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EFFECT OF CRITICAL PARAMETERS ON SKIN FRICTION FOR THE FLOW OF A SECOND ORDER FLUID OVER AN INCLINED POROUS PLATE

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ABSTRACT

The unsteady state flow of a visco elastic fluid of a second order type over an inclined porous plate has been examined in this paper when a uniform tangential force F acts on the free surface for a finite interval of time. In the fitness of the situation the gravitational force that acts on the system has been taken into account. Throughout the analysis, the angle of inclination is held constant at an angle of $\pi/6$. It is noticed that as the visco elasticity increases, the skin friction is found to be gradually decreasing and thereafter remains almost constant. An interesting situation is that, as the porosity of the bounding surface increases, the skin friction is found to be in the decreasing trend. Initially, for the small values of visco elasticity, a relatively significant change in the skin friction is observed. The effect of various flow entities that affects the skin friction has been analyzed and they are illustrated graphically.

Keywords: Second order fluid, porous boundary, visco elasticity, skin friction

INTRODUCTION:

Due to wide ranging applications in the fields of Physics, Chemistry, and Chemical Technology and in situations demanding efficient transfer of mass over inclined beds, the viscous drainage over an inclined rigid plane has been the subject of considerable interest to theoretical and experimental investigators during the last several years. In all experiments, where the transfer of viscous liquid from one container to another is involved, the rate at which the transfer takes place and the thin film adhering to the surfaces of the container is to be taken into account for the purpose of chemical calculations. Failure to do so leads to experimental error. Hence there is need for such analysis.

In many chemical processing industries generally slurry adheres to the reactor vessels and gets consolidated. As a result of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and then consuming more reaction time. The slurry thus formed inside the reactor vessel often acts as a porous boundary for the next cycle of chemical processing.

Flow through porous media has been the subject of considerable research activity in recent years because of its several important applications notably in the flow of oil through porous rock, the extraction of geothermal energy from the deep interior of the earth to the shallow layers, the evaluation of the capability of heat removal from particulate nuclear fuel debris that may result from a hypothetical accident in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion-exchange beds, drug permeation through human skin, chemical reactor for economical separation or purification of mixtures and so on.

Flow in a porous medium can be considered as an ordered flow in a disordered geometry. The transport process of fluid through a porous medium involves two substances, the fluid and the porous matrix, and therefore it will be characterized by specific properties of these two substances. A porous medium usually consists of a large number of interconnected pores each of which is saturated with the fluid. The exact form of the structure, however, is highly complicated and differs from medium to medium. A porous medium may be either an aggregate of a large number of particles such as sand or gravel or solid containing many capillaries such as a porous rock. When the fluid percolates through a porous material, because of the complexity of microscopic flow in the pores, the actual path of an individual fluid particle cannot be followed analytically. In all such cases, one has to consider the gross effect of the phenomena represented by a macroscopic view applied to the masses of fluid, large compared to the dimensions of the pore structure of the medium. The process can be described in terms of equilibrium of forces. The driving force necessary to move a specific volume of fluid at a certain speed through a porous medium is in equilibrium with the resistance force generated by internal friction between the fluid and the pore structure. This resistance force is characterized by Darcy's semi-empirical law established by Darcy [1]. The simplest model for flow through a porous medium is the one-dimensional model derived by Darcy [1]. Obtained from empirical evidence, Darcy's law indicates that for an incompressible fluid flowing through a channel filled with a fixed, uniform and isotropic porous matrix, the flow speed varies linearly with longitudinal pressure variation. Subsequently, Dupuit and Frochheimer presented empirical evidence that, the Darcy law, or the linearity between speed and pressure variation, breaks down for large enough flow speed (a compilation of several experimental results) is presented by MacDonald *et al.* [2]. This was emphasized later by Joseph *et al.* [3] who stressed that, the form drag

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force modeled by the Frochheimer acts in a direction opposite to the velocity vector. It follows that, in multidimensional flow, the momentum equations for each velocity component derived using the Frochheimer extended Darcy equation is at least speculative. Later Knupp and Lage [4] analyzed the theoretical generalization to the tensor permeability case (anisotropic medium) of the empirically obtained Frochheimer extended Darcy unidirectional flow model.

