(2) $g(v) = g(v')$, $v, v' \in S'_n$, then there exists an edge belongs to E_g lies in the frontier of v and v'.

Thus u and u' are incident with an edge belongs to E_f and v, v' are incident with an edge belongs to E_g.

Then E will be take the form $E = (V_f \times E_g) \cup (V_g \times E_f)$. Therefore

$$\Gamma_{f \times g}{}^{*} = \Gamma_f^{*} \times \Gamma_g^{*}.$$

The above theorem can be generalized for a finite number of cellular foldings.

3-3 Examples

(a) Consider K and L are complexes such that $|K\} = |L| = S^1$, with cellular subdivision given as shown in Fig. (21), and let $f : K \to K,\ g : L \to L$ be neat cellular foldings which squash K and L respectively to a 1-cell and two 0-cells.

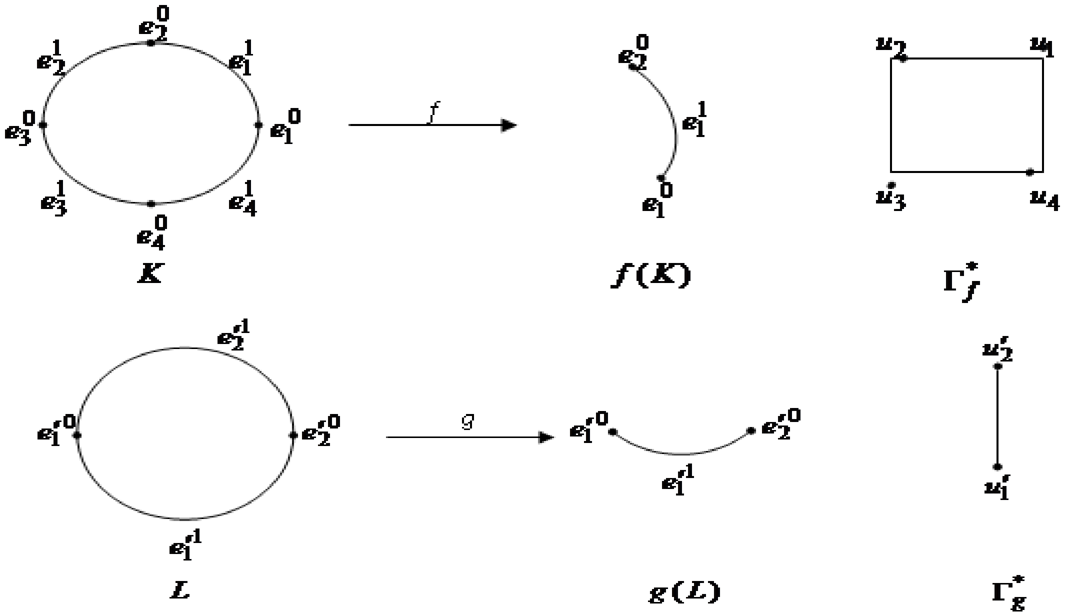

Fig. (21)

Now, $f \times g : K \times L \to K \times L$ is a neat cellular folding with dual folding graph $\Gamma^{*}_{f\times g}$ is as shown in Fig. (22).

