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HALL EFFECT ON THERMAL INSTABILITY OF COMPRESSIBLE COUPLE-STRESS FLUID IN PRESENCE OF SUSPENDED PARTICLES

V. Singh* & Shaily Dixit**

****Department of Applied Science, Moradabad Institute of Technology, Moradabad (U.P.) India***
*****Department of Mathematics, Hindu College, Moradabad (U.P.) India***
*****E–mail: shailydixit15@gmail.com***

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ABSTRACT

Effect of Hall current on the the thermal instability of a compressible couple stress fluid with suspended particles is considered in this paper. It has been found that suspended particles have destabilizing effect on compressed couple stress fluid. Also dispersion relations including the effect of Hall Current and compressibility on the thermal instability of couple stress fluid in the presence of suspended particles are analyzed. Stability of the system and oscillatory modes are considered and it is found that due to the presence of magnetic field and suspended particles oscillatory modes are produced which were non–existent in their absence.

1. INTRODUCTION

The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in geophysics, interior of the Earth, Oceanography and atmospheric physics etc. A detailed account of the theoretical and experimental study of the onset of Benard convection in Newtonian Fluids, under varying assumptions of hydrodynamics, has been given by Chandrasekhar [1]. The use of Boussinesq approximation has been made through out which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. Sharma [5] has considered the effect of rotation and magnetic field on the thermal instability in compressible fluids. The fluid has been considered to be Newtonian in all the above studies while Scanlon and Segel [6] have considered the effect of suspended particles on the onset of Benard convection and found that the critical Rayleigh number was reduced solely because of the heat capacity of the pure fluid.

With the growing importance of non-Newtonian fluids in modern technologies and industries, the investigations on such fluids are desirable.

Stokes [7] proposed and postulated the theory of couple-stress fluids. One of the applications of the couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded – bearing synovial joints of the human body and these joints have a low-friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is viscous, non-Newtonian fluid. According to the theory of Stokes [7], couple-stresses are found to appear in noticeable magnitude in fluids with very large molecules. Since the long chain of hylauronic acid molecules are found as additives in synovial fluid, Walicki and Walicka [17] modeled synovial fluid as a couple-stress fluid in human joints. Goel et al. [2] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture having vertical temperature and concentration gradients with rotation. Sharma and Sharma [8] have studied the couple – stress fluid heated from below in porous medium. An electrically conducting couple stress fluid heated from below in porous medium in the presence of uniform horizontal magnetic field has also been studied by Sharma and Sharma [9]. The use of magnetic field is being made for the clinical purposes in detection and cure of certain diseases with the help of magnetic field devices / instruments. Sharma and Thakur [10] have studied the thermal convection in couple stress fluid in porous medium in hydromagnetics.

Environmental pollution is the main cause of dust to enter into human body. The metal dust which filters into the blood stream of those working near furnace causes extensive damage to the chromozomes and genetic mutations so observed are likely to breed cancer as malformations in the coming progeny. Therefore, it is very essential to study the blood flow with

Corresponding author: Shaily Dixit**, *E-mail: shailydixit15@gmail.com

dust particles. Considering blood as couple-stress fluid and dust particles as micro-organisms, Rathod and Thippeswamy [4] have studied the gravity flow of pulsatile blood through closed rectangular inclined channel with micro-organisms.