A numerical and experimental investigation of the effects of the presence of a solid boundary and initial forces on mass transfer in porous media was presented by Vafai and Tien [5]. The local volume averaging technique has been used to establish the governing equations. The numerical solution of the governing equations is used to investigate the mass concentration field inside a porous media close to an impermeable boundary. In conjunction with the numerical solution, a transient mass transfer experiment has been conducted to demonstrate the boundary and inertia effects on mass transfer. This was accomplished by measuring the time and space averaged mass flux through a porous medium. The results clearly indicate the presence of these effects on mass transfer through porous media.

When the bounding surface is porous, then the rate of percolation of the fluid is found experimentally to be directly proportional to the cross sectional area of the filter bed and the total force, say the sum of pressure gradient and the gravity force. In the sense of Darcy

$$q = CA\left[\frac{P_1 - P_2}{H_1 - H_2} + \rho G\right] \tag{1}$$

where A is the cross sectional area of the filter bed, $C = \frac{K}{\mu}$ in which K is the permeability of the material and μ is the coefficient of viscosity and q is the flux of the fluid. A straight forward generalization of the above equation yields

$$V = -\frac{K}{\mu}\left[\nabla P + \rho G\eta\right] \tag{2}$$

where V is the velocity vector and η is the unit vector along the gravitational force taken in the negative direction. If any other external forces are acting on the system, instead of gravitational force, then we have

$$V = -\frac{K}{\mu}\left[\nabla P - \rho F\right] \tag{3}$$

In the absence of external forces, $V = \frac{-K}{\mu}\nabla P$ as a result of which $\nabla P = \frac{-\mu}{K} V$.

The problem of flow of viscous incompressible fluid moving under gravity down a fixed inclined plane with the assumption that the velocity of the fluid at the free surface is given has been examined earlier by Sneddon. Jeffreys [6] initiated the problem of steady state profile over a vertical flat plate which was further examined by Green [7]. Later, Gutfinger and Tallmadge [8] investigated steady state drainage over a vertical cylinder. Later Bhattacharya [9] examined the problem when uniform tangential force $'F'$ acts on the upper surface for a finite interval of time. These authors examined the problem in the absence of fluid inertia.

The constitutive relation of a second order fluid is given by:

$$S_{ij} = -P\delta_{ij} + \phi_1 E_{ij}^{(1)} + \phi_2 E_{ij}^{(2)} + \phi_3 E_{ij}^{(1)^2} \tag{4}$$

where $$E_{ij}^{(1)} = U_{i,j} + U_{j,i} \tag{5}$$

and $$E_{ij}^{(2)} = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \tag{6}$$

In the above equations, S_{ij} is the stress-tensor, U_i , A_i are the components of velocity and acceleration in the direction of the i th co-ordinate X_i , P is indeterminate hydrostatic pressure and the coefficients ϕ_1 , ϕ_2 and ϕ_3 are material constants.

In view of several industrial and technological importance, Ramacharyulu [10] studied the problem of the exact solutions of two dimensional flows of a second order incompressible fluid by considering the rigid boundaries. Later, Lekoudis *et al* [11] presented a linear analysis of the compressible boundary layer flow over a wall. Subsequently, Shankar and Sinha [12] studied the problem of Rayleigh for wavy wall. The effect of small amplitude wall waviness upon the stability of the laminar boundary layer had been studied by Lessen and Gangwani [13]. Further, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a vertical flat wall was examined by Vajravelu and Shastri [14] and thereafter by Das and Ahmed [15]. The free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls was investigated by Patidar and Purohit [16]. Rajeev Taneja and Jain [17] had examined the problem of MHD flow with slip effects and temperature dependent heat in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate.