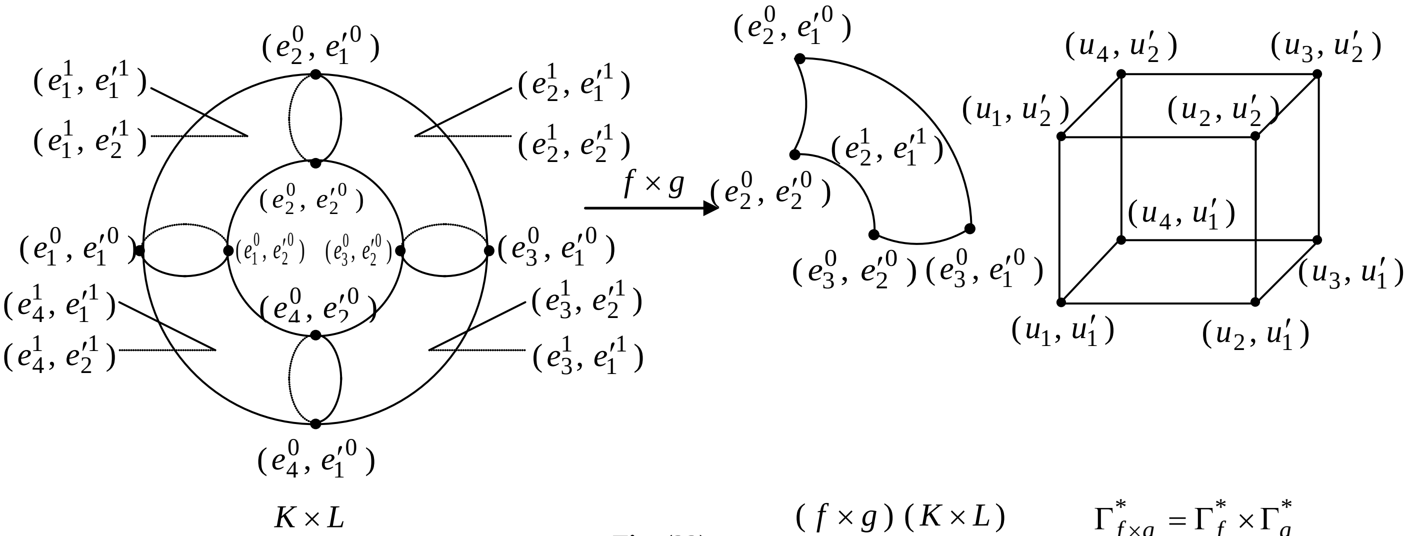

Fig. (22).

Corollary 2: Let M, N, M_1, M_2, N_1, N_2 be complexes of the same dimension and let $f: M \to M_1$, $g: N \to N_1$, $h: M_1 \to M_2$ and $k: N_1 \to N_2$ be cellular foldings with dual folding graphs $\Gamma_f^*, \Gamma_g^*, \Gamma_h^*$ and Γ_k^* respectively. Then $(h\times k)\circ(f\times g) = (h\circ f)\times(k\circ g)$ is a cellular folding with dual folding graphs $\Gamma^*_{(h\times k)\circ(f\times g)} = \Gamma^*_{f\times g} \cup (f\times g)^{-1}(\Gamma^*_{h\times k}) = \Gamma^*_{(h\circ f)\times(k\circ g)} = \Gamma^*_{h\circ f} \times \Gamma^*_{k\circ g}$.

3-4 Example

Suppose M, N, M_1, M_2, N_1, N_2 are complexes such that $|M| = S^1$, $|N| = |M_1| = |M_2| = |N_1| = |N_2| = I$, with cell decompositions shown in Fig. (23). Suppose $f: M \to M_1$, $h: M_1 \to M_2$, $g: N \to N_1$ and $k: N_1 \to N_2$ are cellular foldings.

