International Journal of Mathematical Archive-2(1), Jan. - 2011, Page: 169-173
Available online through www.ijma.info ISSN 2229 – 5046

SOME NEW CLASS OF FUNCTIONS VIA $\delta\hat{g}$-SETS

[*]M. Lellis Thivagar and [**]B. Meera Devi

[*]Department of Mathematics, Arul Anandar College, Karumathur, Madurai-625514, Tamil Nadu
E -mail: mlthivagar@yahoo.co.in

[**]Department of Mathematics, Sri. S. R. N. M. College, Sattur-626203, Tamil Nadu, INDIA.
E -mail: abmeeradevi@gmail.com

(Received on 03-01-11: Accepted on: 18-01-11)

ABSTRACT

In this paper we introduce a new class of functions called $\delta\hat{g}$ -closed maps. We obtain several characterizations and some their properties. We also investigate its relationship with other types of functions. Further we introduce and study a new class of functions namely weaker forms of $\delta\hat{g}$ -closed maps.

Keywords and Phrases: *$\delta\hat{g}$-closed sets, δg ^-continuous, $\delta\hat{g}$-closed maps, $\delta\hat{g}$-regular, $\alpha\hat{g}$-closed sets.*

AMS subject classification: *54C05, 54C10*

1.INTRODUCTION:

Malghan [7] introduced generalised closed functions and Devi et al.[1] intro- duced αg -closed functions. T. Noiri [9] and Veerakumar[12] introduced δ -closed functions and $\hat{g}$ -closed functions in topological spaces. In this present paper we use $\delta\hat{g}$ -closed sets to define a new class of functions called $\delta\hat{g}$ -closed functions and obtain some properties of these functions. We further introduce and study a new class of functions namely weakly $\delta\hat{g}$ -closed functions and we introduce a new space called $\delta\hat{g}$ -regular space.

2. PRELIMINARIES:

Throughout this paper (X, τ) and, $(Y, \sigma)$and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition: 2.1A subset A of a space (X, τ) is called a

(i) semi-open set [3] if $A \subseteq cl(int(A))$.
(ii) α -open set [8] if $A \subseteq int(cl(int(A)))$.
(iii) regular open set [11] if $A= int(cl(A))$.
(iv) δ-open set [13] if $A= \delta int(A)$.

The complement of a semi-open (resp. α-open, regular open) set is called semi- closed (resp. α -closed, regular closed).

The δ –interior [13] of a subset A of X is the union of all regular open set of X contained in A and is denoted by $Int\ \delta(A)$. The subset A is called δ -open [13] if $A = Int_{\delta}(A)$, i.e. a set is δ -open if it is the union of regular open sets. the complement of a δ -open is called δ -closed. Alternatively, a set $A \subseteq (X, \tau)$ is called δ -closed [13] if $A= cl_{\delta}(A)$, where $cl_{\delta}(A) = \{ x \in X: int(cl(U)) \cap A = \varphi, U \in \tau$ and $x \in U \}$.

Definition: 2.2 A subset A of (X, τ) is called
(i) generalized closed (briefly g -closed) set[4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ).
(ii) δ -generalized closed (briefly δg -closed) set[2] if $cl_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ).
(iii) $\hat{g}$ -closed set [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi-open set in (X, τ).
(iv) α - $\hat{g}$ -closed (briefly $\alpha\hat{g}$ -closed) set [6] if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$-open set in (X, τ).
(v) $\delta\hat{g}$ -closed set [5] if $cl_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$-open in (X, τ).

The complement of a g -closed (resp. δg -losed, $\hat{g}$ -closed, $\alpha\hat{g}$ -closed and $\delta\hat{g}$ -closed) set is called g -open (resp. δg -open, $\hat{g}$-open, $\alpha\hat{g}$-open and $\delta\hat{g}$-open).

Definition: 2.3 A function $f: (X, \tau) \to (Y, \sigma)$ is called:

(i) δ -closed [9] if $f(V)$ is δ -closed in (Y, σ) for every δ -closed set V of (X, τ)..
(ii) δ -continuous [10] if $f^{-1}(V)$ is δ -open in (X, τ) for every δ -open set V of (Y, σ).
(iii) $\delta\hat{g}$ -continuous [5] if $f^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ) for every closed set V of (Y, σ).
(iv) $\delta\hat{g}$ -irresolute [5] if $f^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ)

*Corresponding author: [**]B. Meera Devi
E-mail: abmeeradevi@gmail.com

for every $\delta\hat{g}$ closed set V of (Y,σ).