Recently, Ramana Murthy *et al* [18] presented a detailed investigation of the second order fluid flow between two parallel plates with the lower plate possessing natural permeability. In their analysis it is observed that the skin friction on the upper plate is almost linear with respect to the visco elasticity of the fluid. However, the situation seems to be not stated as above at the lower plate. But it is certain that as the visco elasticity increases, the skin friction on the plate increases. The class of exact solutions due to sinusoidal forced oscillations on the porous boundary has been investigated by Ramana Murthy *et al* [19]. Recently Ramana Murthy and Kavitha [20] presented a linear analysis of the flow of second order fluid over an inclined porous plate in which the effect of various flow entities on the velocity profiles were presented. Similar such analysis was presented by Ramana Murthy and Gowthami *et al* [21], but of course the problem was confined only to a rigid plane. In such an analysis the nature of velocity profiles and the magnitude of amplification factor were discussed with respect to critical parameters affecting the flow field. Though the concept of skin friction was of prime importance, the factor has been totally ignored in the above analysis.

EQUATIONS OF THE MOTION IF THE BOUNDING SURFACE IS POROUS:

By taking the porosity into account, the equations of motion in X, Y and Z directions can be expressed as:

$$\rho \frac{DU_1}{DT} = \rho F_X + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \frac{\partial S_{XZ}}{\partial Z} - \frac{\mu}{K} U_1 \tag{7}$$

$$\rho \frac{DU_2}{DT} = \rho F_Y + \frac{\partial S_{YX}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} + \frac{\partial S_{YZ}}{\partial Z} - \frac{\mu}{K} U_2 \tag{8}$$

$$\rho \frac{DU_3}{DT} = \rho F_Z + \frac{\partial S_{ZX}}{\partial X} + \frac{\partial S_{ZY}}{\partial Y} + \frac{\partial S_{ZZ}}{\partial Z} - \frac{\mu}{K} U_3 \tag{9}$$

The aim of the present note is to examine the unsteady state flow (by considering the gravitational force) of a visco elastic fluid of a second order type over an inclined porous plate when a uniform tangential force F acts on the free surface for a finite interval of time and to examine the effect of various flow entities on the skin friction.

MATHEMATICAL FORMULATION OF THE PROBLEM:

The constitutive relation for a visco elastic fluid of second order type as stated in eqn. (4) is considered. The problem is examined hereunder with reference to the rectangular Cartesian co-ordinate system with the x - axis along the plate in the direction of the motion and y - axis into the fluid perpendicular to this direction. The motion is assumed to be unidirectional i.e., along x - axis and hence the components of the velocity can be regarded as $\left[u\left(y,t\right),0,0\right]$.

The equation of motion in the dimensional form is given by the relation:

$$\frac{\partial u}{\partial t} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\left[\phi_1 + \phi_2 \frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial y^2} - g \sin \alpha - \frac{u}{k} \tag{10}$$

where α is the angle of inclination of the plane with the horizontal, ρ is the fluid density which is assumed to be constant throughout and g is the acceleration due to gravity.

The condition of no slip on the boundary would yield $u = 0$, when $y = 0$. Further, the condition of uniform tangential force F on the free surface for a finite interval of time is

$$\left[\phi_1 + \phi_2 \frac{\partial}{\partial t}\right]\frac{\partial u}{\partial y} = F\left[H(t) - H\left(t - t_0\right)\right],\ t > 0 \tag{11}$$

together, with the initial condition $u\left(y,0\right) = 0$. (12)

Introducing the following non - dimensionalization scheme

$$\left.\begin{array}{lll} X = x/H\,, & Y = y/H\,, & \phi_2 = \rho H^2 \beta \\ t = \rho H^2 T/\phi_1, & u = \phi_1^2 U/(\rho H)\,, & k = \rho H^3/(\phi_1^2 K) \\ g = \phi_1^2 G/(\rho^2 H^3)\,, & P = \phi_1^2\ p/(\rho H^2) & \end{array}\right\} \tag{13}$$

where β is the non-dimensional visco elastic parameter.

