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A STUDY OF FRACTIONAL DIFFERENTIAL OPERATOR INVOLVING THE MULTIVARIABLE I-FUNCTION WITH APPLICATIONS AND OTHER DISCIPLINES

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ABSTRACT

*In the present paper we use a fractional differential operator **D** to study the Multivariable I- function with application and other disciplines.*

In this paper we establish two Multiplication formulas for the Multivariable I-function. We determine some new and known results for the above function, a number of known results for other simple function follows as special case of our results.

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1. INTRODUCTION:

The Multivariable I-function is defined by Prasad by Prasad [5] and Prasad [6] and further studied by Sasena [9], Ronghe [7,8] are given extension, summation certain properties etc.

The multivariable I-function defined in a following manner.

$I_R\ [Z_1, Z_2, \ldots\ldots, Z_n] = O.\ N\ \ M_1,\ N_1 \ldots\ldots M_r,\ N_r$

$= I\,[P_i, Q_i: R]\,[P_i, Q_i : R] \ldots\ldots [P_i^{(r)}, Q_i^{(r)} : R^{(r)}]$

$$\left[\begin{array}{l} Z_1\ [(a_j : \alpha_j, \ldots\ldots \alpha_j^{(r)})_{1,N}\ [(a_{ji} : \alpha_{ji} \ldots\ldots \alpha_{ji}^{(r)})_{N+1, P_i}];[(b_{ji} : \beta_{ji}, \ldots\ldots\beta_{ji}^{(r)})_{1,Q}] :[(a_j, \xi_j)_{1,M}\,[(\alpha_{ji}, \xi_{ji})\ M_{i+1}, Q_1] \\ .Z_r\ [(c', r'_{ji})_{1.N2}]\ [9c'_{ji}, \delta'_{ji})_{N1+1}, p_1]\ [c_{ji}^{(r)}(r), \delta_{ji}^{(r)}(r)]_{N+1, Pir}\,[(d_j^{(r)}, \delta_j^{(r)})_{1,Mr}], [d_{ji}^{(r)}(r), \delta_{ji}^{(r)}]_{Mr+1, Qir} \end{array}\right]$$

$$= \frac{1}{(2\pi w)^r} \int_{L1} \ldots\ldots \int_{Lr} \phi(\xi_i) \ldots\ldots \phi_r(\xi_r)\, \psi(\xi_i \ldots\ldots \xi_r)\, z_1^{\xi 1} \ldots\ldots z_r^{\xi r}\, d\xi_i \ldots\, d\xi_r \tag{1.1}$$

where, $\omega = \sqrt{-1}$ and $(0,N) = 0, n_2), (0, n_3) \ldots\ldots (0, n_r)$

$$\Phi_k(\xi_k) = \frac{\prod_{j=1}^{M_k} \Gamma(s_1) \prod_{j=1}^{N_k} \Gamma(s_2)}{\left(\sum_{j=1}^{R_k^{(k)}} \prod_{j=M_{k+1}}^{Q_1^{(k)}} \Gamma(s_3) \prod_{j=N_{k+1}}^{P_1^{(k)}} \Gamma(s_4)\right)} \tag{1.2}$$

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$$\psi_K(\xi_1,\ldots\ldots\ldots\xi_r)=\frac{\prod_{j=1}^{N}\Gamma(S_5)}{\left(\sum_{i=1}^{r}\prod_{i=N+1}^{P_1}\Gamma(S_6)\prod_{i=N_1}^{Q_1}\Gamma(S_7)\right)} \tag{1.3}$$

Where S_1, S_2, S_3, ...S_7 are set of parameters and short notation are given below and will be used in throughout the present paper.

(i) $S_1=(d_j^{(K)}-\delta d_j^{(K)}\xi_K)$

(ii) $S_2=(1-c_j^{(K)}+\delta d_j^{(K)}\xi_K)$

(iii) $S_3=(1-d_{j_i}^{(K)}(K)-\delta_{j_i}^{(K)}(K)\xi_K)$

(iv) $S_4=(c_{j_i}^{(K)}(K)-\delta_{j_i}^{(r)}(K)\xi_K)$

(v) $S_5=(1-a_j+\sum_{K-1}^{r}(\alpha_j^{(K)}\xi_K)$

(vi) $S_6=(a_{j_i}-\sum_{K=1}^{r}(\alpha_{j_i}^{(k)}\xi_K)$

(vii) $S_7=(1-b_{j_i}+\sum_{K=1}^{r}\beta_{j_i}^{(K)}\xi_K)$

And short notation

$$\lambda^*=\sum_{i=1}^{r}\lambda_i\,;\mu^*=\sum_{i=1}^{r}\mu_i\xi_i;\xi_1=\Omega(K_1.......K_r),\prod_{j=1}^{r}\frac{X^r}{K_i\,!}$$