(v) generalized closed (briefly g -closed) (resp.g -open) [7] if the image of every closed (resp. Open) set in (X, τ) is g-closed (resp. g-open) in (Y,σ).

(vi) $\hat{g}$-open [12] if f(V) is $\hat{g}$-open in (Y, σ) for every open set V of (X,τ).

(vii)$\alpha\hat{g}$-closed [6] if the image of every closed set in (X, τ) is $\alpha\hat{g}$-closed in (Y,σ).

3. $\delta\hat{g}$-CLOSED MAPS:

We introduce the following definitions:

Definition: 3.1 A map $f : (X, \tau) \to (Y, \sigma)$ is called $\delta\hat{g}$-closed (resp. $\delta\hat{g}$-open) if the image of each closed (resp. Open) set in (X, τ) is $\delta\hat{g}$-closed (resp. $\delta\hat{g}$-open) in (Y, σ).

Remark: 3.2 $\delta\hat{g}$ -openness and δg^ -continuity are independent as shown by the following examples.

Example: 3.3 Let X = {a, b, c} = Y; τ = {φ, {a}, {c}, {a, c}, {b, c}, X } and, σ = {φ, {b}, Y }. Define f: $(X, \tau) \to (Y, \sigma)$ by f(a) = b, f(b) = c and f(c) = a. Then f is $\delta\hat{g}$-continuous but not $\delta\hat{g}$ -open, because {b, c} is open in (X, τ) but f({b, c}) = {a, c} is not $\delta\hat{g}$-open in (Y,σ).

Example: 3.4 Let X = {a, b, c} =Y with topologies τ = {φ, {a}, {b, c}, X} and, σ = {φ, {a}, Y}. Define $f:(X, \tau) \to (Y, \sigma)$ by f(a) = b, f(b) = a and f(c)=c. Then f is $\delta\hat{g}$-open but not $\delta\hat{g}$-continuous, because {b,c} is closed in (Y,σ) bu $f^{-1}(\{b, c\}) = \{a,c\}$ is not δg^-closed in (X, τ).

Remark: 3.5 The composite mapping of two $\delta\hat{g}$ -closed maps is not in $\delta\hat{g}$ -closed maps as shown in following example.

Example: 3.6 Let X ={a, b, c} = Y = Z ; τ = {φ, {a}, {c}, {a, c}, X } , σ = {φ, {a}, Y } and η ={ φ, {a}, {a, b}, {a, c}, Z } . Define a map $f:(X, \tau) \to (Y, \sigma)$ by f(a) = a, f(b) = c and f(c)= b and let $g:(Y, \sigma) \to (Z, \eta)$ be the identity function. Clearly f and g are δg^ -closed maps. But $g \circ f : (X, \tau) \to (Z, \eta)$ is not an $\delta\hat{g}$-closed map because $(g \circ f)$ ({b}) = {c}is not an $\delta\hat{g}$-closed set of (Z, η) where {b} is a closed set of (X, τ).

Theorem:3.7 If $f:(X, \tau) \to (Y, \sigma)$ is closed and $g:(Y, \sigma) \to (Z, \eta)$ is $\delta\hat{g}$ -closed map then $g \circ f :(X, \tau) \to (Z, \eta)$ is $\delta\hat{g}$-closed.

Proof: Let G be a closed subset of X. Since f is closed, f(G) is closed set of Y. On the other hand, $\delta\hat{g}$ -closeness of g implies g(f(G)) is $\delta\hat{g}$ -closed in Z. Hence $g \circ f:(X, \tau) \to (Z, \eta)$ is $\delta\hat{g}$-closed map.

Remark: 3.8. If $f:(X, \tau) \to (Y, \sigma)$ is $\delta\hat{g}$ -closed and $g:(Y, \sigma) \to (Z, \eta)$ is closed map then $g \circ f:(X, \tau) \to (Z, \eta)$ may not be $\delta\hat{g}$ -closed. In an example 3.6, f is $\delta\hat{g}$ -closed and g is closed but $g \circ f$ is not $\delta\hat{g}$-closed map.