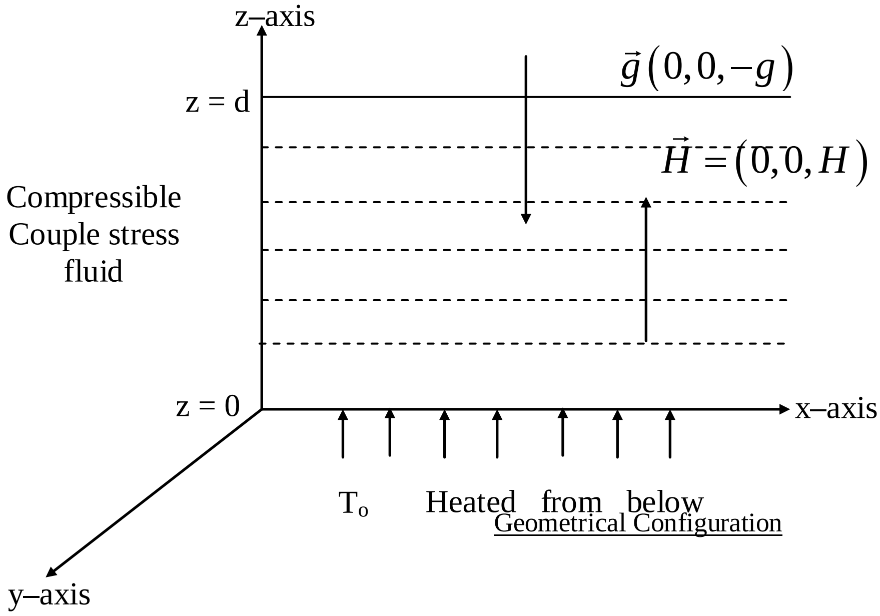
If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current to flow across an electric field in the presence of magnetic field is called Hall Effect. The Hall Current is likely to be important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. Sherman and Sutton [11] have considered the effect of Hall currents on the efficiency of a magnetohydro-fluid dynamic (MHD) generator while Gupta [3] studied the effect of Hall currents on the thermal instability of electrically conducting fluid in the presence of uniform vertical magnetic field. Sharma and Gupta [12] investigated the effect of Hall currents on thermosolutal instability of a rotating plasma and established the destabilizing influence of Hall currents. For compressible fluids, the equations governing the system become quite complicated. Spiegel and Veronis [13] have simplified the set of equations governing the flow of compressible fluids assuming that the depth of the fluid layer is much smaller than the scale height as defined by them and the motions of infinitesimal amplitude are considered. Sharma [14] investigated the thermal instability of compressible fluids in the presence of rotation and magnetic field while Sharma and Gupta [15] studied the effect of finite Larmor radius on thermal instability of compressible rotating plasma. Thermal instability of compressible finite Larmar radius Hall plasma has been studied by Sharma and Sunil [16] in a porous medium. Sharma and Sharma have studied the Effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field.

Keeping in mind the importance in paper industry, petroleum industry, geophysics and bio-mechanics and various applications mentioned above, Hall effect on the thermal instability of a compressible couple-stress fluid with suspended particles has been considered in the present paper.]

2. FORMULATION OF THE PROBLEM

Consider an infinite, horizontal, compressible couple-stress fluid layer of thickness d, heated from below so that, the temperature and density at the bottom surface $z = 0$ are T_0, ρ_o respectively and at the upper surface $z = d$ are T_d , ρ_d and that a uniform adverse temperature gradient $\beta = \left|\frac{dT}{dZ}\right|$ is maintained. Let ρ, P, T and $\vec{v}(u, v, w)$ denote respectively the density, pressure, temperature and velocity of the fluid, $\vec{v}_d(\overline{x}, t)$ and $N(\overline{x}, t)$ denote the velocity and number density of suspended particles respectively. μ_e is the magnetic permeability, $\overline{x} = (x, y, z)$.

The layer is acted upon by the gravity force $\vec{g}(0, 0, -g)$ and uniform vertical magnetic field $\vec{H} = (0, 0, H)$.



Geometrical Configuration

Then the momentum balance, mass balance equations of the couple – stress fluid in presence of suspended particles and Hall currents are

$$\frac{\partial \vec{v}}{\partial t}+(\vec{v}.\nabla)\vec{v}=-\frac{\nabla P}{\rho}+\vec{g}\left(1+\frac{\delta\rho}{\rho}\right)+\left(\nu-\frac{\mu'}{\rho}\nabla^2\right)\nabla^2\vec{v}+\frac{KN}{\rho}(\vec{v}_d-\vec{v})+\frac{\mu_e}{4\pi\rho}\left(\nabla\times\vec{H}\right)\times\vec{H} \tag{1}$$

$$\nabla.\vec{v}=0 \tag{2}$$

Where $K = 6\,\pi\,\mu\,\eta'$, η' being particle radius, is the Stokes' drag coefficient. Assuming uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term, in the equation of motion (1) proportional to the velocity difference between particles and fluid.

