

**A STUDY OF FRACTIONAL DIFFERENTIAL OPERATOR INVOLVING  
THE MULTIVARIABLE I-FUNCTION  
WITH APPLICATIONS AND OTHER DISCIPLINES**

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**ABSTRACT**

*In the present paper we use a fractional differential operator  $D$  to study the Multivariable I- function with application and other disciplines.*

*In this paper we establish two Multiplication formulas for the Multivariable I-function. We determine some new and known results for the above function, a number of known results for other simple function follows as special case of our results.*

**1. INTRODUCTION:**

The Multivariable I-function is defined by Prasad by Prasad [5] and Prasad [6] and further studied by Sasena [9], Ronghe [7,8] are given extension, summation certain properties etc.

The multivariable I-function defined in a following manner.

$$I_R [ Z_1, Z_2, \dots, Z_n ] = O. N M_1, N_1 \dots M_r, N_r$$

$$= I [ P_i, Q_i; R ] [ P_i, Q_i; R ] \dots [ P_i^{(r)}, Q_i^{(r)}; R^{(r)} ]$$

$$\left[ Z_1 [(a_j : \alpha_j, \dots, \alpha_j^{(r)})_1, N [(a_{ji} : \alpha_{ji}, \dots, \alpha_{ji}^{(r)})_{N+1}, P_i] : [(b_{ji} : \beta_{ji}, \dots, \beta_{ji}^{(r)})_1, Q] : [(a_j, \xi_j)_{1,M} [(a_{ji}, \xi_{ji})_{M_{i+1}}, Q_i] \right.$$

$$\left. \cdot Z_r [(c', r'_{ji})_{1,N_2} ] [ 9c'_{ji}, \delta'_{ji} )_{N_1+1}, P_i ] [ c_{ji}^{(r)}(r), \delta_{ji}^{(r)}(r) ]_{N+1}, P_r [ (d_j^{(r)}, \delta_j^{(r)})_{1,M_r} ], [ d_{ji}^{(r)}(r), \delta_{ji}^{(r)}(r) ]_{M_r+1}, Q_r \right]$$

$$= \frac{1}{(2\pi w)^r} \int_{L_1} \dots \int_{L_r} \phi(\xi_i) \dots \phi_r(\xi_r) \psi(\xi_i, \dots, \xi_r) z_1^{\xi_1} \dots z_r^{\xi_r} d \xi_i \dots d \xi_r \quad (1.1)$$

where,  $\omega = \sqrt{-1}$  and  $(0, N) = (0, n_2), (0, n_3) \dots (0, n_r)$

$$\Phi_k(\xi_k) = \frac{\prod_{j=1}^{M_k} \Gamma(s_1) \prod_{j=1}^{N_k} \Gamma(s_2)}{\left( \sum_{j=1}^{R_k} \prod_{j=M_{k+1}}^{Q_k} \Gamma(s_3) \prod_{j=N_{k+1}}^{P_k} \Gamma(s_4) \right)} \quad (1.2)$$

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$$\psi_K(\xi_1, \dots, \xi_r) = \frac{\prod_{j=1}^N \Gamma(s_j)}{\left( \sum_{l=1}^r \prod_{i=N+1}^{P_i} \Gamma(s_{\theta_i}) \prod_{i=N_l}^{Q_i} \Gamma(s_{\gamma_i}) \right)} \quad (1.3)$$

Where  $S_1, S_2, S_3, \dots, S_7$  are set of parameters and short notation are given below and will be used in throughout the present paper.

- (i)  $S_1 = (d_j^{(K)} - \delta d_j^{(K)} \xi_K)$
- (ii)  $S_2 = (1 - c_j^{(K)} + \delta d_j^{(K)} \xi_K)$
- (iii)  $S_3 = (1 - d_{j_i}^{(K)}(K) - \delta_{j_i}^{(K)}(K) \xi_K)$
- (iv)  $S_4 = (c_{j_i}^{(K)}(K) - \delta_{j_i}^{(r)}(K) \xi_K)$
- (v)  $S_5 = (1 - a_j + \sum_{K=1}^r (\alpha_j^{(K)} \xi_K))$
- (vi)  $S_6 = (a_{j_i} - \sum_{K=1}^r (\alpha_{j_i}^{(K)} \xi_K))$
- (vii)  $S_7 = (1 - b_{j_i} + \sum_{K=1}^r \beta_{j_i}^{(K)} \xi_K)$

