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SOME NEW CLASS OF FUNCTIONS VIA $\delta\hat{g}$-SETS

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ABSTRACT

In this paper we introduce a new class of functions called $\delta\hat{g}$ -closed maps. We obtain several characterizations and some their properties. We also investigate its relationship with other types of functions. Further we introduce and study a new class of functions namely weaker forms of $\delta\hat{g}$ -closed maps.

Keywords and Phrases: *$\delta\hat{g}$-closed sets, δg ^-continuous, $\delta\hat{g}$-closed maps, $\delta\hat{g}$-regular, $\alpha\hat{g}$-closed sets.*

AMS subject classification: *54C05, 54C10*

1.INTRODUCTION:

Malghan [7] introduced generalised closed functions and Devi et al.[1] intro- duced αg -closed functions. T. Noiri [9] and Veerakumar[12] introduced δ -closed functions and $\hat{g}$ -closed functions in topological spaces. In this present paper we use $\delta\hat{g}$ -closed sets to define a new class of functions called $\delta\hat{g}$ -closed functions and obtain some properties of these functions. We further introduce and study a new class of functions namely weakly $\delta\hat{g}$ -closed functions and we introduce a new space called $\delta\hat{g}$ -regular space.

2. PRELIMINARIES:

Throughout this paper (X, τ) and, $(Y, \sigma)$and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition: 2.1A subset A of a space (X, τ) is called a

(i) semi-open set [3] if $A \subseteq cl(int(A))$.
(ii) α -open set [8] if $A \subseteq int(cl(int(A)))$.
(iii) regular open set [11] if $A= int(cl(A))$.
(iv) δ-open set [13] if $A= \delta int(A)$.

The complement of a semi-open (resp. α-open, regular open) set is called semi- closed (resp. α -closed, regular closed).

The δ –interior [13] of a subset A of X is the union of all regular open set of X contained in A and is denoted by Int δ (A) . The subset A is called δ -open [13] if A =Int $_{\delta}$ (A) , i.e. a set is δ -open if it is the union of regular open sets. the complement of a δ -open is called δ -closed. Alternatively, a set $A \subseteq (X, \tau)$ is called δ -closed [13] if A= cl_{δ} (A), where $cl_{\delta}(A) = \{ x \in X: int(cl(U)) \cap A = \varphi ,U \in \tau$ and $x \in U \}$.

Definition: 2.2 A subset A of (X, τ) is called
(i) generalized closed (briefly g -closed) set[4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ).
(ii) δ -generalized closed (briefly δg -closed) set[2] if $cl_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ).
(iii) $\hat{g}$ -closed set [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi-open set in (X, τ).
(iv) α - $\hat{g}$ -closed (briefly $\alpha\hat{g}$ -closed) set [6] if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$-open set in (X, τ).
(v) $\delta\hat{g}$ -closed set [5] if $cl_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$-open in (X, τ).

The complement of a g -closed (resp. δg -losed, $\hat{g}$ -closed, $\alpha\hat{g}$ -closed and $\delta\hat{g}$ -closed) set is called g -open (resp. δg - open, $\hat{g}$-open, $\alpha\hat{g}$-open and $\delta\hat{g}$-open).

Definition: 2.3 A function $f: (X, \tau) \to (Y, \sigma)$ is called:

(i) δ -closed [9] if $f(V)$ is δ -closed in (Y, σ) for every δ -closed set V of (X, τ)..
(ii) δ -continuous [10] if $f^{-1}(V)$ is δ -open in (X, τ) for every δ -open set V of (Y, σ).
(iii) $\delta\hat{g}$ -continuous [5] if $f^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ) for every closed set V of (Y, σ).
(iv) $\delta\hat{g}$ -irresolute [5] if $f^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ)

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for every $\delta\hat{g}$ closed set V of (Y,σ).
(v) generalized closed (briefly g -closed) (resp.g -open) [7] if the image of every closed (resp. Open) set in (X, τ) is g-closed (resp. g-open) in (Y,σ).
(vi) $\hat{g}$-open [12] if f(V) is $\hat{g}$-open in (Y, σ) for every open set V of (X,τ).
(vii)$\alpha\hat{g}$-closed [6] if the image of every closed set in (X, τ) is $\alpha\hat{g}$-closed in (Y,σ).