The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. Inter particle reactions are ignored for we assume that the distances between the particles are large compared with their diameters. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are:

$$mN\left[\frac{\partial \vec{v}_d}{\partial t}+(\vec{v}_d.\nabla)\vec{v}_d\right]=KN[\vec{v}-\vec{v}_d] \tag{3}$$

$$\frac{\partial N}{\partial t}+\nabla.(N\vec{v}_d)=0 \tag{4}$$

Let C_v, C_{pt} denote the heat capacity of the fluid at constant volume and the heat capacity of the particles. Assuming that the particles and fluid are in thermal equilibrium, the equation of heat conduction gives.

$$\frac{\partial T}{\partial t}+(\vec{v}.\nabla)T+\left(\frac{mNC_{pt}}{\rho C_v}\right)\left(\frac{\partial}{\partial t}+\vec{v}_d.\nabla\right)T=\kappa\nabla^2 T \tag{5}$$

The kinematic viscosity ν, couple – stress viscosity μ', thermal diffusivity κ and coefficient of thermal expansion α are all assumed to be constants. The Maxwell's equations in the presence of Hall currents yield.

$$\frac{\partial \vec{H}}{\partial t}=\nabla\times(\vec{v}\times\vec{H})+\eta\nabla^2\vec{H}-\frac{1}{4\pi N'e}\nabla\times[(\nabla\times\vec{H})\times\vec{H}] \tag{6}$$

$$\nabla.\vec{H}=0 \tag{7}$$

where η, N' and e denote, respectively, the resistivity, the electron number density and the charge of an electron.

Spiegel and Veronis (1960) defined f as any of the state variables (pressure (P), density (ρ) or temperature (T)) and expressed these in the form

$$f\,(x, y, z, t) = f_m + f_o\,(z) + f'\,(x, y, z, t) \tag{8a}$$

Where f_m is the constant space average of f, f_o is the variation in the absence of motion and f' is the fluctuation resulting from the motion.

The initial state is, therefore, a state in which the density, pressure, temperature and velocity in the fluid are given by

$$\rho=\rho(z), P=P(z), T=T(z), \vec{v}=(0,0,0) \tag{8b}$$

$\vec{v}_d=(0,0,0)$, $\vec{H}=(0,0,\vec{H})$, $N=N_0$ a constant (8b)

where following Spiegel and Veronis [13], we have

$$P(z)=P_m-g\int_0^z(\rho_m+\rho_0)\,dz$$

$$\rho(z)=\rho_m\left[1-\alpha_m\left(T-T_m\right)+K_m\left(\rho-\rho_m\right)\right]$$
$$T(z)=-\beta z+T_0,$$
$$\alpha_m=-\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)_m \quad (=\alpha,\ \text{say}), \tag{8c}$$
$$k_m=\left(\frac{1}{\rho}\frac{\partial\rho}{\partial P}\right)_m,$$

where P_m and ρ_m stand for a constant space distribution of P and ρ while ρ_o and T_o stand for density and temperature of the fluid at the lower boundary. Following the assumptions given by Spiegel and Veronis [13] and results for compressible fluids, the flow equations are found to be the same as those of incompressible fluids except that the static temperature gradient β is replaced by its excess over the adiabatic $(\beta - g / c_p)$, c_p being specific heat of the fluid at constant pressure.

3. PERTURBATION EQUATIONS

Assume small perturbations around the basic solution and let δP, $\delta\rho$, N, θ, $\vec{v}$ $(u, \vec{v}, w)$, $\vec{v}_d$ (l, r, s), $\vec{h}\left(h_x, h_y, h_z\right)$ denote the perturbations in fluid pressure, density, particle number density N_0, temperature, couple stress fluid velocity, particle velocity, magnetic field $\vec{H}$ respectively. The equation of state is

$$\rho=\rho_m\left[1-\alpha\left(T-T_0\right)\right] \tag{9}$$

where α is the coefficient of thermal expansions as density variations arise mainly due to temperature variations. Thereafter the change in density $\delta\rho$ caused mainly by the perturbation θ in temperature is given by