And short notation

$$\lambda^* = \sum_{i=1}^r \lambda_i; \mu^* = \sum_{i=1}^r \mu_i \xi_i; \xi_1 = \Omega(K_1, \dots, K_r), \prod_{j=1}^r \frac{X^r}{K_j!}$$

$$\xi_2 = \prod_{j=1}^r \frac{(-N_j^*) M_j^*, K_j^*}{K_j^*!} A[N_1^*, K_1^*, \dots, N_r^*, K_r^*]$$

$$\xi_3 = \prod_{j=1}^r \frac{(P_j K_j)_{\lambda_j} [a_j x]^{\lambda_j}}{\lambda_j!} : \xi_4 = \prod_{j=1}^{r-1} \frac{(P_j K_j)_{\lambda_j} [a_j x]^{\lambda_j}}{\gamma_j!} \frac{[a_r x]^{\lambda_r}}{\gamma_r!}$$

$$x_j = \frac{x_i}{(1 - a_j x)^{p_i}} : Y_j^* = y_j \xi^{\sigma_j} x^{\sigma_j}, \forall j \in \{1, \dots, r\} \quad (1.4)$$

$$\sum_{K_i=0}^{\infty} \equiv \sum_{K_i: \dots : K_r=0}^{\infty} : \sum_{\lambda_i=0}^{\infty} \equiv \sum_{\gamma_i: \dots : \lambda_r=0}^{\infty}$$

and

$$\sum_{K_i^*=0}^{[N_i^*/M_i^*]} = \sum_{K_i^*=0}^{[N_i^*/M_i^*]} = \dots = \sum_{K_r^*=0}^{[N_r^*/M_r^*]}, i = 1, 2, \dots, r.$$

For further details and asymptotic expansions of the above function (1.1) refer to [5] and [6].

## 2. PRELIMINATIES:

We use the fractional derivative operator's defined in the following manner.

$$D_K^\alpha (ax + b)^{\mu-1} = \frac{a^\mu \Gamma(\mu) (ax + b)^{\mu-\alpha-1}}{\Gamma(\mu - \alpha)}, \quad (2.1)$$

where  $\alpha \neq \mu$

$$D_{K,\alpha,x}(ax+b)^\mu = \frac{\Gamma(1+\mu)a^\alpha(ax+b)^{\mu+K}}{\Gamma(\mu-\alpha+1)} \quad (2.2)$$

where  $\alpha \neq \mu$

$$D_{K,\alpha,x}^\alpha(ax+b)^\mu = [a]^{\frac{n\mu+n(n-1)K}{2}} \prod_{w=0}^{n-1} \frac{\Gamma(1+\mu+wk)}{\Gamma(1+\mu+wk-\alpha)} x(ax+b)^{\mu+nK} \quad (2.3)$$

In (2.3) by taking  $K = \alpha$

$$D_{\alpha,\alpha,x}^n(ax+b)^\mu = [a]^{\frac{n\mu+n(n-1)K}{2}} \frac{\Gamma(1+\mu+w\alpha)}{\Gamma(\mu-\alpha+1)} (ax+b)^{\mu+nK} \quad (2.4)$$

Taking  $a = 1, b = 0$  in (2.1) to (2.4) we get the results due to Oldhem and Spnier [4,P.49] We also assume throughout this paper that,

$$\varphi_1 \equiv (-\mu - \omega k + \lambda_1, \dots, \lambda_r)_{\omega=0, n-1}$$

$$\varphi_2 \equiv (-\mu - rk + \lambda_1, \dots, \lambda_r)_{\omega=0, n-1}$$

$$\varphi_3 \equiv (-\mu - \omega k + \lambda_1, \dots, \lambda_r)_{\omega=0, n-1} \text{ and } (ax+b) = T$$

### 3. FRACTIONAL DERIVATIVES OF MULTIVARIABLE I-FUNCTION:

In this section we obtain an interesting result with the help of fractional derivative operator indicated in the preceding section

$$D_{k,\alpha,x}^n(T)^\mu I[T_1^{\lambda_1}, T_2^{\lambda_2}, \dots, T_n^{\lambda_r}] = (T)^{\mu+nk+\lambda} [a]^{\frac{n\mu+n(n-1)k+\lambda}{2}} \times I_{[P_i+N, Q_i+n; R][P_i, Q_i; R], \dots, [P_i^{(r)}, Q_i^{(r)}; R^{(r)}]}^{O, N+n; M_1, N_1, \dots, M_r, N_r} \left[ \begin{matrix} a^{\lambda_1} (T)^{\lambda_1} \\ a^{\lambda_r} (T)^{\lambda_r} \end{matrix} \middle| \begin{matrix} \varphi_1, (1-S_5), (S_6), \dots \\ (1-S_7), \varphi_2 \end{matrix} \right]$$