3. $\delta\hat{g}$-CLOSED MAPS:

We introduce the following definitions:

Definition: 3.1 A map $f : (X, \tau) \to (Y, \sigma)$ is called $\delta\hat{g}$-closed (resp. $\delta\hat{g}$-open) if the image of each closed (resp. Open) set in (X, τ) is $\delta\hat{g}$-closed (resp. $\delta\hat{g}$-open) in (Y, σ).

Remark: 3.2 $\delta\hat{g}$ -openness and δg^ -continuity are independent as shown by the following examples.

Example: 3.3 Let X = {a, b, c} = Y; $\tau = \{\varphi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$ and, $\sigma = \{\varphi, \{b\}, Y\}$. Define f: $(X, \tau) \to (Y, \sigma)$ by f(a) = b, f(b) = c and f(c) = a. Then f is $\delta\hat{g}$-continuous but not $\delta\hat{g}$-open, because {b, c} is open in (X, τ) but f({b, c}) = {a, c} is not $\delta\hat{g}$-open in (Y,σ).

Example: 3.4 Let X = {a, b, c} =Y with topologies $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$ and, $\sigma = \{\varphi, \{a\}, Y\}$. Define f:$(X, \tau) \to (Y, \sigma)$ by f(a) = b, f(b) = a and f(c)=c. Then f is $\delta\hat{g}$-open but not $\delta\hat{g}$-continuous, because {b,c} is closed in (Y,σ) bu $f^{-1}(\{b, c\}) = \{a,c\}$ is not δg^-closed in (X, τ).

Remark: 3.5 The composite mapping of two $\delta\hat{g}$ -closed maps is not in $\delta\hat{g}$ -closed maps as shown in following example.

Example: 3.6 Let X ={a, b, c} = Y = Z ; $\tau = \{\varphi, \{a\}, \{c\}, \{a, c\}, X\}$, $\sigma = \{\varphi, \{a\}, Y\}$ and $\eta = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, Z\}$. Define a map f:$(X, \tau) \to (Y, \sigma)$ by f(a) = a, f(b) = c and f(c)= b and let g:$(Y, \sigma) \to (Z, \eta)$ be the identity function. Clearly f and g are δg^ -closed maps. But $g \circ f : (X, \tau) \to (Z, \eta)$ is not an $\delta\hat{g}$-closed map because $(g \circ f)(\{b\}) = \{c\}$ is not an $\delta\hat{g}$-closed set of (Z, η) where {b} is a closed set of (X, τ).

Theorem:3.7 If f:$(X, \tau) \to (Y, \sigma)$ is closed and g:$(Y, \sigma) \to (Z, \eta)$ is $\delta\hat{g}$-closed map then $g \circ f : (X, \tau) \to (Z, \eta)$ is $\delta\hat{g}$-closed.

Proof: Let G be a closed subset of X. Since f is closed, f(G) is closed set of Y. On the other hand, $\delta\hat{g}$-closeness of g implies g(f(G)) is $\delta\hat{g}$-closed in Z. Hence $g \circ f:(X, \tau) \to (Z, \eta)$ is $\delta\hat{g}$-closed map.

Remark: 3.8. If f:$(X, \tau) \to (Y, \sigma)$ is $\delta\hat{g}$ -closed and g:$(Y, \sigma) \to (Z, \eta)$ is closed map then $g \circ f:(X, \tau) \to (Z, \eta)$ may not be $\delta\hat{g}$ -closed. In an example 3.6, f is $\delta\hat{g}$ -closed and g is closed but $g \circ f$ is not $\delta\hat{g}$-closed map.

Definition: 3.9. A map f:$(X, \tau) \to (Y, \sigma)$ is called δg-closed. (resp. δg -open) if the image of each closed (resp. open) set in (X, τ) is δg-closed in (Y, σ)

Theorem 3.10. Every $\delta\hat{g}$-closed map is δg-closed.
Proof: It is true that every $\delta\hat{g}$-closed set is δg-closed.

Remark: 3.11. The converse of theorem 3.10 need not be true as shown in the following example.