$$\partial\rho=-\alpha\rho_m\theta \tag{10}$$

Then the linearized hydromagnetic perturbation equations for thermal convection in a compressible couple stress fluid under speigel and Veronis assumptions are

$$\frac{\partial\vec{v}}{\partial t}=-\frac{1}{\rho_m}\nabla\delta P+\left(\nu-\frac{\mu'}{\rho_m}\nabla^2\right)\nabla^2\vec{v}+\vec{g}\frac{\delta\rho}{\rho_m}+\frac{KN_0}{\rho_m}\left(\vec{v}_d-\vec{v}\right)+\frac{\mu_e}{4\pi\rho_m}[(\nabla\times\vec{h})\times\vec{H}] \tag{11}$$

$$\nabla.\vec{v}=0 \tag{12}$$

$$mN_0\frac{\partial}{\partial t}\vec{v}_d=KN_0(\vec{v}-\vec{v}_d) \tag{13}$$

$$\left(1+h\right)\frac{\partial\theta}{\partial t}=\left(\beta-\frac{g}{C_p}\right)\left(w+hs\right)+\kappa\nabla^2\theta \tag{14}$$

Where $\kappa=\dfrac{\nu}{\rho_m C_v}$ and $h=\dfrac{mN_0C_{pt}}{\rho_m C_v}$

and also $\alpha_m = \dfrac{1}{T_m} = \alpha$, say $\nu=\dfrac{\mu}{\rho_m}$ and g/C_p, ν, κ stand for the adiabatic gradient, kinematic viscosity and thermal diffusivity respectively.

$$\nabla.\vec{h}=0 \tag{15}$$

$$\frac{\partial \vec{h}}{\partial t}=\left(\vec{H}.\nabla\right)\vec{v}+\eta\nabla^2 h-\frac{1}{4\pi N'e}\nabla\times\left[\left(\nabla\times\vec{h}\right)\times\vec{H}\right] \tag{16}$$

Eliminating $\vec{V}_d$ between equations (11) – (13),

$$\left[1+\frac{mN_o/\rho_m}{1+\tau\frac{\partial}{\partial t}}\right]\frac{\partial}{\partial t}\vec{v}=-\frac{1}{\rho_m}\nabla\delta P+(\nu-\frac{\mu'}{\rho_m}\nabla^2)\nabla^2\vec{v}+\vec{g}\frac{\delta\rho}{\rho_m}+\frac{\mu_e}{4\pi\rho_m}\left[\left(\nabla\times h\right)\times H\right] \tag{17}$$

Where $\tau=\dfrac{m}{K}$

Writing the scalar component of equation (17)

$$\left[1+\frac{mN_o/\rho_m}{1+\tau\frac{\partial}{\partial t}}\right]\frac{\partial u}{\partial t}=-\frac{1}{\rho_m}\left(\frac{\partial}{\partial x}\delta P\right)+\left(\nu-\frac{\mu'}{\rho_m}\nabla^2\right)\nabla^2 u-\frac{\mu_e}{4\pi\rho_m}H\left(\frac{\partial h_z}{\partial x}-\frac{\partial h_x}{\partial z}\right) \tag{18}$$

$$\left[1+\frac{mN_o/\rho_m}{1+\tau\frac{\partial}{\partial t}}\right]\frac{\partial v}{\partial t}=-\frac{1}{\rho_m}\left(\frac{\partial}{\partial y}\delta P\right)+\left(\nu-\frac{\mu'}{\rho_m}\nabla^2\right)\nabla^2 v-\frac{\mu_e}{4\pi\rho_m}H\left(\frac{\partial h_z}{\partial y}-\frac{\partial h_y}{\partial z}\right) \tag{19}$$

$$\left[1+\frac{mN_o/\rho_m}{1+\tau\frac{\partial}{\partial t}}\right]\frac{\partial w}{\partial t}=-\frac{1}{\rho_m}\left(\frac{\partial}{\partial z}\delta P\right)+g\alpha\theta+\left(\nu-\frac{\mu'}{\rho_m}\nabla^2\right)\nabla^2 w \tag{20}$$