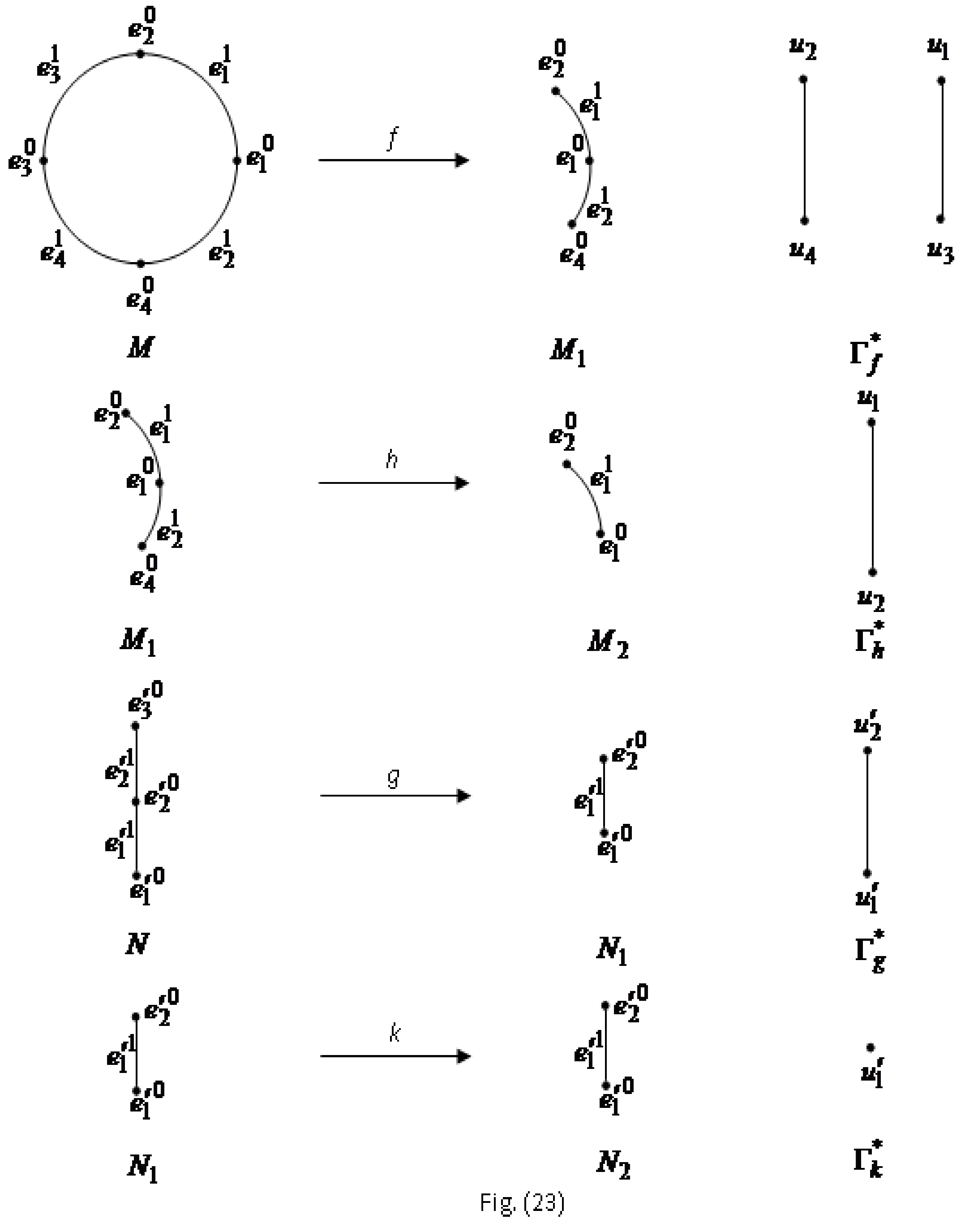

Fig. (23)

The cellular foldings $(f \times g)$, $(h \times k)$ and the dual folding graphs $\Gamma^*_{f\times g}$, $\Gamma^*_{h\times k}$, $\Gamma^*_{(h\times k)\circ(f\times g)}$ are as shown in Fig. (24).