The governing equation for the fluid motion in the non-dimensional form together with the required conditions reduces to

$$\frac{\partial U}{\partial T}=\frac{-\partial P}{\partial X}+\frac{\partial^2 U}{\partial Y^2}+\beta\frac{\partial^3 U}{\partial Y^2\partial T}-G\sin\alpha-\frac{U}{K} \tag{14}$$

with the condition $U=0$ when $Y=0$, (15)
together with:

$$\frac{\partial U}{\partial Y}+\beta\frac{\partial^2 U}{\partial Y\partial T}=F\left[H(T)-H(T-T_0)\right] \text{ at } Y=1. \tag{16}$$

$H(T)$ in eqn. (11) represents the Heavisides Unit step function given by

$$\left.\begin{aligned} H(T)&=0 \text{ for } T<0\\ &=1 \text{ for } T>0\end{aligned}\right\} \tag{17}$$

Eqn. (14) after applying Laplace transforms will be:

$$[1+\beta s]\frac{d^2\overline{U}}{dY^2}-\left(s+\frac{1}{K}\right)\overline{U}=\frac{1}{s}\left(\frac{\partial P}{\partial X}+G\sin\alpha\right) \tag{18}$$

together with the conditions on the boundary as

$$\overline{U}=0 \text{ when } Y=0, \tag{19}$$

and $$[1+\beta s]\frac{d\overline{U}}{dY}=\frac{F}{s}\left[1-\exp(-T_0 s)\right] \text{ at } Y=1 \tag{20}$$

The solution of eqn. (18) satisfying conditions eqn. (19) and eqn. (20) is given by:

$$\overline{U}=\left(\frac{1}{C^2}\right)\left(\frac{D+G\sin\alpha}{s(1+\beta s)}\right)\cosh CY+\left[\frac{F\left(1-e^{-T_0(s)}\right)}{Cs(1+\beta s)\cosh C}-\frac{1}{C^2}\left(\frac{D+G\sin\alpha}{s(1+\beta s)}\right)\tanh C\right]\sinh CY$$
$$-\frac{1}{C^2}\left(\frac{D+G\sin\alpha}{s(1+\beta s)}\right) \tag{21}$$

where $C^2=\dfrac{s+\left(\frac{1}{K}\right)}{1+\beta s}$ and $D=\dfrac{\partial P}{\partial X}$ (22)

Taking inverse Laplace transform of eqn. (21) the velocity field is obtained as

$$U=K(D+G\sin\alpha)\left(\cosh\sqrt{\frac{1}{K}}Y-1\right)-K(D+G\sin\alpha)\left(\tanh\sqrt{\frac{1}{K}}\cdot\sinh\sqrt{\frac{1}{K}}Y\right)$$
$$-2F\sum\frac{(-1)^n\sin(2n+1)\frac{\pi}{2}Y}{(2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}}\left[\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)T}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)-\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)(T-T_0)}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)\right]$$
$$+2(D+G\sin\alpha)\sum\frac{\sin(2n+1)\frac{\pi}{2}Y}{\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)\left((2n+1)\frac{\pi}{2}\right)}\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)T}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right) \tag{23}$$

Skin friction on the plate is $\psi = \dfrac{\partial U}{\partial Y} + \beta\left[\dfrac{\partial^2 U}{\partial Y \partial T} - \dfrac{\partial^2 U}{\partial Y^2}\right]$ when $Y = 0$ (24)