using equation (12) and on solving (18), (19) and (20) we get

$$\left[1+\frac{mN_o/\rho_m}{1+\tau\frac{\partial}{\partial t}}\right]\frac{\partial}{\partial t}\nabla^2 w=\left(\nu-\frac{\mu'}{\rho_m}\nabla^2\right)\nabla^4 w+g\alpha\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\theta+\frac{\mu_e H}{4\pi\rho_m}\nabla^2\frac{\partial h_z}{\partial z} \tag{21}$$

$$\left[1+\frac{mN_o/\rho_m}{1+\tau\frac{\partial}{\partial t}}\right]\frac{\partial\zeta}{\partial t}=\left(\nu-\frac{\mu'}{\rho_m}\nabla^2\right)\nabla^2\zeta+\frac{\mu_e H}{4\pi\rho_m}\frac{\partial\xi}{\partial z} \tag{22}$$

Equation (14) and (16) can be written as

$$\left[H_d\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta=\left(\beta-\frac{g}{C_p}\right)\left(\frac{H_d+\tau\frac{\partial}{\partial t}}{1+\tau\frac{\partial}{\partial t}}\right)w \tag{23}$$

Where $1 + h = H_d$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z=H\frac{\partial w}{\partial z}-\frac{H}{4\pi N'e}\frac{\partial\xi}{\partial z} \tag{24}$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)\xi=H\frac{\partial\zeta}{\partial z}+\frac{H}{4\pi N'e}\frac{\partial}{\partial z}\left(\nabla^2 h_z\right) \tag{25}$$

where $\zeta=\dfrac{\partial v}{\partial x}-\dfrac{\partial u}{\partial y}$ is the z component of vorticity

$\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is the z – component of current density.

4. NORMAL MODE ANALYSIS METHOD AND DISPERSION RELATION

Analyze the perturbation quantities into normal modes by seeking solutions in the form.

$$[w, \theta, h_z\ \xi, \zeta] = [\, w(z), \Theta(z), K(z), X(z), Z(z)] \exp(ik_x x + ik_y y + nt) \tag{26}$$

where k_x and k_y are the wave numbers along x and y directions and the resultant wave number is given by $k^2 = k_x^2 + k_y^2$ and n is the growth rate.

Using expression (26) equation (21) – (25), in non – dimensional form becomes

$$\left[\sigma\left(1+\frac{f}{1+\tau_1\sigma}\right)+F(D^2-a^2)^2-(D^2-a^2)\right](D^2-a^2)W = -\frac{g\alpha d^2 a^2 \Theta}{\nu}+\frac{\mu_e Hd}{4\pi\rho_m \nu}(D^2-a^2)DK \tag{27}$$

$$\left[\sigma\left(1+\frac{f}{1+\tau_1\sigma}\right)+F(D^2-a^2)^2-(D^2-a^2)\right]Z = \frac{\mu_e Hd}{4\pi\rho_m \nu}DX \tag{28}$$

$$\left(D^2-a^2-H_d P_1\sigma\right)\Theta = -\frac{\beta d^2}{\kappa}\left(\frac{G-1}{G}\right)\frac{\left(H_d+\tau_1\sigma\right)}{\left(1+\tau_1\sigma\right)}W \tag{29}$$

$$\left(D^2-a^2-P_2\sigma\right)K = -\left(\frac{Hd}{\eta}\right)DW+\frac{Hd}{4\pi N'e\eta}DX \tag{30}$$

$$\left(D^2-a^2-P_2\sigma\right)X = -\left(\frac{Hd}{\eta}\right)DZ-\frac{H}{4\pi N'e\eta d}D\left(D^2-a^2\right)K \tag{31}$$

Where we have non – dimensionalized various parameters as follows

$a = kd$, $\sigma = \frac{nd^2}{\nu}, \tau_1 = \frac{\tau\nu}{d^2}, f = \frac{mN_o}{\rho_m}$

$p_1 = \frac{\nu}{k}$, $\quad F = \frac{\mu'}{\rho_m d^{\partial}\nu}$, $\quad p_2 = \frac{\nu}{\eta}$

$D = \frac{d}{dz}$, $1 + h = H_d$, $G = \frac{C_p\beta}{g}$ is the dimensionless compressibility.