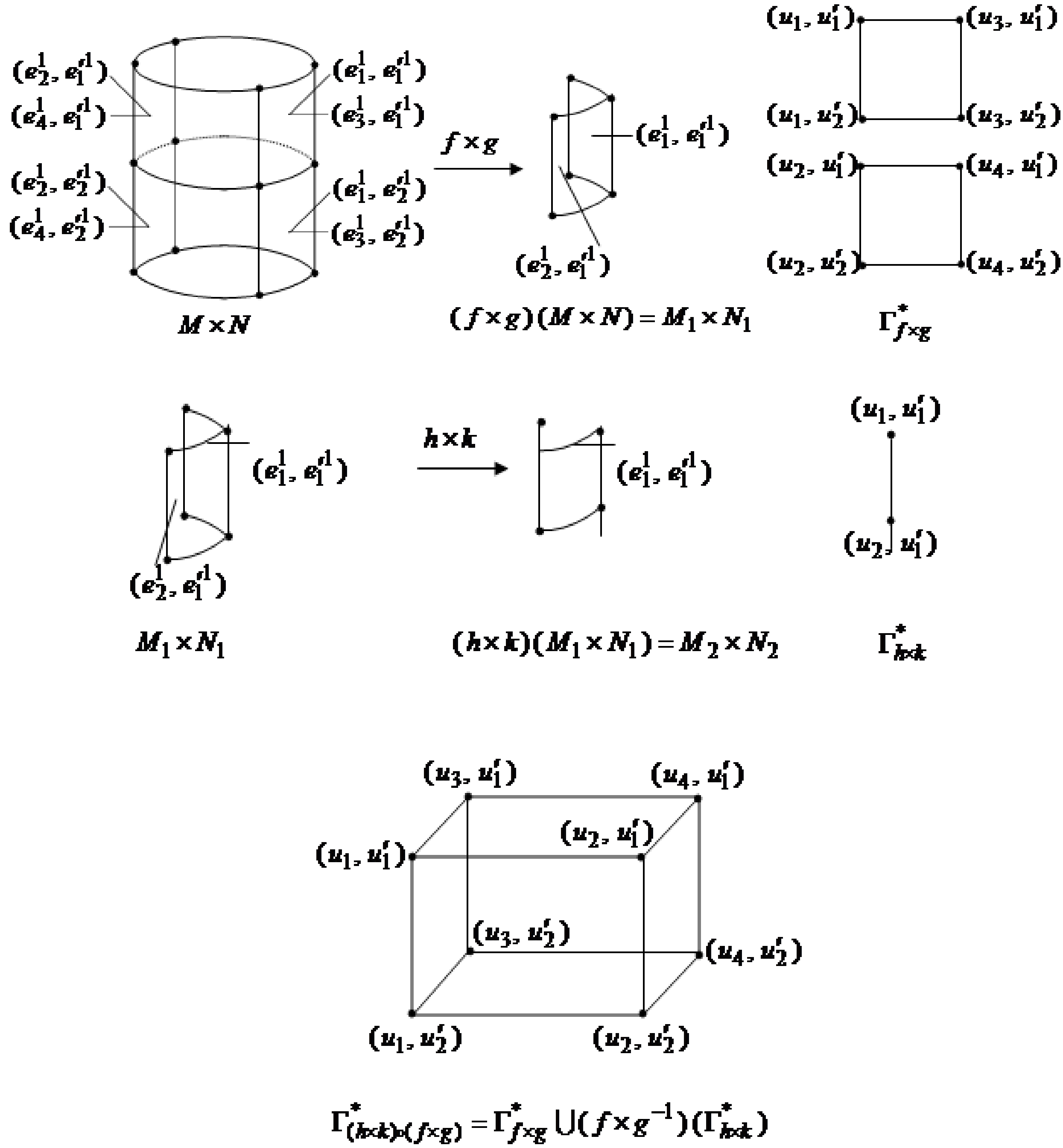

Fig. (24)

Also the cellular foldings $h \circ f$, $k \circ g$ and the dual folding graphs $\Gamma^*_{h\circ f}$, $\Gamma^*_{k\circ g}$, $\Gamma^*_{(h\circ f)\times(k\circ g)}$ are as shown in Fig. (25).

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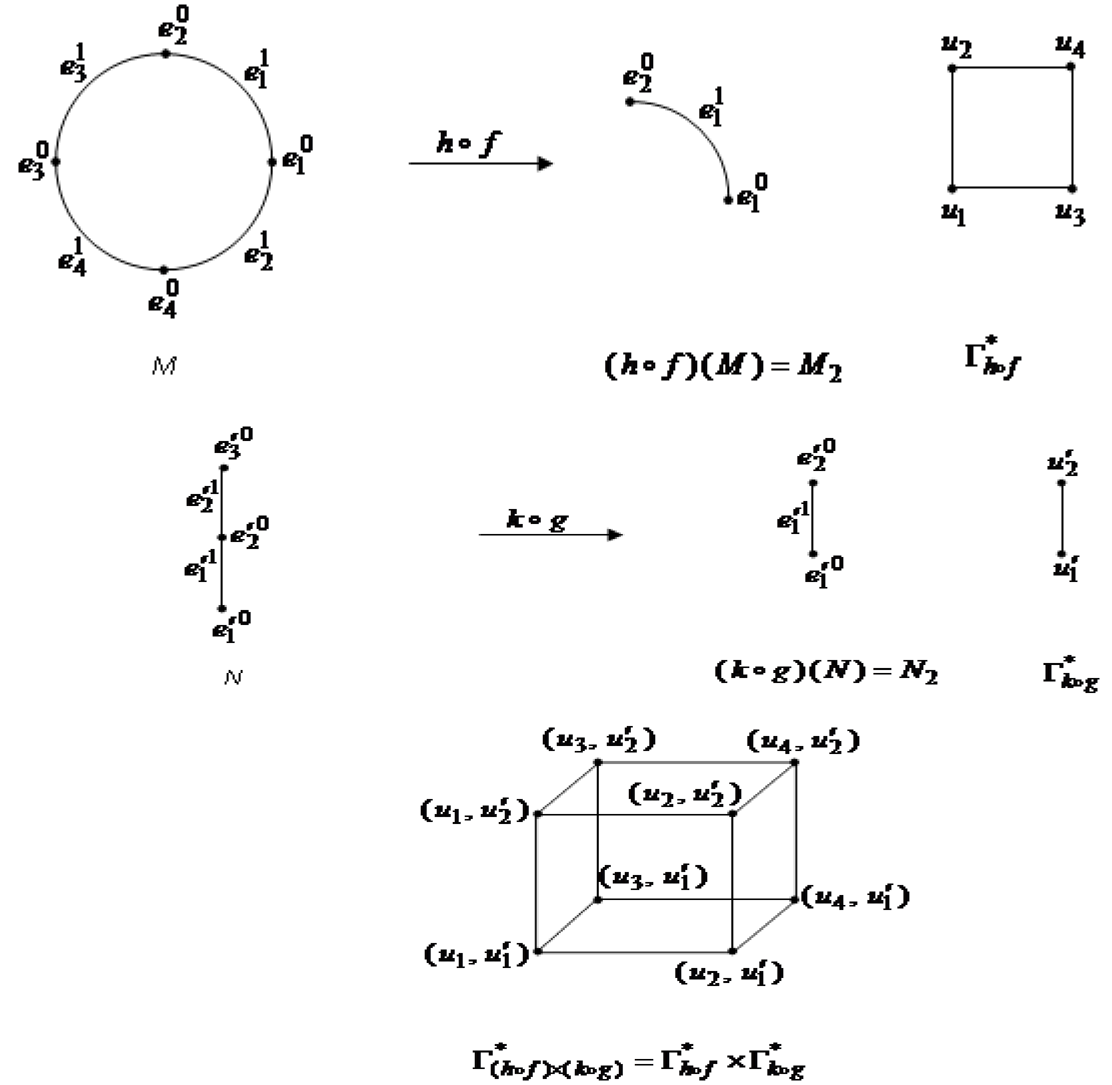