Example: 3.12. Let X ={a, b, c} = Y with topologies $\tau = \{\varphi, \{b, c\}, X\}$ and $\sigma = \{\varphi, \{c\}, \{a, b\}, Y\}$. Define a function f:$(X, \tau) \to (Y, \sigma)$ by f(a) = b, f(b)=a, f(c) = c. Then f is not $\delta\hat{g}$ -closed map because {a} is closed in (X, τ) but f({ a }) = {b} is not $\delta\hat{g}$ -closed in (Y, σ). However f is $\delta\hat{g}$ -closed.

Theorem: 3.13. Every $\delta\hat{g}$-closed map is $\alpha\hat{g}$-closed.
Proof: It is true that every $\delta\hat{g}$-closed set is $\alpha\hat{g}$-closed.

Remark: 3.14. The converse of Theorem 3.13 need not be true as shown in the following example.

Example: 3.15. Let X ={a, b, c} = Y with topologies $\tau = \{\varphi, \{b\}, \{b, c\}, X\}$ and $\sigma = \{\varphi, \{b\}, \{a, b\}, \{b, c\}, Y\}$ .Let f:$(X, \tau) \to (Y, \sigma)$ be a function defined by f(a) = c, f(b) = b and f(c)=a. Then f is not $\delta\hat{g}$ -closed map because {a} is closed
in (X, τ) but f({a}) ={c} is not $\delta\hat{g}$ -closed in (Y, σ). However f is $\alpha\hat{g}$-closed.

Theorem: 3.16. Every $\delta\hat{g}$-closed map is g-closed.
Proof: It is true that every $\delta\hat{g}$-closed set is g-closed.

Remark: 3.17. The converse of the above theorem need not be true as shown in the following example.

Example: 3.18. Let X= {a, b, c} = Y with topologies $\tau = \{\varphi, \{b\}, \{a, c\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, Y\}$.Define a map f:$(X, \tau) \to (Y, \sigma)$ be a function defined by f(a) = c, f(b) = c and f(c)=b. Then f is $\delta\hat{g}$ -closed map. However it is not $\delta\hat{g}$ -closed because {b} is closed in (X, τ) but f({b}) = {c} is not $\delta\hat{g}$-closed in (Y,σ).
Remark: 3.19 The following examples show that $\delta\hat{g}$ -closeness and $\hat{g}$-closeness are independent notions.

Example: 3.20. Let X ={a, b, c} = Y with topologies $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\varphi, \{a\}, Y\}$. Define a function f:$(X, \tau) \to (Y, \sigma)$ by f(a) = a, f(b) = c and f(c) = b. Then f is $\delta\hat{g}$ -closed map but not $\hat{g}$- closed because f({c} = {b} is not$\hat{g}$- closed in (Y, σ) where {c} is closed set in (X, τ).

Example: 3.21. Let X = {a, b, c} = Y with topologies $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{b, c\}, Y\}$. Define a function f: $(X, \tau) \to (Y, \sigma)$ by f(a) = a, f(b) = c and f(c)=b. Then f is $\hat{g}$ -closed map, However f is not $\delta\hat{g}$ -closed because {c} is not closed in (X, τ) but f({c}) = {b} is not $\delta\hat{g}$-closed in (Y, σ).

Remark: 3.22. The following table shows the relationships of $\delta\hat{g}$ -closed maps with other known existing maps. The symbol"1"in a cell means that a map implies the other maps. Finally the symbol" 0" means that a map not implies the other maps.

TABLE- 1

closed functions	$\delta\hat{g}$	δg	g	$\alpha\hat{g}$	$\hat{g}$
$\delta\hat{g}$	1	1	1	1	0
δg	0	1	1	0	0
g	0	0	1	0	0
$\alpha\hat{g}$	0	0	0	1	0
$\hat{g}$	0	0	1	0	1

Theorem: 3.23. A map $f:(X, \tau) \to (Y, \sigma)$ is $\delta\hat{g}$-closed if and only if for each subset G of (Y, σ) and for each open set U of (X, τ) containing $f^{-1}(G)$,there exists an $\delta\hat{g}$-open set B of (Y, σ) such that $G \subset V$ and $f^{-1}(V) \subset U$.