After eliminating Θ, Z, X and K from equation (27)-(31) we obtain

$$\left[\sigma\left(1+\frac{f}{(1+\tau_1\sigma)}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W$$
$$-\left(\frac{G-1}{G}\right)Ra^2\frac{\left(H_d+\tau_1\sigma\right)}{(1+\tau_1\sigma)\left(D^2-a^2-H_dP_1\sigma\right)}+\left[\left\{\sigma\left(1+\frac{f}{(1+\tau_1\sigma)}\right)\right.\right.$$
$$\left.\left.\times F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right\}\left(D^2-a^2-P_2\sigma\right)+QD^2\right]$$

$$\left[\left\{\sigma\left(1+\frac{f}{(1+\tau_1\sigma)}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right\}\left\{\left(D^2-a^2-P_2\sigma\right)\right.\right.$$
$$\left.\left.+MD^2\left(D^2-a^2\right)\right\}+D^2\left(D^2-a^2-P_2\sigma\right)Q\right]^{-1}QD^2\left(D^2-a^2\right)W=0 \tag{32}$$

Where

$Q=\dfrac{\mu_e H^2 d^2}{4\pi\rho_m \nu\eta}$ is the Chandrasekhar number

$R=\dfrac{g\alpha\beta d^4}{\nu k}$ is the thermal Rayleigh number,

$M=\left(\dfrac{H}{4\pi N' e\eta}\right)^2$ is the non-dimensional number accounting for Hall currents.

Since both the boundaries are maintained at constant temperature, the perturbation in the temperature are zero at the boundaries. The case of two free boundaries is little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions with respect to which equations (27) to (31) must be solved are $W = 0$, $DZ = 0$, $\Theta = 0$ at $z = 0$ and 1.

$K = 0$ and $DX = 0$, on perfectly conducting boundaries (33)

and h_x, h_y, h_z are continuous. Since the components of magnetic field are continuous and the tangential components are zero outside the fluid, we have

$DK = 0$, on the boundaries (34)

The constitutive equations for the couple – stress fluid are

$$\tau_{ij} = (2\mu - 2\mu'\,\nabla^2)\,e_{ij},$$

$$e_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)$$

The conditions on a free surface are

$$\tau_{xz} = (\mu - \mu'\,\nabla^2)\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)=0,$$
$$\tau_{yz} = (\mu - \mu'\,\nabla^2)\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=0 \tag{35}$$

From the equation of continuity (12) differentiate with respect to z, we conclude that

$$\left[\mu-\mu'\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\right]\frac{\partial^2 w}{\partial z^2}=0 \tag{36}$$

which implies that

$$\frac{\partial^2 w}{\partial z^2}=0,\ \frac{\partial^4 w}{\partial z^4}=0,\ \text{at}\ \ z=0\ \text{and}\ \ d \tag{37}$$

Using equation (26), the boundary conditions (37) in non– dimensional form transform to

$D^2W = D^4W = 0$ at $z = 0$ and 1 (38)

Using boundary conditions (33), (34) and (38), we can see that all the even order derivatives of W must vanish for $z = 0$ and 1. Therefore the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{39}$$

where W_0 is a constant.

substituting the proper solution $W = W_0 \sin\pi z$ in equation (32) and letting $R_1 = \frac{R}{\pi^4}$

$$Q_1 = \frac{Q}{\pi^2}, \quad x = \frac{a^2}{\pi^2}, \quad i\sigma_1 = \frac{\sigma}{\pi^2}, \quad F_1 = \pi^2 F$$