$$\xi_2=\prod_{j=1}^{r}\frac{(-N_j^*)M_j^*,K_j^*}{K_j^*\,!}A[N_1^*,K_1^*..........N_r^*,K_r^*]$$

$$\xi_3=\prod_{j=1}^{r}\frac{(P_iK_i)_{\lambda_i}}{\lambda_i}[a_ix]^{\lambda_i}:\xi_4=\prod_{j=1}^{r-1}\frac{(P_iK_i)_{\lambda_i}[a_jx]^{\lambda_i}}{\gamma_i\,!}\frac{[a_rx]^{\lambda_i}}{\gamma_r\,!}$$

$$x_j=\frac{x_i}{(1-a_jx)^{pi}}:Y_j^*=y_j\xi^{\sigma_j}x^{\sigma_j},\forall j\varepsilon\{1,........r\} \tag{1.4}$$

$$\sum_{K_i=0}^{\infty}\equiv\sum_{K_i:.........:K_r=0}^{\infty}:\sum_{\lambda_i=0}^{\infty}\equiv\sum_{\gamma_i............\lambda_r=0}^{\infty}$$

and

$$\sum_{K_i^*=0}^{[N_i^*/M_i^*]}=\sum_{K_1^*=0}^{[N_1^*/M_1^*]}=........=\sum_{K_r^*=0}^{[N_r^*/M_r^*]},i=1,2,.....,r.$$

For further details and asymptotic expansions of the above function (1.1) refer to [5] and [6].

2. PRELIMINATIES:

We use the fractional derivative operator's defined in the following manner.

$$D_K^{\alpha}(ax+b)^{\mu-1}=\frac{a^{\mu}\Gamma(\mu)(ax+b)^{\mu-\alpha-1}}{\Gamma(\mu-\alpha)}, \tag{2.1}$$

where $\alpha\neq\mu$

$$D_{K,\alpha,x}(ax+b)^{\mu}=\frac{\Gamma(1+\mu)a^{\alpha}(ax+b)^{\mu+K}}{\Gamma(\mu-\alpha+1)} \tag{2.2}$$

where $\alpha \neq \mu$

$$D^{\alpha}_{K,\alpha,x}(ax+b)^{\mu}=[a]^{\frac{n\mu+n(n-1)K}{2}}\prod_{w=0}^{n-1}\frac{\Gamma(1+\mu+wk)}{\Gamma(1+\mu+wk-\alpha)}x(ax+b)^{\mu+nK} \tag{2.3}$$

In (2.3) by taking $K=\alpha$

$$D^{n}_{\alpha,\infty,x}(ax+b)^{\mu}=[a]^{\frac{n\mu+n(n-1)K}{2}}\frac{\Gamma(1+\mu+w\alpha)}{\Gamma(\mu-\alpha+1)}(ax+b)^{\mu+nK} \tag{2.4}$$

Taking a = 1, b = 0 in (2,1) to (2,4) we get the results due to Oldhem and Spnier [4,P.49] We also assume throughout this paper that,

$$\varphi_1 \equiv (-\mu-\omega k+\lambda_1,......,\lambda_r)_{\omega=0,n-1}$$
$$\varphi_2 \equiv (-\mu-rk+\lambda_1,......,\lambda_r)_{\omega=0,n-1}$$
$$\varphi_3 \equiv (-\mu-\omega k+\lambda_1,......,\lambda_r)_{\omega=0,n-1} and (ax+b)=T$$

3. FRACTIONAL DERIVATIVES OF MULTIVARIABLE I-FUNCTION:

In this section we obtain an interesting result with the help of fractional derivative operator indicated in the preceding section

$$D^{'n}_{k,\alpha,x}(T)^{\mu}\,\mathrm{I}[T_I^{\lambda_1},T_2^{\lambda_2},..........T_n^{\lambda_r}]$$

$$=(T)^{\mu+nk+\lambda}[a]\frac{n\mu+n(n-1)k+\lambda}{2}\times I^{O,N+n:M_I,N_I........M_r,N_r}_{[P_i+n,Q_i+n:R]:[P_i,Q_i:R],.....,[P_i^{(r)},Q_i^{(r)}:R^{(r)}}\left[\begin{matrix} a^{\lambda_1}(T)^{\lambda_1} \\ M \\ a^{\lambda r}(T)^{\lambda r}\end{matrix}\middle| \begin{matrix}\varphi_1,(1-S_5),(S_6)...... \\ \\ (1-S_7),\varphi_2\end{matrix}\right]$$