Provided that  $\mu$  and  $\lambda_1, \lambda_2, \dots, \lambda_r$  are positive integers and

$$R_e(\mu) + \sum_{k=1}^r \lambda_k \min \left\langle R_e \left( d_j^{(k)/\delta_j^k} \right) \right\rangle > -1 \quad (3.1)$$

**Proof:** To establish (3.1) replacing the Multivariable I-function in the derivative as Melline-Barnes type contour integral (1, 1), and changing the order of integration with the help of differential operator (2.2) we get.

$$= \frac{1}{(2\tau\omega)^r} \int_{L_1} \dots \int_{L_r} \varphi_1(\xi_1), \dots, \varphi_r(\xi_r) \psi(S_1, \dots, S_r) \times D_{k,\alpha,x}^{n+\lambda} \left\langle (ax+b)^{\mu+\lambda_1 S_1 + \dots + \lambda_r S_r} \right\rangle dS_1 \dots dS_r$$

Now evaluating inner differential operator with the help of (2.3) and interpreting the result thus obtained with the help of (1.1) we get r.h.s. of (3.1).

**PARTICULAR CASES OF 3.1:** the multivariable I-function, being in generalized form, contains a large number of special functions as its particular cases.

(a) In (3.1) putting  $a = 1, b = 0$ , we get

$$D_{k,\alpha,x}^{n+\lambda}(X)^\mu I[X^{\lambda_1} \cdot X^{\lambda_2} \dots X^{\lambda_r}] = I_{[P_i+N, Q_i+n; R][P_i, Q_i; R], \dots, [P_i^{(r)}, Q_i^{(r)}; R^{(r)}]}^{O, N+n; M_1, N_1, \dots, M_r, N_r} \left[ \begin{matrix} X^{\lambda_1} \\ X^{\lambda_r} \end{matrix} \middle| \begin{matrix} \varphi_1, (1-S_5), (S_6), \dots \\ (1-S_7), \varphi_2 \end{matrix} \right] \quad (3.2)$$

$$D_{k,\alpha,x}^{n+\lambda} (T)^\mu H_{P_i+n,Q_i+n:P,Q_iP_i^{(r)},Q_i^{(r)}}^{O,N+n:M_1,N_1,\dots,M_r,N_r} [T^{\lambda_1} \dots T^{\lambda_r}] = T^{\mu+nk+\lambda} [a] \frac{n\mu + n(n-1)k + \lambda}{2} \times H_{P_i+n,Q_i+n:P,Q_iP_i^{(r)},Q_i^{(r)}}^{O,N+n:M_1,N_1,\dots,M_r,N_r} \left[ \begin{matrix} a^{\lambda_1} (T)^{\lambda_1} \\ a^{\lambda_r} (T)^{\lambda_r} \end{matrix} \middle| \begin{matrix} \varphi_1, (1-S_5), (S_6) \\ (1-S_7), \varphi_2 \end{matrix} \right] \quad (3.3)$$

(b) In (3.1) if  $R^{(r)} = 1, r=1$  and  $i = 1$ . If reduce to Multivariable H – function.

The condition of validity of (3.2) and (3.3) are directly obtainable form (3.1)

#### 4. MULTIPLICATION FORMULAS

In this section we establish two multiplication formulae involving I- function of Multivariable