Hence,

$$\psi = -\left(D + G\sin\alpha\right)\left(\sqrt{K}\tan h\sqrt{\frac{1}{K}} + \beta\right) +$$

$$\left[2\left(D + G\sin\alpha\right)\sum \exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4} + \frac{1}{K}\right)T}{1 + (2n+1)^2\frac{\pi^2}{4}\beta}\right)\frac{\left(1 - \frac{\beta}{K}\right)}{\left(1 + (2n+1)^2\frac{\pi^2}{4}\beta\right)\left((2n+1)^2\frac{\pi^2}{4} + \frac{1}{K}\right)}\right]$$

$$-2F\sum\left[\frac{(-1)^n(2n+1)\frac{\pi}{2}\left(1 - \frac{\beta}{K}\right)}{\left(1 + (2n+1)^2\frac{\pi^2}{4}\beta\right)\left((2n+1)^2\frac{\pi^2}{4} + \frac{1}{K}\right)}\right]$$

$$\left[\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4} + \frac{1}{K}\right)T}{1 + (2n+1)^2\frac{\pi^2}{4}\beta}\right) - \exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4} + \frac{1}{K}\right)(T - T_0)}{1 + (2n+1)^2\frac{\pi^2}{4}\beta}\right)\right] \tag{25}$$

RESULTS AND CONCLUSIONS:

1. Fig. 1 illustrates the combined effect of porosity and visco elasticity of the fluid on skin friction. It is noticed that as the visco elasticity increases, the skin friction is found to be decreasing gradually and thereafter, remains almost constant. Further, it is noticed that, when the visco elasticity of the fluid medium is held constant and as the porosity increases, the skin friction decreases.

2. The effect of porosity on the skin friction is illustrated in fig. 2 and fig. 3. It is noticed that, as the porosity of the bounding surface increases, the skin friction is found to be in the decreasing trend. Further, as the visco elasticity increases, the skin friction is found to be on raise and subsequently converges to a common point. It is further noticed that the contribution of porosity of the bounding surface (K) has not much of influence on the profiles as seen in both situations inspite of variation in visco elaticity.

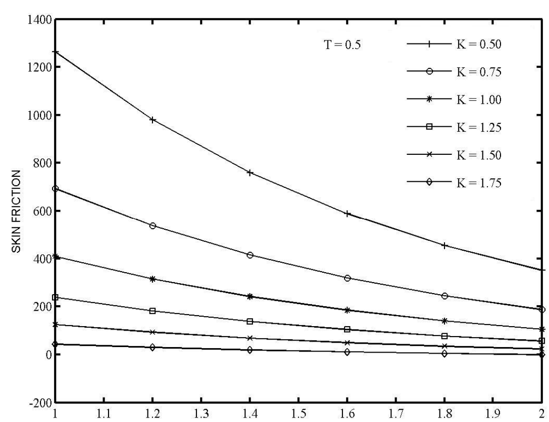

Fig. 1: Profiles for skin friction Vs porosity values when T =0.5

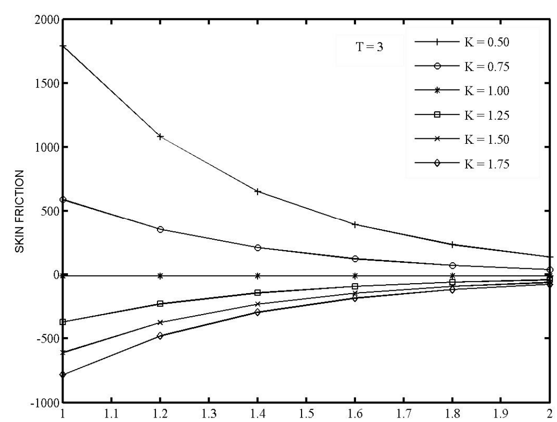

Fig. 2: Profiles for skin friction as the porosity of the boundary varies when T =3