Provided that μ and λ_1, λ_2 .... λ_r are positive integers and

$$R_e(\mu)+\sum_{k=1}^{r}\lambda_k \min\left\langle R_e\left(d_j^{(k)}/\delta_j^k\right)\right\rangle > -1 \tag{3.1}$$

Proof: To establish (3.1) replacing the Multivariable I-function in the derivative as Melline-Barnes type contour integral (1, 1), and changing the order of integration with the help of differential operator (2.2) we get.

$$=\frac{1}{(2\tau\omega)^r}\int_{L_i}......\int_{L_i}\varphi_1(\xi_1),.......\varphi_r(\xi_r)\psi(S_1.....S_r)\times D^{'n+\lambda}_{k,\alpha,x}\left\langle(ax+b)^{\mu+\lambda_1 S_1+......+\lambda_r S}\right\rangle dS_1.....dS_r$$

Now evaluating inner differential operator with the help of (2.3) and interpreting the result thus obtained with the help of (1.1) we get r.h.s. of (3.1).

PARTICULAR CASES OF 3.1: the multivariable I-function, being in generalized form, contains a large number of special functions as its particular cases.

(a) In (3.1) putting a = 1, b = 0, we get

$$D^{n+\lambda}_{k,\alpha,x}(X)^{\mu}I\left[x^{\lambda_1}.x^{\lambda 2}.....x^{\lambda r}\right]=I^{O,N+n:[M_1,N_1:......M_r,N_r}_{[P_i+n,Q_i+n:R]:[P_i,Q_i:R].....[P_i^{(r)},Q_i^{(r)}:R^{(r)}]}\left[\begin{matrix}X^{\lambda_1} \\ M \\ X^{\lambda r}\end{matrix}\middle|\begin{matrix}\varphi_1,(1-S_5),(S_6)....... \\ \\ (1-S_7),\varphi_2\end{matrix}\right] \tag{3.2}$$

$$D_{k,\alpha,x}^{n+\lambda}(T)^{\mu} H_{P_i+n,Q_i+n:P,Q_iP_i^{(r)},Q_i^{(r)}}^{O,N+n:M_1,N_1:\ldots\ldots M_r,N_r}\left[T^{\lambda_1}\ldots\ldots T^{\lambda_r}\right]$$

$$=T^{\mu+nk+\lambda}[a]\frac{n\mu+n(n-1)k+\lambda}{2}\times H_{P_i+n,Q_i+n:P,Q_iP_i^{(r)},Q_i^{(r)}}^{O,N+n:M_1,N_1:\ldots\ldots M_r,N_r}\left[\begin{matrix} a^{\lambda_1}(T)^{\lambda_1} \\ a^{\lambda_r}(T)^{\lambda_r}\end{matrix}\middle|\begin{matrix}\varphi_1,(1-S_5),(S_6) \\ (1-S_7),\varphi_2\end{matrix}\right] \quad (3.3)$$

(b) In (3.1) if $R^{(r)} = 1$, r=1 and i = 1. If reduce to Multivariable H – function.

The condition of validity of (3.2) and (3.3) are directly obtainable form (3.1)

4. MULTIPLICAION FORMULAS

In this section we establish two multiplication formulae involving I- function of Multivariable

$$\prod_{w=0}^{n-1}(\mu+wk+a_1-1)I_{[P_i+n,Q_i=n:R]:[P_i,Q_i:R]\ldots\ldots[P_i^r,Q_i^r:R^r]}^{O,N+n:M_1,N_1:\ldots\ldots\ldots M_r,N_r}\left[(T)^{\lambda_1}\ldots.(T)^{\lambda_r}\middle|\begin{matrix}\varphi_1(a_1:\lambda_1+\lambda_2+\ldots.+\lambda_r)(1-S_5)(S_6)\\(1-S_7),\varphi_2\end{matrix}\right]$$