Definition: 3.9. A map $f:(X, \tau) \to (Y, \sigma)$ is called δg-closed. (resp. δg -open) if the image of each closed (resp. open) set in (X, τ) is δg-closed in (Y, σ)

Theorem 3.10. Every $\delta\hat{g}$-closed map is δg-closed.

Proof: It is true that every $\delta\hat{g}$-closed set is δg-closed.

Remark: 3.11. The converse of theorem 3.10 need not be true as shown in the following example.

Example: 3.12. Let X ={a, b, c} = Y with topologies τ = {φ, {b, c}, X} and σ = {φ, {c}, {a, b}, Y }. Define a function $f:(X, \tau) \to Y, \sigma)$ by f(a) = b, f(b)=a, f(c) = c. Then f is not $\delta\hat{g}$ -closed map because {a} is closed in (X, τ) but f({ a }) = {b} is not $\delta\hat{g}$ -closed in (Y, σ). However f is $\delta\hat{g}$ -closed.

Theorem: 3.13. Every $\delta\hat{g}$-closed map is $\alpha\hat{g}$-closed.

Proof: It is true that every $\delta\hat{g}$-closed set is $\alpha\hat{g}$-closed.

Remark: 3.14. The converse of Theorem 3.13 need not be true as shown in the following example.

Example: 3.15. Let X ={a, b, c} = Y with topologies τ = {φ, {b}, {b, c}, X } and σ = {φ, {b}, {a, b}, {b, c}, Y } .Let $f:(X, \tau) \to (Y, \sigma)$ be a function defined by f(a) = c, f(b) = b and f(c)=a. Then f is not $\delta\hat{g}$ -closed map because {a} is closed
in (X, τ) but f ({a}) ={c} is not $\delta\hat{g}$ -closed in (Y, σ). However f is $\alpha\hat{g}$-closed.

Theorem: 3.16. Every $\delta\hat{g}$-closed map is g-closed.

Proof: It is true that every $\delta\hat{g}$-closed set is g-closed.

Remark: 3.17. The converse of the above theorem need not be true as shown in the following example.

Example: 3.18. Let X= {a, b, c} = Y with topologies τ = {φ, {b}, {a, c}, X} and σ = {φ, {a}, {a, b}, {a, c}, Y}.Define a map $f:(X, \tau) \to (Y, \sigma)$ be a function defined by f(a) = c, f(b) = c and f(c)=b. Then f is $\delta\hat{g}$ -closed map. However it is not $\delta\hat{g}$ -closed because {b} is closed in (X, τ) but f({b}) = {c} is not $\delta\hat{g}$-closed in (Y,σ).

Remark: 3.19 The following examples show that $\delta\hat{g}$ -closeness and $\hat{g}$-closeness are independent notions.

Example: 3.20. Let X ={a, b, c} = Y with topologies τ = {φ, {a}, {a, b}, X } and σ = {φ, {a}, Y }. Define a function $f:(X, \tau) \to (Y, \sigma)$ by f(a) = a, f(b) = c and f(c) = b. Then f is $\delta\hat{g}$ -closed map but not $\hat{g}$– closed because f({c} = {b} is not $\hat{g}$– closed in (Y, σ) where {c} is closed set in (X, τ).

Example: 3.21. Let X = {a, b, c} = Y with topologies τ = {φ, {a}, {a, b}, X } and σ = {φ, {a}, {b, c}, Y } . Define a function f: $(X, \tau) \to (Y, \sigma)$ by f(a) = a, f(b) = c and f(c)=b. Then f is $\hat{g}$ -closed map, However f is not $\delta\hat{g}$ -closed because {c} is not closed in (X, τ) but f({c}) = {b} is not $\delta\hat{g}$-closed in (Y, σ).

Remark: 3.22. The following table shows the relationships of $\delta\hat{g}$ -closed maps with other known existing maps. The symbol"1"in a cell means that a map implies the other maps. Finally the symbol" 0" means that a map not implies the other maps.