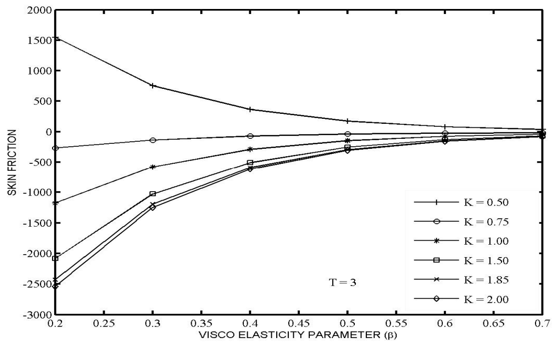

Fig. 3: Variation of skin friction for different porosity values when T = 3

3 Fig. 4 illustrates the effect of visco elasticity of the fluid and porosity of the bounding surface on the skin friction. Initially, for the small values of visco elasticity, relatively significant change in the skin friction is observed. However, as it increases, the skin friction is found to be almost absent. It is noticed that, the skin friction takes negative values which indicates that the friction is offered by the plate to the fluid than by the fluid on the plate.

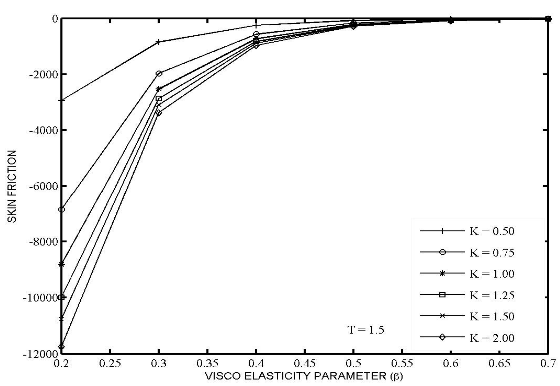

Fig. 4: Effect of porosity on skin friction for T =1.5

4 The effect of porosity, relatively for higher values of visco elasticity parameter is illustrated in fig. 5. As found in the earlier cases, even if visco elasticity increases, significant drop in the skin friction is noticed. In addition to the above, as the porosity increases, a decreasing trend in the skin friction is observed.

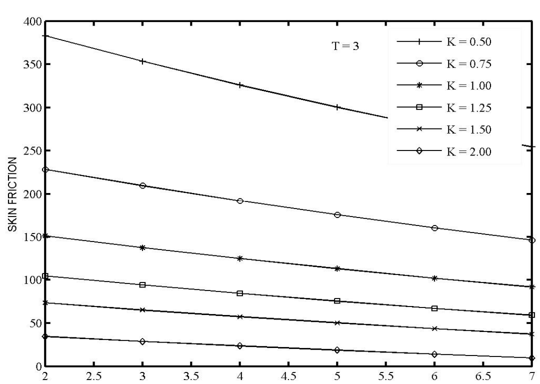

Fig. 5: Skin friction Vs porosity values when T = 3

5 Figs. 6, 7 and 8 illustrates the combined effect of time as well as porosity on the skin friction. It is noticed that as the time parameter T advances, the skin friction increases. However, as the pore size of the fluid bed increases, the skin friction is found to be on decreasing trend. Such a pattern can be attributed to the fact that the fluid particles are trapped in the pores of the fluid bed resulting in the lower skin friction.