Fig. (25)

The following theorem gives the condition satisfied by the dual graphs in order to have the wedge sum of two cellular folding is a cellular folding.

Theorem 3: Let *M*, *N*, *K* and *L* be regular *CW*-complexes of the same dimension, let $f: M \to M,\ f(M) = N \neq M$ and $g: K \to K,\ g(K) = L \neq K$ be cellular foldings with dual folding graphs $\Gamma_f^* = (V_f, E_f)$ and $\Gamma_g^* = (V_g, E_g)$ respectively. Then the wedge sum $f \vee g : M \vee K \to N \vee L$ is a cellular folding with dual folding graph $\Gamma_{f \vee g}^* = (V, E)$ such that $\Gamma_{f \vee g}^* = \Gamma_f^* + \Gamma_g^*$.

Proof: Let $f: K \to M,\ g: L \to N$ be cellular foldings, then *K* and *L* have stratifications $S,\ S'$ respectively such that $V_f = \{n - \text{cells of } S\}$ and $V_g = \{n - \text{cells of } S'\}$ where $S \cap S' =$ one 0-cell.

The map $f \vee g : K \vee L \to M \vee N$ is a cellular folding iff there exists another stratification S'' on $K \vee L$ such that $S_n'' = S_n + S_n'$, thus $V_{f \vee g} = V_f + V_g$.

Now let $e \in E$, then $e \in S_{n-1}''$ and lies in the frontier of exactly two *n*-cells $\gamma, \gamma' \in S_n''$ such that $(f \vee g)(\gamma) = (f \vee g)(\gamma')$. Then there are two cases:

(1) $f(\gamma) = f(\gamma'),\ \gamma, \gamma' \in S_n$, then there exists an edges to E_f lies in the frontier of γ and γ'.

(2) $g(\gamma) = g(\gamma'),\ \gamma, \gamma' \in S_n'$, then there exists an edge belongs to E_g lies in the frontier of γ and γ'.

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We conclude from the above possibilities that $E = E_f + E_g$. Thus $\Gamma^*_{f\vee g} = \Gamma^*_f + \Gamma^*_g$.

The above theorem can be generalized for a finite number of cellular foldings.

3-5 Example

Consider *M* and *K* are complexes such that $|M| = |K| = S^1$ with cell decompositions given in Fig. (26) and let $f : M \to M$, $g : K \to K$ be neat cellular foldings which squash *M* and *K* respectively to a 1-cell and two 0-cells.