$$=I_{[P_i+n,Q_i+n:R]:[P_i,Q_i:R]\ldots..[P_i^{(r)},Q_i^{(r)}:R^{(r)}]}^{O,N=n:M_1,N_1\ldots\ldots.M_r,N_r}\left[(T)\middle|\begin{matrix}\varphi_1(a_1:\lambda_1+\ldots.+\lambda_r)(1-S_5)(S_6)\\(-S_7),\varphi_2\end{matrix}\right]$$
$$-I_{[P_i+n,Q_i+n:R]:[P_i,Q_i:R]\ldots..[P_i^{(r)},Q_i^{(r)}:R^{(r)}]}^{O,N+n:M_1,N_1:\ldots..M_r,N_r}\left[(T)\middle|\begin{matrix}\varphi_3(a_1:\lambda_1+\ldots.+\lambda_r)(1-S_5)(S_6)\\(-S_7),\varphi_2\end{matrix}\right] \quad (4.1)$$

$$\prod_{w=0}^{n-1}(\mu+wk+b_1)I_{[P_i+n,Q_i+n:R]:[P_i,Q_i:R]\ldots..[P_i^{(r)},Q_i^{(r)}:R^{(r)}]}^{O,N+n:M_1,N_1:\ldots..M_r,N_r}\left[(T)\middle|\begin{matrix}\varphi_3(1-S_5)(S_6)\\\varphi_2,(b_1:\lambda_1+\ldots.+\lambda_r)\end{matrix}\right]$$

$$=I_{[P_i+n,Q_i+n:R]:[P_i,Q_i:R]\ldots..[P_i^{(r)},Q_i^{(r)}:R^{(r)}]}^{O,N+n:M_1,N_1:\ldots..M_r,N_r}\left[(T)\middle|\begin{matrix}\varphi_3(1-S_5)(S_6)\\\varphi_2,(b_1=1:\lambda_1+\ldots.+\lambda_r)(1-S_7)\end{matrix}\right]$$
$$+I_{[P_i+n,Q_i+n:R]:[P_i,Q_i:R]}^{O,N+n:M_1,N_1:\ldots..M_r,N_r}\left[(T)\middle|\begin{matrix}\varphi_3(1-S_5)(S_6)\\\varphi_2,(b_1,\lambda_1+\ldots.+\lambda_r)(1-S_7)\end{matrix}\right] \quad (4.2)$$

To prove (4.1) and (4.2) we use the definition (1.3),(3.1) and used of Gamma function i.e. Γ (Z+1) = Γ(Z).Z.

5. APPLICATION:

in (4.1) putting and r=1, it reduce in term of the I-function of one variable [7, P.98]

$$\prod_{w=0}^{n-1}(\mu+wk+d_1-1)I_{[P_i+n,Q_i+n:R]}^{O,N+n}\left[(T)\middle|\begin{matrix}\varphi_1(a_1:\lambda)\ldots\ldots\ldots\ldots\\\varphi_2,\ldots\ldots\ldots\ldots\ldots\end{matrix}\right]$$

$$=I_{[P_i+n,Q_i+n:R]}^{O,N+n}\left[(T)\middle|\begin{matrix}\varphi_3,(a_1:\lambda)\ldots\ldots\ldots\ldots\\\varphi_2,\ldots\ldots\ldots\ldots\ldots\end{matrix}\right]-I_{[P_i+n,Q_i+n:R]}^{O,N+n}\left[(T)\middle|\begin{matrix}\varphi_3(a_1-1,\lambda),\ldots\ldots\\\varphi_2,\ldots\ldots\ldots\ldots\ldots\end{matrix}\right] \quad (5.1)$$

$$\prod_{w=0}^{n-1}(\mu+wk+b_1)I_{[P_i+n,Q_i+n:R]}^{O,N+n}\left[(T)\middle|\begin{matrix}\varphi_3,\ldots\ldots\ldots\\\varphi_2,(b_1,\lambda)\ldots..\end{matrix}\right]I_{[P_i+n,Q_i+n:R]}^{O,N+n}\left[(T)\middle|\begin{matrix}\varphi_3,\ldots\ldots\ldots\\\varphi_2,(b_1,\lambda)\end{matrix}\right]+I_{[P_i+n,Q_i+n:R]}^{O,N+n}\left[(T)\middle|\begin{matrix}\varphi_3,\ldots\ldots\ldots\\\varphi_2,(b_1,\lambda)\ldots..\end{matrix}\right] \quad (5.2)$$

The condition (5.1) and (5.2) given in (4.1) and (4.2) are satisfied.

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