We obtain the dispersion relation

$$\begin{aligned} R_1 = &\left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)\left(1+x+iH_d\sigma_1 P_1\right) \\ &\left(\left\{i\sigma_1\left(1+\frac{f}{1+i\sigma_1\tau_1\pi^2}\right)+F_1(1+x)^2+(1+x)\right\}+Q_1\right. \\ &\left[\left\{i\sigma_1\left(1+\frac{f}{1+i\sigma_1\tau_1\pi^2}\right)+F_1(1+x)^2+(1+x)\right\}\left(1+x+i\sigma_1 P_2\right)+Q_1\right] \\ &\left(\left\{i\sigma_1\left(1+\frac{f}{1+i\sigma_1\tau_1\pi^2}\right)+F_1(1+x)^2+(1+x)\right\}\left\{(1+x+i\sigma_1 P_2)^2+M(1+x)\right\}\right. \\ &\left.\left.+Q_1\left(1+x+i\sigma_1 P_2\right)\right)^{-1}\right)\times\frac{(1+i\sigma_1\tau_1\pi^2)}{(H_d+i\sigma_1\tau_1\pi^2)} \end{aligned} \tag{40}$$

Equation (40) is the required dispersion relation including the effects of Hall currents and compressibility on the thermal instability of couple stress fluid in the presence of suspended particles.

5. STATIONARY CONVECTION:

Let us consider the case when instability sets in the form of stationary convection. For stationary convection $\sigma_1 = 0$ and the dispersion relation (40) reduces to

$$R_1 = \left(\frac{G}{G-1}\right)\left[\left\{(1+x)^3 F_1 + (1+x)^2\right\}\frac{Q_1\left\{(1+x)^3 F_1 + (1+x)^2 + Q_1\right\}}{\left[\left\{(1+x)^2 F_1 + (1+x)\right\}\left\{1+x+M\right\}+Q_1\right]}\right] \tag{41}$$

which expresses the modified Rayleigh number R_1 as a function of dimensionless wave number x and the parameters Q_1, M, F_1, G and H_d.

Let the non-dimensional number G accounting for compressibility effect is kept as fixed, then we get

$$\bar{R}_c = \left(\frac{G}{G-1}\right)R_c, \tag{42}$$

Where $\bar{R}_c$ and R_c denote, respectively, the critical Rayleigh numbes in the presence and absence of compressibility. Thus the effect of compressibility is to postpone the onset of thermal instability. The cases $G < 1$ and $G = 1$ correspond to negative and infinite value of Rayleight number which are not relevant in the present study. Hence, compressibility has a stabilizing effect on thermal convection.

To study the effects of magnetic field, Hall currents, couple-stress and suspended particles, we examine the natures of

$\frac{dR_1}{dQ_1}, \frac{dR_1}{dM}, \frac{dR_1}{dF_1}$ and $\frac{dR_1}{dH_d}$ analytically.

From equation (41) we have

$$\frac{dR_1}{dQ_1} = \frac{G}{G-1}\left(\frac{1+x}{xH_d}\right)$$
$$\left[\frac{\left[(1+x)^2 F_1 + (1+x)(1+x+M)\right]\left\{(1+x)^3 F_1 + (1+x)^2 + 2Q_1\right\} + Q_1^2}{\left[\left\{(1+x)^2 F_1 + (1+x)\right\}(1+x+M) + Q_1\right]^2}\right] \tag{43}$$

From equation [43] it follows that magnetic field has a stabilizing effect on thermal convection of a compressible couple stress fluid.

$$\frac{dR_1}{dM} = -\left(\frac{G}{G-1}\right)\left(\frac{(1+x)^2}{xH_d}\right)\{(1+x)F_1+1\}$$
$$\left[\frac{Q_1\{(1+x)^3 F_1 + (1+x)^2\} + Q_1^2}{[\{(1+x)^2 F_1 + (1+x)\}(1+x+M) + Q_1]^2}\right] \tag{44}$$

From equation [44] we see Hall currents have destabilizing effects on thermal convection.

$$\frac{dR_1}{dF_1} = \frac{G}{G-1}\left(\frac{(1+x)^3}{xH_d}\right)\left[(1+x) - \frac{MQ_1^2}{[\{(1+x)^2 F_1 + (1+x)\}(1+x+M) + Q_1]^2}\right] \tag{45}$$

From equation [45] shows that couple stress has both stabilizing and destabilizing effect if

$(1 + x) >$ or $<$ $\frac{MQ_1^2}{[\{(1+x)^2 F_1 + (1+x)\}(1+x+M) + Q_1]^2}$

$$\frac{dR_1}{dH_d} = \frac{-G}{G-1}\left(\frac{1+x}{xH_d^2}\right)\left[\{(1+x)^3 F_1 + (1+x)^2\} + \frac{Q_1\{(1+x)^3 F_1 + (1+x)^2 + Q_1\}}{[\{(1+x)^2 F_1 + (1+x)\}(1+x+M) + Q_1]}\right] \tag{46}$$

Equation [46] follows that suspended particles have destabilizing effect on compressible couple-stress fluid.