Proof: Let f be an $\delta\hat{g}$-closed map and let G be an subset of (Y, σ) and U be an open set of (X, τ) containing $f^{-1}(G)$. Then X-U is closed in (X, τ). Since f is $\delta\hat{g}$-closed map, f(X-U) is $\delta\hat{g}$-closed set in (Y, σ). Hence Y- f(X - U) is $\delta\hat{g}$-open set in (Y, σ). Take V = Y- f(X - U). Then V is $\delta\hat{g}$-open set in (Y, σ) containing G. Such that $f^{-1}(V) \subset U$. Conversely, let F be an closed subset of (X, τ). Then $f^{-1}(Y-f(F)) \subset X - F$ and X - F is open. By hypothesis there is an $\delta\hat{g}$-open set V of (Y, σ) such that $Y- f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore, $F \subset X - f^{-1}(V)$. Hence $Y-V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$ which implies f (F) = Y - V and hence f (F) is $\delta\hat{g}$- closed in (Y, σ). Thus f is an $\delta\hat{g}$-closed map.

Theorem: 3.24. Let $f:(X, \tau) \to (Y, \sigma)$ and $g:(Y, \sigma) \to (Z, \eta)$ be any two maps:

(i) If $g \circ f : (X, \tau) \to (Z, \eta)$ is $\delta\hat{g}$-closed map and g is $\delta\hat{g}$-irresolute injective map then f is $\delta\hat{g}$ closed
(ii) If $g \circ f : (X, \tau) \to (Z, \eta)$ is $\delta\hat{g}$-irresolute and g is $\delta\hat{g}$-closed injective map then f is $\delta\hat{g}$-continuous.

Proof: (i) Let U be closed in (X, τ). Since $g \circ f$ is $\delta\hat{g}$-closed, $(g \circ f)(U)$ is $\delta\hat{g}$-closed in (Z, η). Therefore g(f(U)) is $\delta\hat{g}$-closed in (Z, η). Since g is irresolute, $g^{-1}(g(f(U)))$ is $\delta\hat{g}$-closed in (Y, σ). That is f(U) is $\delta\hat{g}$-closed in (Y, σ). Hence f is $\delta\hat{g}$-closed.
(ii) Let U be closed in (Y, σ). Since g is $\delta\hat{g}$-closed, g(U) is $\delta\hat{g}$-closed in (Z, η). Since $g \circ f$ is $\delta\hat{g}$-irresolute, $(g \circ f)^{-1}(g(U))$ is $\delta\hat{g}$-closed in (X, τ).

Therefore, $(f^{-1} \circ g^{-1}) g(U))$ is $\delta\hat{g}$-closed in (X, τ).
Hence $f^{-1}(U)$ is $\delta\hat{g}$-closed in (X, τ).
This shows that f is $\delta\hat{g}$-continuous.

Theorem: 3.25. Let $f:(X, \tau) \to (Y, \sigma)$ and $g:(Y, \sigma) \to (Z, \eta)$ be any two maps and $g \circ f :(X, \tau) \to (Z, \eta)$ be an $\delta\hat{g}$ - closed map. If f is continuous then g is $\delta\hat{g}$-closed.

Proof: Let V be closed in (Y, σ). Since f is continuous, $f^{-1}(V)$ is closed in (X, τ). Since $g \circ f$ is $\delta\hat{g}$-closed, $(g \circ f)(f^{-1}(V))$ is $\delta\hat{g}$-closed in (Z, η). That is g(V) is $\delta\hat{g}$-closed in (Z, η). Hence g is $\delta\hat{g}$-closed.

Theorem: 3.26. A bijection $f:(X, \tau) \to (Y, \sigma)$ is $\delta\hat{g}$-closed map iff f(U) is $\delta\hat{g}$-open in (Y, σ) for every open set U in (X, τ).