TABLE- 1

closed functions	$\delta\hat{g}$	δg	g	$\alpha\hat{g}$	$\hat{g}$
$\delta\hat{g}$	1	1	1	1	0
δg	0	1	1	0	0
g	0	0	1	0	0
$\alpha\hat{g}$	0	0	0	1	0
$\hat{g}$	0	0	1	0	1

Theorem: 3.23. A map $f:(X,\tau)\to(Y,\sigma)$ is $\delta\hat{g}$-closed if and only if for each subset G of (Y,σ) and for each open set U of (X,τ) containing $f^{-1}(G)$, there exists an $\delta\hat{g}$-open set B of (Y,σ) such that $G\subset V$ and $f^{-1}(V)\subset U$.

Proof: Let f be an $\delta\hat{g}$-closed map and let G be an subset of (Y,σ) and U be an open set of (X,τ) containing $f^{-1}(G)$. Then X-U is closed in (X,τ). Since f is $\delta\hat{g}$-closed map, f(X-U) is $\delta\hat{g}$-closed set in (Y,σ). Hence Y- f(X - U) is $\delta\hat{g}$-open set in (Y,σ). Take V = Y- f(X - U). Then V is $\delta\hat{g}$-open set in (Y,σ) containing G. Such that $f^{-1}(V)\subset U$. Conversely, let F be an closed subset of (X,τ). Then $f^{-1}(Y-f(F))\subset X-F$ and X - F is open. By hypothesis there is an $\delta\hat{g}$-open set V of (Y,σ) such that $Y-f(F)\subset V$ and $f^{-1}(V)\subset X-F$. Therefore, $F\subset X-f^{-1}(V)$. Hence $Y-V\subset f(F)\subset f(X-f^{-1}(V))\subset Y-V$ which implies f (F) = Y - V and hence f (F) is $\delta\hat{g}$- closed in (Y,σ). Thus f is an $\delta\hat{g}$-closed map.

Theorem: 3.24. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be any two maps:

(i) If $g\circ f:(X,\tau)\to(Z,\eta)$ is $\delta\hat{g}$-closed map and g is $\delta\hat{g}$-irresolute injective map then f is $\delta\hat{g}$ closed
(ii) If $g\circ f:(X,\tau)\to(Z,\eta)$ is $\delta\hat{g}$-irresolute and g is $\delta\hat{g}$-closed injective map then f is $\delta\hat{g}$-continuous.

Proof: (i) Let U be closed in (X,τ). Since $g\circ f$ is $\delta\hat{g}$-closed, $(g\circ f)(U)$ is $\delta\hat{g}$-closed in (Z,η). Therefore g(f(U)) is $\delta\hat{g}$-closed in (Z,η). Since g is irresolute, $g^{-1}(g(f(U)))$ is $\delta\hat{g}$-closed in (Y,σ). That is f(U) is $\delta\hat{g}$-closed in (Y,σ). Hence f is $\delta\hat{g}$-closed.
(ii) Let U be closed in (Y,σ). Since g is $\delta\hat{g}$-closed, g(U) is $\delta\hat{g}$-closed in (Z,η). Since $g\circ f$ is $\delta\hat{g}$-irresolute, $(g\circ f)^{-1}(g(U))$ is $\delta\hat{g}$-closed in (X,τ).

Therefore, $(f^{-1}\circ g^{-1})\,g(U))$ is $\delta\hat{g}$-closed in (X,τ).
Hence $f^{-1}(U)$ is $\delta\hat{g}$-closed in (X,τ).
This shows that f is $\delta\hat{g}$-continuous.

Theorem: 3.25. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be any two maps and $g\circ f:(X,\tau)\to(Z,\eta)$ be an $\delta\hat{g}$ - closed map. If f is continuous then g is $\delta\hat{g}$-closed.

Proof: Let V be closed in (Y,σ). Since f is continuous, $f^{-1}(V)$ is closed in (X,τ). Since $g\circ f$ is $\delta\hat{g}$-closed, $(g\circ f)(f^{-1}(V))$ is $\delta\hat{g}$-closed in (Z,η). That is g(V) is $\delta\hat{g}$-closed in (Z,η). Hence g is $\delta\hat{g}$-closed.

Theorem: 3.26. A bijection $f:(X,\tau)\to(Y,\sigma)$ is $\delta\hat{g}$-closed map iff f(U) is $\delta\hat{g}$-open in (Y,σ) for every open set U in (X,τ).