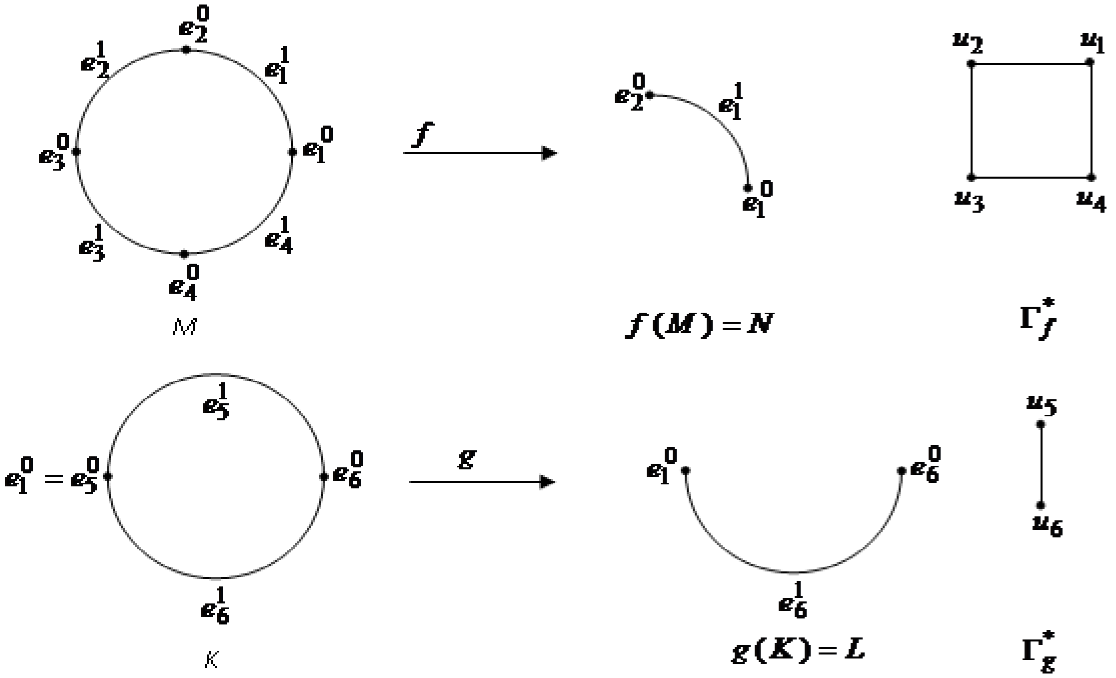

Fig. (26)

Then the wedge sum $f \vee g : M \vee K \to N \vee L$ is a cellular folding with dual folding graph $\Gamma^*_{f\vee g}$ is as shown in Fig. (27).

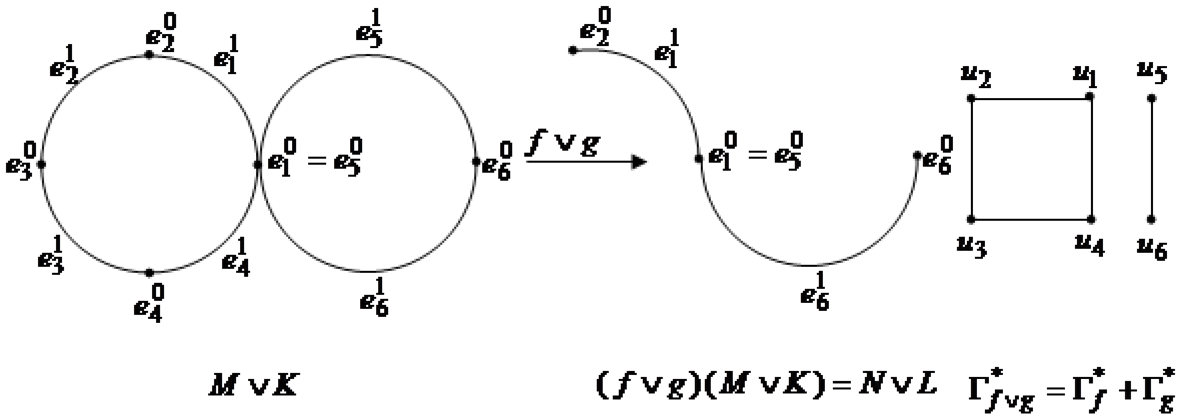

Fig. (27)

Lemma 1: Let *M* be a regular *CW*-complex of dimension 2, $N \subset M$, let $f : M \to M$ be a cellular folding with dual folding graph $\Gamma^*_f = (V_f, E_f)$. Then $g : M/N \to M/N$ is a cellular folding with dual folding graph $\Gamma^*_g = (V_g, E_g)$ such that $\Gamma^*_g = \Gamma^*_f$.