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an $\delta\hat{g}$-closed map and U be an open set in (X, τ) .Then U^c is closed in (X, τ). Since f is $\delta\hat{g}$-closed map, $f(U^c)$ is $\delta\hat{g}$-closed set in(Y, σ). But $f(U^c) = [f(U)]^c$ and hence $[f(U)]^c$ is $\delta\hat{g}$-closed in (Y, σ). Hence f(U) is $\delta\hat{g}$-open in (Y, σ). Conversely, f(U) is $\delta\hat{g}$-open in (Y, σ) for every open set U of (X, τ) then U^c is closed set in (X, τ) and $[f(U)]^c$ is $\delta\hat{g}$-closed in (Y, σ). But $[f(U)]^c = f(U^c)$ and hence $f(U^c)$ is $\delta\hat{g}$-closed in (Y, σ). Therefore , f is $\delta\hat{g}$-closed map.

4.WEAKLY $\delta\hat{g}$-CLOSED MAPS

We introduce the following definition:

Definition: 4.1 A map $f:(X, \tau) \to (Y, \sigma)$ is called weakly $\delta\hat{g}$-closed (resp. weakly $\delta\hat{g}$-open) if the image of every δ-closed (resp. δ-open) set in (X, τ) is $\delta\hat{g}$-closed (resp. $\delta\hat{g}$-open) set in (Y, σ).

Theorem: 4.2 Every $\delta\hat{g}$-closed map is weakly $\delta\hat{g}$-closed.

Proof: Let $f:(X, \tau) \to (Y, \sigma)$ be an $\delta\hat{g}$-closed map and G be a δ-closed set in (X, τ). Every δ-closed set is closed, G is closed set in (X, τ). Since f is $\delta\hat{g}$-closed, f(G) is $\delta\hat{g}$-closed in (Y, σ). Hence f is weakly $\delta\hat{g}$-closed map.

Remark: 4.3. The converse of the above theorem need not be true as shown in the following example.

Example:4.4. Let X ={a, b, c} = Y ; $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$, $\sigma = \{\varphi, \{b\}, \{a, c\}, Y\}$. Let $f:(X, \tau) \to (Y, \sigma)$ be the identity map. Then f is weakly $\delta\hat{g}$-Closed map but f is not $\delta\hat{g}$-closed. Since f ({b, c}) = {b, c} is not $\delta\hat{g}$-closed in (Y, σ) where {b, c} is closed in (X, τ).

Theorem: 4.5 Every δ-closed map is weakly $\delta\hat{g}$-closed map

Proof: It is true that every δ-closed set is $\delta\hat{g}$-closed.

Remark: 4.6 The converse of the above theorem need not be true as shown in the following example.
Example: 4.7 Let X ={a, b, c} = Y ; $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$, $\sigma = \{\varphi, \{a\}, Y\}$. Let $f :(X, \tau) \to (Y, \sigma)$ be the identity map. Then f is weakly $\delta\hat{g}$-closed map but f is not δ-closed map because f ({a, c}) = {a, c} is not δ –

closed in (Y, σ) where $\{a, c\}$ is closed in (X, τ).

Proposition: 4.8. The composite mapping of weakly $\delta\hat{g}$-closed maps need not be weakly $\delta\hat{g}$-closed as shown in the following example.

Example 4.9. Let X ={a, b, c}=Y=Z with topologies $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, Y\}$, $\eta = \{\varphi, \{c\}, \{a, b\}, Z\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by f(a) = a, f(b) = a and f(c) = b and let $g : (Y, \sigma) \to (Z, \eta)$ be the identity function. Clearly f and g are weakly $\delta\hat{g}$-closed map but the $g \circ f : (X, \tau) \to (Z, \eta)$ is not an weakly $\delta\hat{g}$-closed map because {b} is δ-closed in (X, τ) but $(g \circ f)(\{b\}) = g(f(\{b\})) = \{a\}$ is not $\delta\hat{g}$-closed set in (Z, η).

Theorem : 4.10 Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be any two maps. Then

(i) $g \circ f:(X, \tau) \to (Z, \eta)$ is weakly $\delta\hat{g}$-closed map, if f is δ-closed map and g is weakly $\delta\hat{g}$-closed map.
(ii) If $g \circ f : (X, \tau) \to (Z, \eta)$ is weakly $\delta\hat{g}$-closed and g is $\delta\hat{g}$-irresolute injective map then f is weakly $\delta\hat{g}$-closed.