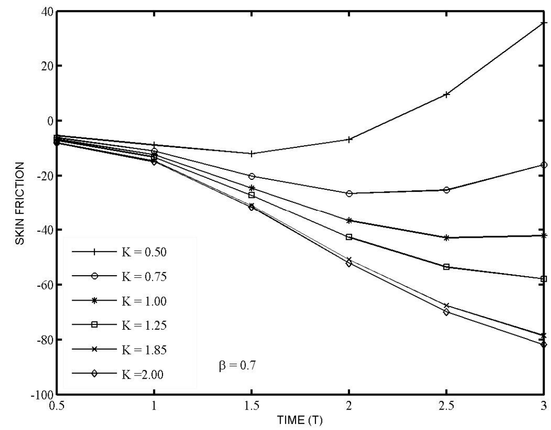

Fig. 6: Skin friction Vs porosity values when β = 0.7

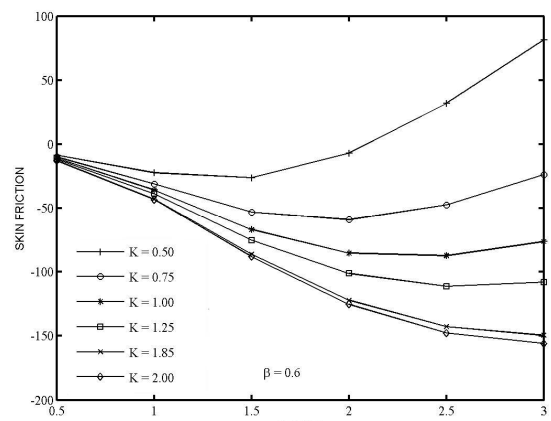

Fig. 7: Effect of porosity on skin friction as time (T) advances for β = 0.6

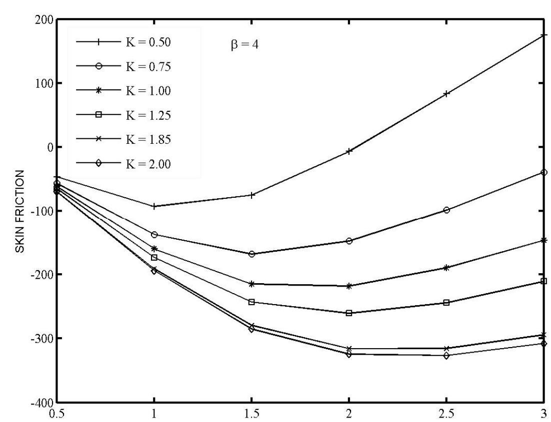

Fig. 8: Profiles for skin friction for different porosity values when β = 4

6 The effect of time on the skin friction is exhibited in figs. 9 and 10. It is noticed that when the time parameter T is held constant and as the porosity increases, the skin friction decreases. However, as T increases, the skin friction is found to be inversely related.

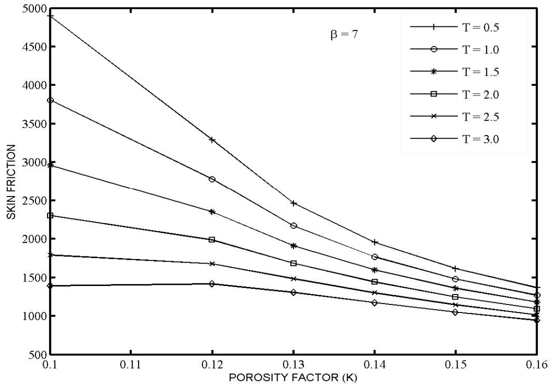

Fig. 9: Effect of Time on skin friction for different porosity values when β = 7

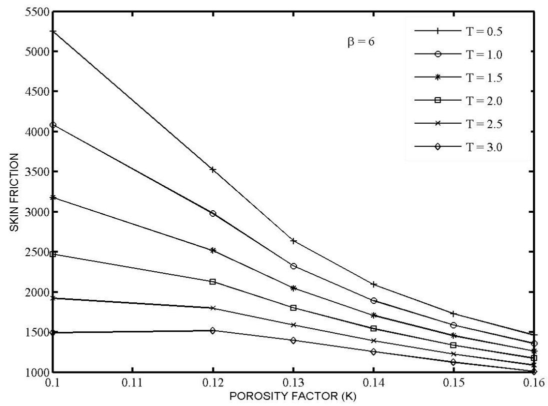

Fig. 10: Effect of Time on skin friction for different porosity values when β = 6

7 Figs. 11 and 12 illustrates the effect of visco elasticity on skin friction. It is noticed that, as the visco elasticity increases, the skin friction decreases. Such a situation is due to the strong intra molecular forces that exists in between the fluid particles. Greater the visco elasticity, stronger is the intra molecular forces.