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an $\delta\hat{g}$-closed map and U be an open set in (X,τ). Then U^c is closed in (X,τ). Since f is $\delta\hat{g}$-closed map, $f(U^c)$ is $\delta\hat{g}$-closed set in (Y,σ). But $f(U^c)=[f(U)]^c$ and hence $[f(U)]^c$ is $\delta\hat{g}$-closed in (Y,σ). Hence f(U) is $\delta\hat{g}$-open in (Y,σ). Conversely, f(U) is $\delta\hat{g}$-open in (Y,σ) for every open set U of (X,τ) then U^c is closed set in (X,τ) and $[f(U)]^c$ is $\delta\hat{g}$-closed in (Y,σ). But $[f(U)]^c=f(U^c)$ and hence $f(U^c)$ is $\delta\hat{g}$-closed in (Y,σ). Therefore, f is $\delta\hat{g}$-closed map.

4. WEAKLY $\delta\hat{g}$-CLOSED MAPS

We introduce the following definition:

Definition: 4.1 A map $f:(X,\tau)\to(Y,\sigma)$ is called weakly $\delta\hat{g}$-closed (resp. weakly $\delta\hat{g}$-open) if the image of every δ-closed (resp. δ-open) set in (X,τ) is $\delta\hat{g}$-closed (resp. $\delta\hat{g}$-open) set in (Y,σ).

Theorem: 4.2 Every $\delta\hat{g}$-closed map is weakly $\delta\hat{g}$-closed.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an $\delta\hat{g}$-closed map and G be a δ-closed set in (X,τ). Every δ-closed set is closed, G is closed set in (X,τ). Since f is $\delta\hat{g}$-closed, f(G) is $\delta\hat{g}$-closed in (Y,σ). Hence f is weakly $\delta\hat{g}$-closed map.

Remark: 4.3. The converse of the above theorem need not be true as shown in the following example.

Example: 4.4. Let X ={a, b, c} = Y ; τ = {φ, {a}, {b}, {a, b}, {a, c}, X }, σ = {φ, {b}, {a, c}, Y }. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. Then f is weakly $\delta\hat{g}$-Closed map but f is not $\delta\hat{g}$-closed. Since f ({b, c}) = {b, c} is not $\delta\hat{g}$-closed in (Y,σ) where {b, c} is closed in (X,τ).

Theorem: 4.5 Every δ-closed map is weakly $\delta\hat{g}$-closed map

Proof: It is true that every δ-closed set is $\delta\hat{g}$-closed.

Remark: 4.6 The converse of the above theorem need not be true as shown in the following example.

Example: 4.7 Let X ={a, b, c} = Y ; τ = {φ, {a}, {b}, {a, b}, {a, c}, X }, σ = {φ, {a}, Y }. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. Then f is weakly $\delta\hat{g}$-closed map but f is not δ-closed map because f ({a, c}) = {a, c} is not δ –

closed in (Y, σ) where {a, c} is closed in (X, τ).

Proposition: 4.8. The composite mapping of weakly $\delta\hat{g}$-closed maps need not be weakly $\delta\hat{g}$-closed as shown in the following example.

Example 4.9. Let X ={a, b, c}=Y=Z with topologies τ = {φ, {a}, {b}, {a, b}, {a, c}, X } and σ = {φ, {b}, {c}, {a, b}, {b, c}, Y }, η = {φ, {c}, {a, b}, Z }. Define a function f : (X, τ) → (Y, σ) by f(a) = a, f(b) = a and f(c) = b and let g : (Y, σ) → (Z, η) be the identity function. Clearly f and g are weakly $\delta\hat{g}$-closed map but the g ∘ f : (X, τ) → (Z, η) is not an weakly $\delta\hat{g}$-closed map because {b} is δ-closed in (X, τ) but (g ∘ f)({b}) = g(f({b})) = {a} is not $\delta\hat{g}$-closed set in (Z, η).

Theorem : 4.10 Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be any two maps. Then

(i) g ∘ f:(X, τ) → (Z, η) is weakly $\delta\hat{g}$-closed map, if f is δ-closed map and g is weakly $\delta\hat{g}$-closed map.
(ii) If g ∘ f : (X, τ) → (Z, η) is weakly $\delta\hat{g}$-closed and g is $\delta\hat{g}$-irresolute injective map then f is weakly $\delta\hat{g}$-closed.