$$\prod_{w=0}^{n-1} (\mu + wk + a_1 - 1) I_{[P_i+n,Q_i+n:R][P_i,Q_i:R] \dots [P_i^{(r)},Q_i^{(r)}:R^{(r)}]}^{O,N+n:M_1,N_1,\dots,M_r,N_r} \left[ (T)^{\lambda_1} \dots (T)^{\lambda_r} \middle| \begin{matrix} \varphi_1 (a_1: \lambda_1 + \lambda_2 + \dots + \lambda_r) (1-S_5) (S_6) \\ (1-S_7), \varphi_2 \end{matrix} \right] = I_{[P_i+n,Q_i+n:R][P_i,Q_i:R] \dots [P_i^{(r)},Q_i^{(r)}:R^{(r)}]}^{O,N+n:M_1,N_1,\dots,M_r,N_r} \left[ (T) \middle| \begin{matrix} \varphi_1 (a_1: \lambda_1 + \dots + \lambda_r) (1-S_5) (S_6) \\ (-S_7), \varphi_2 \end{matrix} \right] - I_{[P_i+n,Q_i+n:R][P_i,Q_i:R] \dots [P_i^{(r)},Q_i^{(r)}:R^{(r)}]}^{O,N+n:M_1,N_1,\dots,M_r,N_r} \left[ (T) \middle| \begin{matrix} \varphi_3 (a_1: \lambda_1 + \dots + \lambda_r) (1-S_5) (S_6) \\ (-S_7), \varphi_2 \end{matrix} \right] \quad (4.1)$$

$$\prod_{w=0}^{n-1} (\mu + wk + b_1) I_{[P_i+n,Q_i+n:R][P_i,Q_i:R] \dots [P_i^{(r)},Q_i^{(r)}:R^{(r)}]}^{O,N+n:M_1,N_1,\dots,M_r,N_r} \left[ (T) \middle| \begin{matrix} \varphi_3 (1-S_5) (S_6) \\ \varphi_2, (b_1: \lambda_1 + \dots + \lambda_r) \end{matrix} \right] = I_{[P_i+n,Q_i+n:R][P_i,Q_i:R] \dots [P_i^{(r)},Q_i^{(r)}:R^{(r)}]}^{O,N+n:M_1,N_1,\dots,M_r,N_r} \left[ (T) \middle| \begin{matrix} \varphi_3 (1-S_5) (S_6) \\ \varphi_2, (b_1=1: \lambda_1 + \dots + \lambda_r) (1-S_7) \end{matrix} \right] + I_{[P_i+n,Q_i+n:R][P_i,Q_i:R]}^{O,N+n:M_1,N_1,\dots,M_r,N_r} \left[ (T) \middle| \begin{matrix} \varphi_3 (1-S_5) (S_6) \\ \varphi_2, (b_1, \lambda_1 + \dots + \lambda_r) (1-S_7) \end{matrix} \right] \quad (4.2)$$

To prove (4.1) and (4.2) we use the definition (1.3),(3.1) and used of Gamma function i.e.  $\Gamma (Z+1) = \Gamma(Z).Z$ .

#### 5. APPLICATION:

in (4.1) putting and  $r=1$ , it reduce in term of the I-function of one variable [7, P.98]

$$\prod_{w=0}^{n-1} (\mu + wk + d_1 - 1) I_{[P_i+n,Q_i+n:R]}^{O,N+n} \left[ (T) \middle| \begin{matrix} \varphi_1 (a_1: \lambda) \dots \dots \dots \\ \varphi_2, \dots \dots \dots \end{matrix} \right] = I_{[P_i+n,Q_i+n:R]}^{O,N+n} \left[ (T) \middle| \begin{matrix} \varphi_3, (a_1: \lambda) \dots \dots \dots \\ \varphi_2, \dots \dots \dots \end{matrix} \right] - I_{[P_i+n,Q_i+n:R]}^{O,N+n} \left[ (T) \middle| \begin{matrix} \varphi_3 (a_1 - 1, \lambda) \dots \dots \dots \\ \varphi_2, \dots \dots \dots \end{matrix} \right] \quad (5.1)$$

$$\prod_{w=0}^{n-1} (\mu + wk + b_1) I_{[P_i+n,Q_i+n:R]}^{O,N+n} \left[ (T) \middle| \begin{matrix} \varphi_3 \dots \dots \dots \\ \varphi_2, (b_1, \lambda) \dots \dots \end{matrix} \right] = I_{[P_i+n,Q_i+n:R]}^{O,N+n} \left[ (T) \middle| \begin{matrix} \varphi_3 \dots \dots \dots \\ \varphi_2, (b_1, \lambda) \end{matrix} \right] + I_{[P_i+n,Q_i+n:R]}^{O,N+n} \left[ (T) \middle| \begin{matrix} \varphi_3 \dots \dots \dots \\ \varphi_2, (b_1, \lambda) \dots \dots \end{matrix} \right] \quad (5.2)$$

The condition (5.1) and (5.2) given in (4.1) and (4.2) are satisfied.

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