Proof: (i) Let V be δ-closed in (X, τ). Since f is δ-closed map, f(V) is δ-closed in (Y, σ). Since g is weakly $\delta\hat{g}$-closed map, g(f(V)) is $\delta\hat{g}$-closed in (Z, η). That is $(g \circ f)(V)$ is $\delta\hat{g}$-closed in (Z, η). Hence $(g \circ f)$ is weakly $\delta\hat{g}$-closed map.
(ii) Let U be the δ-closed in (X, τ). Since $g \circ f$ is weakly $\delta\hat{g}$-closed map, $(g \circ f)(U)$ is $\delta\hat{g}$-closed in (Z, η). Therefore g(f(U)) is $\delta\hat{g}$-closed in (Z, η). Since g is $\delta\hat{g}$-irresolute, $g^{-1}(g(f(U)))$ is $\delta\hat{g}$-closed in (Y, σ). That is f(U) is $\delta\hat{g}$-closed in (Y, σ). Hence f is weakly $\delta\hat{g}$-closed map.

Remark: 4.11 Weakly $\delta\hat{g}$-closeness and $\delta\hat{g}$-irresoluteness are independent notions as shown in the following example.

Example: 4.12 Let X ={a, b, c} = Y with topologies $\tau = \{\varphi, \{a, b\}, X\}$, $\sigma = \{\varphi, \{b\}, Y\}$. Define a function $f :(X, \tau) \to (Y, \sigma)$ by f(a) = b, f(b)=c and f(c) = a. Then f is weakly $\delta\hat{g}$-closed map but not $\delta\hat{g}$-irresolute because $f^{-1}(\{b, c\}) = \{a, b\}$ is not $\delta\hat{g}$-closed in (X, τ) where {b,c} is $\delta\hat{g}$-closed in (Y, σ).

Example: 4.13. Let X ={a, b, c} = Y with topologies $\tau = \{\varphi, \{a\}, \{c\}, \{a, c\}, X\}$, $= \{\varphi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be an identity function. Clearly f is $\delta\hat{g}$-irresolute but not weakly $\delta\hat{g}$-closed map because f({c}) = {c} is not $\delta\hat{g}$-closed in (Y, σ) where {c} is δ-closed in (X, τ).

Theorem: 4.14. A bijection $f : (X, \tau) \to (Y, \sigma)$ is weakly $\delta\hat{g}$-closed map, iff f(U) is $\delta\hat{g}$-open in (Y, σ) for every δ-open set U in (X, τ).

Proof: Let $f:(X, \tau) \to (Y, \sigma)$ is weakly $\delta\hat{g}$-closed map and U be an δ-open set in (X, τ). Then U^c is δ-closed set in (X, τ). Since f is weakly $\delta\hat{g}$-closed map, $f(U^c)$ is $\delta\hat{g}$-closed set in (Y, σ). But $f(U^c) = [f(U)]^c$ and hence $[f(U)]^c$ is $\delta\hat{g}$-closed set in (Y, σ). Hence f(U) is $\delta\hat{g}$-open in (Y, σ). Conversely, f(U) is $\delta\hat{g}$-open in (Y, σ) for every δ-open set U of (X, τ). Then U^c is δ-closed set in (X, τ) and $[f(U)]^c$ is $\delta\hat{g}$-closed in (Y, σ). Hence $f(U^c)$ is $\delta\hat{g}$-closed in (Y, σ). Thus f is weakly $\delta\hat{g}$-closed map.

Theorem: 4.15. A map $f:(X, \tau) \to (Y, \sigma)$ is weakly $\delta\hat{g}$-closed map, iff for each subset B of (Y, σ) and for each δ-open set U of (X, τ) containing $f^{-1}(B)$, there exists an $\delta\hat{g}$-open set V of (Y, σ) such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof: Necessity, suppose f is weakly $\delta\hat{g}$-closed map. Let B be any subset of (Y, σ) and U be an δ-open set of (X, τ) containing $f^{-1}(B)$. Then X – U is δ-closed subset of (X, τ). Since f is weakly $\delta\hat{g}$-closed map, f(X –U) is $\delta\hat{g}$-closed set in (Y, σ). That is Y–f(X–U) is $\delta\hat{g}$-open in (Y, σ). Put V = Y-f(X-U). Then V is an $\delta\hat{g}$-open set in (Y, σ) containing B such that $f^{-1}(V) \subset U$. Sufficiency. Let F be any δ-closed subset of (X, τ). Then $f^{-1}(Y - f(F)) \subset X - F$ and X-F is δ-open in (X, τ). Put B = Y-f(F). Then $f^{-1}(B) \subset X - F$ There exists an $\delta\hat{g}$-open set V of (Y, σ) such that $B = Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore we obtain f(F)= Y - V and hence f(F) is $\delta\hat{g}$-closed in (Y, σ). Thus f is weakly $\delta\hat{g}$-closed map.