Proof: (i) Let V be δ-closed in (X, τ). Since f is δ-closed map, f(V) is δ-closed in (Y, σ). Since g is weakly $\delta\hat{g}$-closed map, g(f(V)) is $\delta\hat{g}$-closed in (Z, η). That is (g ∘ f)(V) is $\delta\hat{g}$-closed in (Z, η). Hence (g ∘ f) is weakly $\delta\hat{g}$-closed map.
(ii) Let U be the δ-closed in (X, τ). Since g ∘ f is weakly $\delta\hat{g}$-closed map, (g ∘ f)(U) is $\delta\hat{g}$-closed in (Z, η). Therefore g(f(U)) is $\delta\hat{g}$-closed in (Z, η). Since g is $\delta\hat{g}$-irresolute, g^{-1}(g(f(U))) is $\delta\hat{g}$-closed in (Y, σ). That is f(U) is $\delta\hat{g}$-closed in (Y, σ). Hence f is weakly $\delta\hat{g}$-closed map.

Remark: 4.11 Weakly $\delta\hat{g}$-closeness and $\delta\hat{g}$-irresoluteness are independent notions as shown in the following example.

Example: 4.12 Let X ={a, b, c} = Y with topologies τ = {φ, {a, b}, X }, σ = {φ, {b}, Y }. Define a function f :(X, τ) → (Y, σ) by f(a) = b, f(b)=c and f(c) = a. Then f is weakly $\delta\hat{g}$-closed map but not $\delta\hat{g}$-irresolute because f^{-1}({b, c}) = {a, b} is not $\delta\hat{g}$-closed in (X, τ) where {b,c} is $\delta\hat{g}$-closed in (Y, σ).

Example: 4.13. Let X ={a, b, c} = Y with topologies τ = {φ, {a}, {c}, {a, c}, X }, = {φ, {a}, {c}, {a, b}, {a, c}, Y }. Let f : (X, τ) → (Y, σ) be an identity function. Clearly f is $\delta\hat{g}$-irresolute but not weakly $\delta\hat{g}$-closed map because f ({c}) = {c} is not $\delta\hat{g}$-closed in (Y, σ)
where {c} is δ-closed in (X, τ).

Theorem: 4.14. A bijection f : (X, τ) → (Y, σ) is weakly $\delta\hat{g}$-closed map, iff f(U) is $\delta\hat{g}$-open in (Y, σ) for every δ-open set U in (X, τ).

Proof: Let f:(X, τ) → (Y, σ) is weakly $\delta\hat{g}$-closed map and U be an δ-open set in (X, τ). Then U^c is δ-closed set in (X, τ). Since f is weakly $\delta\hat{g}$-closed map, f(U^c) is $\delta\hat{g}$-closed set in (Y, σ). But f(U^c) = $[f(U)]^c$ and hence $[f(U)]^c$ is $\delta\hat{g}$-closed set in (Y, σ). Hence f(U) is $\delta\hat{g}$-open in (Y, σ). Conversely, f(U) is $\delta\hat{g}$-open in (Y, σ) for every δ-open set U of (X, τ). Then U^c is δ-closed set in (X, τ) and $[f(U)]^c$ is $\delta\hat{g}$-closed in (Y, σ). Hence f(U^c) is $\delta\hat{g}$-closed in (Y, σ). Thus f is weakly $\delta\hat{g}$-closed map.

Theorem: 4.15. A map f:(X, τ) → (Y, σ) is weakly $\delta\hat{g}$-closed map, iff for each subset B of (Y, σ) and for each δ-open set U of (X, τ) containing f^{-1}(B), there exists an $\delta\hat{g}$-open set V of (Y, σ) such that B ⊂ V and f^{-1}(V) ⊂ U.

Proof: Necessity, suppose f is weakly $\delta\hat{g}$-closed map. Let B be any subset of (Y, σ) and U be an δ-open set of (X, τ) containing f^{-1}(B). Then X – U is δ-closed subset of (X, τ). Since f is weakly $\delta\hat{g}$-closed map, f(X –U) is $\delta\hat{g}$-closed set in (Y, σ). That is Y–f(X–U) is $\delta\hat{g}$-open in (Y, σ). Put V = Y-f(X-U). Then V is an $\delta\hat{g}$-open set in (Y, σ) containing B such that f^{-1}(V) ⊂ U. Sufficiency. Let F be any δ-closed subset of (X, τ). Then f^{-1}(Y –f(F)) ⊂X – F and X-F is δ-open in (X, τ). Put B = Y-f(F). Then f^{-1}(B)⊂ X –F There exists an $\delta\hat{g}$-open set V of (Y, σ) such that B = Y- f(F) ⊂V and f^{-1}(V) ⊂ X – F. Therefore we obtain f(F)= Y - V and hence f(F) is $\delta\hat{g}$-closed in (Y, σ). Thus f is weakly $\delta\hat{g}$-closed map.