Stability of the system and oscillatory modes

To determine the possibility of oscillatory modes we multiply equation (27) by W^*, the complex conjugate of W and using equation (28) – (31) together with the boundary condition (33), (34) and (38) we obtain.

$$\left[\sigma\left(1+\frac{f}{1+\tau_1\sigma}\right)I_1 + FI_1 + I_3 + \frac{\mu_e\eta}{4\pi\rho_m\nu}\left(I_6 + P_2\sigma^* I_7\right) - \frac{g\alpha\kappa a^2}{\nu\beta}\left(\frac{G}{G-1}\right)\right.$$
$$\left(\frac{1+\tau_1\sigma^*}{H_d+\tau_1\sigma^*}\right)\left(I_4 + H_d P_1\sigma^* I_5\right) + \frac{\mu_e\eta d^2}{4\pi\rho_m\nu}\left(I_8 + P_2\sigma I_9\right) + d^2$$
$$\left.\sigma^*\left(\frac{f}{1+\tau_1\sigma^*}\right)I_{10} + FI_{11} + I_{12}\right] = 0 \tag{47}$$

Where

$$I_1 = \int_0^1 \left(|DW|^2 + a^2 |W|^2 \right) dz$$

$$I_2 = \int_0^1 \left(|D^3W|^2 + 3a^2 |D^2W|^2 + 3a^4 |DW|^2 + a^6 |W|^2 \right) d$$

$$I_3 = \int_0^1 \left(|D^2W|^2 + 2a^2 |DW|^2 + a^4 |W|^2 \right) dz$$

$$I_4 = \int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2 \right) dz$$

$$I_5 = \int_0^1 |\Theta|^2 \, dz$$

$$I_6 = \int_0^1 \left(|D^2K|^2 + 2a^2 |DK|^2 + a^4 |K|^2 \right) d$$

$$I_7 = \int_0^1 \left(|DK|^2 + a^2 |K|^2 \right) dz$$

$$I_8 = \int_0^1 \left(|DX|^2 + a^2 |X|^2 \right) dz$$

$$I_9 = \int_0^1 |X|^2 \, dz$$

$$I_{10} = \int_0^1 |Z|^2 \, dz$$

$$I_{11} = \int_0^1 \left(|D^2Z|^2 + 2a^2 |DZ|^2 + a^4 |Z|^2 \right) dz$$

$$I_{12} = \int_0^1 \left(|DZ|^2 + a^2 |Z|^2 \right) dz$$

Where integrals I_1, I_2 ….. I_{11}, I_{12} are all positive definite. Putting $\sigma = i\sigma_i$ and equating imaginary part of equation (47), we get

$$\sigma_i \left[\left(1 + \frac{f}{1+\tau_1^2\sigma_i^2} \right) I_1 - \frac{\mu_e \eta}{4\pi\rho_m \nu} P_2 I_7 + \frac{g\alpha\kappa a^2}{\nu\beta} \left(\frac{G}{G-1} \right) \right.$$
$$\left. \times \left[\frac{\tau_1 (H_d - 1)}{H_d^2 + \tau_1^2 \sigma_i^2} I_4 + \frac{H_d + \tau_1^2 \sigma_i^2}{H_d^2 + \tau_1^2 \sigma_i^2} H_d P_1 I_5 \right] + \frac{\mu\eta d^2}{4\pi\rho_m \nu} P_2 I_9 - d^2 \left(1 + \frac{f}{1+\tau_1^2 \sigma_i^2} \right) I_{10} \right] = 0 \qquad (48)$$

From equation (48) σ_i may be zero or non zero, meaning that the modes may be non oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of magnetic field (hence Hall current) and suspended particles, which were non–existence in their absence.

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