5. APPLICATIONS:

Definition: 5.1. [5] A space (X, τ) is called $\hat{T}_{3/4}$-space if every $\delta\hat{g}$-Closed set in it is–closed.

Theorem: 5.2. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ two functions. Let (Y, σ) be $\hat{T}_{3/4}$ spaces. Then

(i) $g \circ f:(X, \tau) \to (Z, \eta)$ is $\delta\hat{g}$-closed map if g is $\delta\hat{g}$-closed and f is $\delta\hat{g}$-closed map.
(ii) $g \circ f :(X, \tau) \to (Z, \eta)$ is weakly $\delta\hat{g}$-closed map if g is weakly $\delta\hat{g}$-closed and f is weakly $\delta\hat{g}$-closed.

Proof: (i) Let V be closed in (X, τ). Since f is $\delta\hat{g}$-closed map, f (V) is $\delta\hat{g}$-closed set in (Y, σ). Since Y is $\hat{T}_{3/4}$ space, f(V) is δ-closed in Y. Since g is $\delta\hat{g}$- closed map, g(f (V)) in (Z, η). That is $(g \circ f)(V)$ is $\delta\hat{g}$-closed in (Z, η). Hence $(g \circ f)$ is $\delta\hat{g}$-closed map.

(ii)Let U be the δ-closed in (X, τ). Since f is weakly $\delta\hat{g}$-closed map, f(U) is $\delta\hat{g}$- closed in (Y, σ). since Y is $\hat{T}_{3/4}$ space, f(U) is δ-closed in (Y, σ). Since g is weakly $\delta\hat{g}$-closed, g(f (U)) is $\delta\hat{g}$-closed in (Z, η). That is $g \circ f(U)$ is $\delta\hat{g}$-closedin (Z, η). Hence $g \circ f$ is weakly δ g^ -closed map.

We introduce the following definition:

Definition: 5.3. A space (X, τ) is said to be $\delta\hat{g}$-regular if for each closed set F of X and each point $x \notin F$ there exists disjoint $\delta\hat{g}$-open sets U and V such that $F \subset U$ and $x \in V$.

Theorem: 5.4. In a topological space (X, τ), the following statements are equiv- alent.

(i) (X, τ) is $\delta\hat{g}$-regular.

(ii) For every point of (X, τ) and every open set V containing x there exists an $\delta\hat{g}$-open set A such that $x \in A \subset cl_\delta(A) \subset V$.

Proof: (i) $\Rightarrow$ (ii) Let $x \in X$ and V be an open set containing x. Then X-V is closed and $x \notin X - V$. By (i) there exists an $\delta\hat{g}$-open set A and B such that $x \in A$ and $X\text{-}V \subset B$. That is $X\text{-}B \subset V$. Since every open set is $\hat{g}$-open, V is $\hat{g}$-open. X-B is $\delta\hat{g}$-closed. Therefore $cl_\delta(X\text{-}B) \subset V$. Since $A \cap B = \varphi$, $A \subset X - B$. Hence $x \in A \subset cl_\delta(A) \subset cl_\delta(X\text{-}B) \subset V$ Thus $x \in A \subset cl_\delta(A) \subset (V)$.(ii) $\Rightarrow$(i) Let F be a closed set and $x \notin F$. This implies that X-F is open set containing x. By (ii), there exists an $\delta\hat{g}$-open set A such that $x \in A \subset cl_\delta(A) \subset X - F$ .That is $F \subset X\text{-}cl_\delta(A)$. Since every closed set is $\delta\hat{g}$-closed, $cl_\delta(A)$ is $\delta\hat{g}$-closed and $X\text{-}cl_\delta(A)$ is $\delta\hat{g}$-open. Therefore, A and $X\text{-}cl_\delta(A)$ are the required $\delta\hat{g}$-open sets.