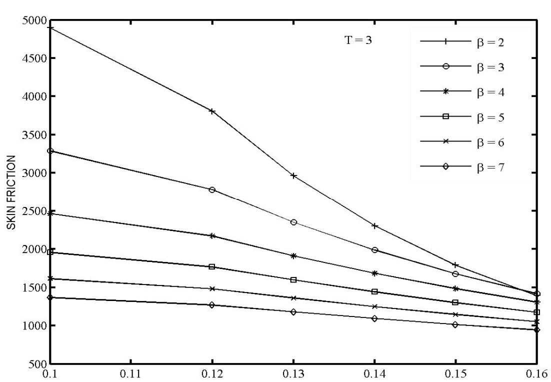

Fig. 11: Profiles for skin friction for different visco elasticity values when T = 3

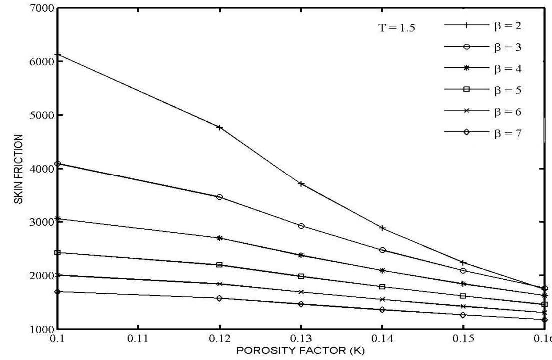

Fig. 12: Skin friction Vs visco elasticity when T =1.5

8 The effect of time on the relatively lower values of visco elasticity parameter which affects the skin friction is observed in fig. 13. Initially for the smaller values of visco elasticity, as time parameter (T) increases, the skin friction is found to be on the lower side. For the visco elasticity parameter slightly above and higher for $\beta \geq$ 0.3 the effect is found to be on reverse.

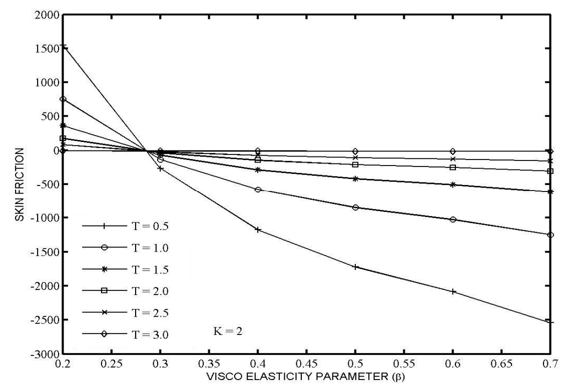

Fig. 13: Effect of time on skin friction at different when K = 2

9 While all other conclusions stated above are found to be in common in many parameters, it is seen that the effect of time parameter T and visco elasticity (β) is found to be zero for β = 0.3 and K= 2 in fig. 13. The situation is found to be similar now when β = 0.25 and K = 1.5 in fig. 14. Such an observation indicates that the nil effect is found to be shifting towards the left as the porosity of the fluid bed decreases.

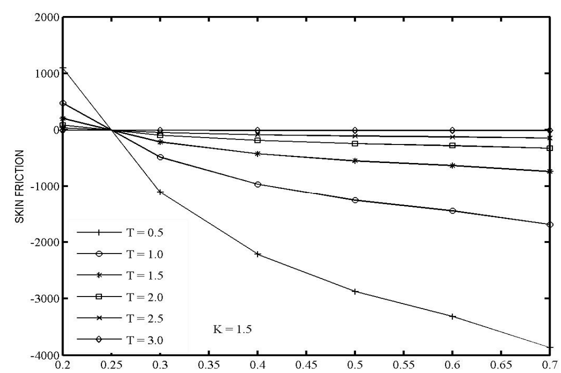

Fig. 14: Skin friction Vs Time when K = 1.5

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