3-6 Example

Let $M = D^2$ be a disc with cellular subdivision consisting of five 0-cells, eight 1-cells and four 2-cells, and let $N = S^1 = \partial(D^2)$, $f : D^2 \to D^2$ be a cellular folding defined by

$$f(e_1^0, e_2^0, e_3^0, e_4^0, e_5^0) = (e_1^0, e_2^0, e_3^0, e_2^0, e_3^0),$$
$$f(e_1^1, e_2^1, e_3^1, e_4^1, e_5^1, e_6^1, e_7^1, e_8^1) = (e_1^1, e_1^1, e_3^1, e_3^1, e_5^1, e_5^1, e_5^1, e_5^1),$$
$$f(e_i^2) = (e_1^2), \qquad i = 1, 2, 3, 4.$$

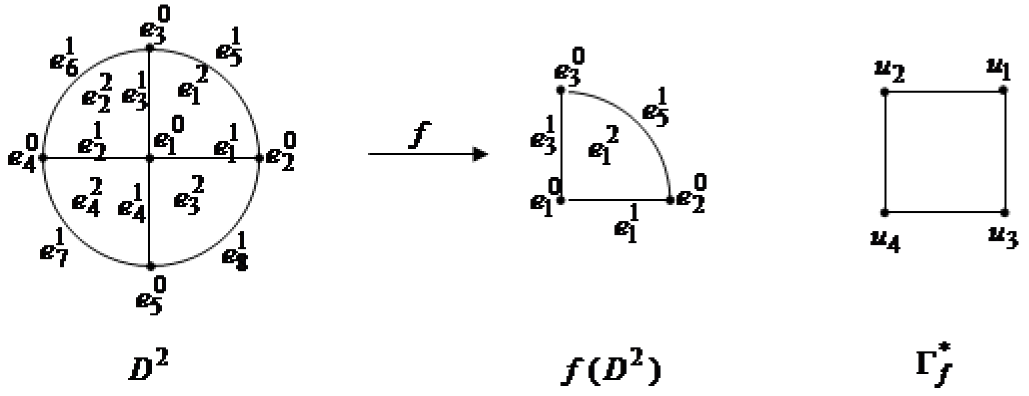

Fig. (28)

Then $g : D^2/N \to D^2/N$ is a cellular folding defined by:

$$g(e_1^0, e_6^0) = (e_1^0, e_6^0),$$
$$g(e_1^1, e_2^1, e_3^1, e_4^1) = (e_1^1, e_1^1, e_3^1, e_3^1),$$
$$g(e_i^2) = (e_1^2), \qquad i = 1, 2, 3, 4.$$

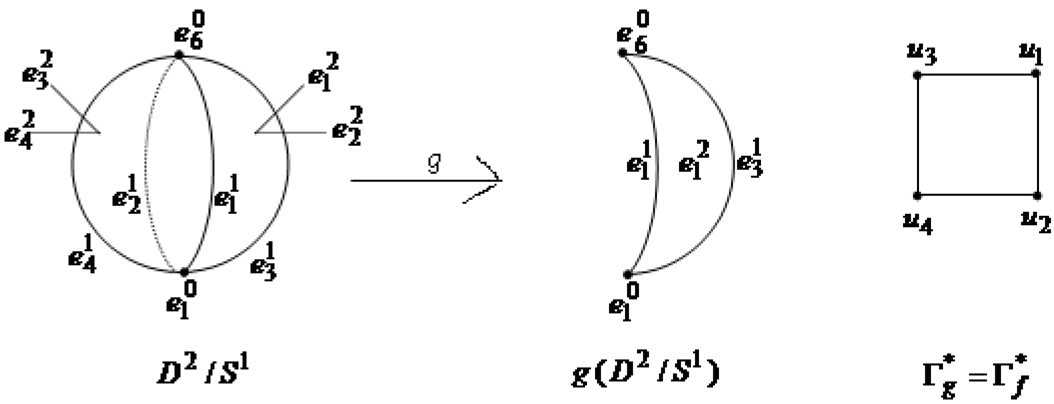

Fig. (29)

Lemma 2: Let *M* and *N* be regular *CW*-complexes of the same dimension, let $f : M \to N$ be cellular folding with dual folding graph $\Gamma_f^* = (V_f, E_f)$. Then the suspension map $g = S\,f : S\,M \to S\,N$ is a cellular folding with dual folding graph $\Gamma_g^* = (V_g, E_g)$ such that $\Gamma_g^* = \Gamma_f^* + \Gamma_f^*$.