Theorem: 5.5. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is a continuous and $\delta\hat{g}$-closed, bijection map and (X, τ) is a regular space then (Y, σ) is $\delta\hat{g}$-regular.

Proof: let $y \in Y$ and B be an open set containing y of (Y, σ) Let x be a point of (X, τ) such that $y = f(x)$. Since f is continuous, $f^{-1}(V)$ is open in (X, τ). since (X, τ) is regular, there exists an open set U such that $x \in U \subset cl(U) \subset f^{-1}(V)$. Hence $y = f(x) \in f(U) \subset f(cl(U)) \subset V$. Since f is an $\delta\hat{g}$-closed map, $f(cl(U))$ is an $\delta\hat{g}$-closed set Contained in the open set V, which is $\hat{g}$-open. Hence we have $cl_\delta(f(cl(U))) \subset V$. Therefore $y \in f(U) \subset cl_\delta(f(U)) \subset cl_\delta(f(cl(U))) \subset (V)$ .This implies $y \in f(U) \subset cl_\delta(f(U)) \subset V$. Since f is $\delta\hat{g}$-closed map, U^c is closed in X, $f(U^c)$ is $\delta\hat{g}$-closed in (Y, σ). But $f(U^c) = [f(U)]^c$ is $\delta\hat{g}$-closed in (Y, σ). Hence f(U) is $\delta\hat{g}$-open in (Y, σ). Thus for every point y of (Y, σ) and every open set V containing y there exists an $\delta\hat{g}$-open set f(U) such that

$$y \in f(U) \subset cl_\delta(f(U)) \subset V.$$

Hence by the above theorem, (Y, σ) is $\delta\hat{g}$-regular.

Theorem: 5.6. If $f:(X, \tau) \rightarrow (Y, \sigma)$ is a continuous and weakly $\delta\hat{g}$-closed bijective map and if (X, τ) is $\hat{T}_{3/4}$-space and regular space then (Y, σ) is $\delta\hat{g}$-regular

Proof: let $y \in (Y, \sigma)$ and V be an open set containing y. Let x be a point of (X, τ), such that $y = f(x)$. Since f is continuous, $f^{-1}(V)$ is open in (X, τ). By assumptions and theorem 5.3, there exists an $\delta\hat{g}$-open set U such that $x \in U \subset cl_\delta(U) \subset f^{-1}(V)$. Then

$$y \in f(U) \subset f(cl_\delta(U)) \subset V$$

We know that

$cl_\delta(U)$ is δ-closed. Since f is weakly $\delta\hat{g}$-closed, $f(cl_\delta(U))$ is $\delta\hat{g}$-closed set in (Y, σ). Every open set is $\hat{g}$-open and hence V is $\hat{g}$-open. Therefore we get $cl_\delta(f(cl_\delta(U))) \subset V$. This implies $y \in f(U) \subset cl_\delta(f(U)) \subset cl_\delta(f(cl_\delta(U))) \subset V$. That is $y \in f(U) \subset cl_\delta(f(U)) \subset V$. Now U is $\delta\hat{g}$-open implies U^c is $\delta\hat{g}$-closed in (X, τ). Since (X, τ) is $\hat{T}_{3/4}$ and f is weakly $\delta\hat{g}$-closed map $f(U^c)$ is $\delta\hat{g}$-closed in (Y, σ). But $f(U^c) = [f(U)]^c$. That is $[f(U)]^c$ is $\delta\hat{g}$-closed in (Y, σ). This implies f(U) is $\delta\hat{g}$-open in (Y, σ). Thus for every point y of (Y, σ) and every open set V containing y, there exists an $\delta\hat{g}$-open set f(U) such that $y \in f(U) \subset cl_\delta(f(U)) \subset V$. Hence by theorem 5.3, (Y, σ) is $\delta\hat{g}$-regular.

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