3-7 Example

Let *M* be a complex such that $|M| = S^1$ with cellular subdivision consisting of four 0-cells and four 1-cells and let $f : M \to M$ be a cellular folding defined by:

$$f(e_1^0, e_2^0, e_3^0, e_4^0) = (e_1^0, e_4^0, e_1^0, e_4^0), \; f(e_1^1, e_2^1, e_3^1, e_4^1) = (e_4^1, e_4^1, e_4^1, e_4^1).$$

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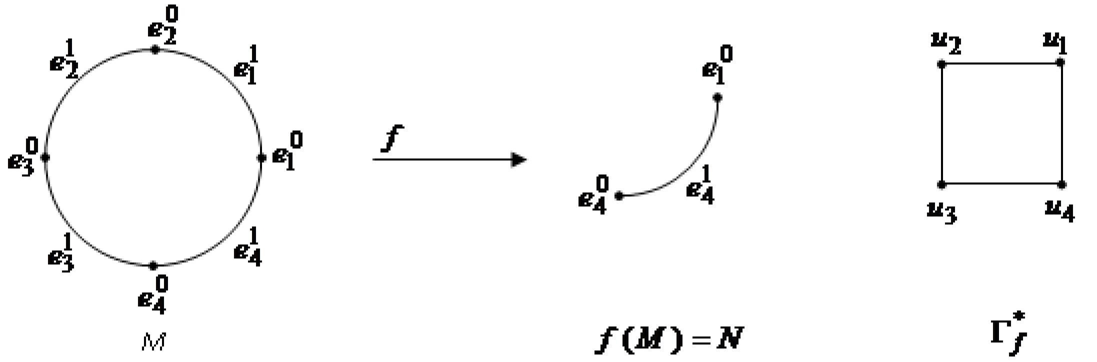

Fig. (30)

Then $g = S\,f : S\,M \to S\,N$ is a cellular folding defined by:

$$g\,(e_1^0, e_2^0, e_3^0, e_4^0, e_5^0, e_6^0) = (e_1^0, e_4^0, e_1^0, e_4^0, e_5^0, e_6^0),$$

$$g\,(e_1^2, ..., e_8^2) = (e_1^2, e_1^2, e_1^2, e_1^2, e_5^2, e_5^2, e_5^2, e_5^2).$$

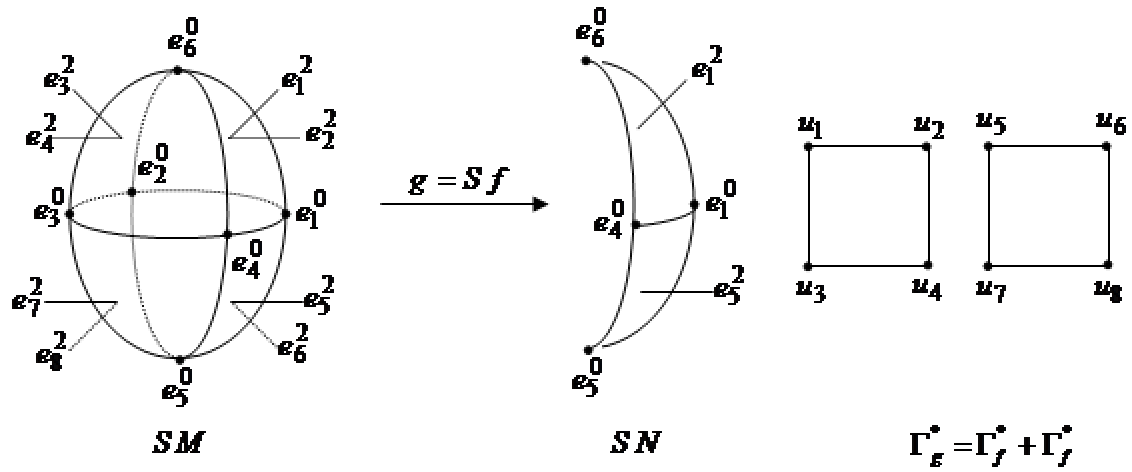

Fig. (31)

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