5. APPLICATIONS:

Definition: 5.1. [5] A space (X, τ) is called $\hat{T}_{3/4}$-space if every $\delta\hat{g}$-Closed set in it is–closed.

Theorem: 5.2. Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) two functions. Let (Y, σ) be $\hat{T}_{3/4}$ spaces. Then

(i) g ∘ f:(X, τ) → (Z, η) is $\delta\hat{g}$-closed map if g is $\delta\hat{g}$-closed and f is $\delta\hat{g}$-closed map.
(ii) g ∘ f :(X, τ) → (Z, η) is weakly $\delta\hat{g}$-closed map if g is weakly $\delta\hat{g}$-closed and f is weakly $\delta\hat{g}$-closed.

Proof: (i) Let V be closed in (X, τ). Since f is $\delta\hat{g}$-closed map, f (V) is $\delta\hat{g}$-closed set in (Y, σ). Since Y is $\hat{T}_{3/4}$ space, f(V) is δ-closed in Y. Since g is $\delta\hat{g}$- closed map, g(f (V)) in (Z, η). That is (g ∘ f)(V) is $\delta\hat{g}$-closed in (Z, η). Hence (g ∘ f) is $\delta\hat{g}$-closed map.

(ii)Let U be the δ-closed in (X, τ). Since f is weakly $\delta\hat{g}$-closed map, f(U) is $\delta\hat{g}$- closed in (Y, σ). since Y is $\hat{T}_{3/4}$ space, f(U) is δ-closed in (Y, σ). Since g is weakly $\delta\hat{g}$-closed, g(f (U)) is $\delta\hat{g}$-closed in (Z, η). That is g ∘ f (U) is $\delta\hat{g}$-closedin (Z, η). Hence g ∘ f is weakly δ g^ -closed map.

We introduce the following definition:

Definition: 5.3. A space (X, τ) is said to be $\delta\hat{g}$-regular if for each closed set F of X and each point x ∉F there exists disjoint $\delta\hat{g}$-open sets U and V such that F ⊂ U and x∈ V.

Theorem: 5.4. In a topological space (X, τ), the

following statements are equiv- alent.
(i) (X, τ) is $\delta\hat{g}$-regular.
(ii) For every point of (X, τ) and every open set V containing x there exists an $\delta\hat{g}$-open set A such that $x \in A \subset cl_\delta(A) \subset V$.

Proof: (i) $\Rightarrow$ (ii) Let $x \in X$ and V be an open set containing x. Then X-V is closed and $x \notin X - V$. By (i) there exists an $\delta\hat{g}$-open set A and B such that $x \in A$ and $X\text{-}V \subset B$. That is $X\text{-}B \subset V$. Since every open set is $\hat{g}$-open, V is $\hat{g}$-open. X-B is $\delta\hat{g}$-closed. Therefore $cl_\delta(X\text{-}B) \subset V$. Since $A \cap B = \varphi$, $A \subset X - B$. Hence $x \in A \subset cl_\delta(A) \subset cl_\delta(X\text{-}B) \subset V$ Thus $x \in A \subset cl_\delta(A) \subset (V)$.(ii) $\Rightarrow$(i) Let F be a closed set and $x \notin F$. This implies that X-F is open set containing x. By (ii), there exists an $\delta\hat{g}$-open set A such that $x \in A \subset cl_\delta(A) \subset X - F$ .That is $F \subset X\text{-}cl_\delta(A)$. Since every closed set is $\delta\hat{g}$-closed, $cl_\delta(A)$ is $\delta\hat{g}$-closed and $X\text{-}cl_\delta(A)$ is $\delta\hat{g}$-open. Therefore, A and $X\text{-}cl_\delta(A)$ are the required $\delta\hat{g}$-open sets.

Theorem: 5.5. Let $f : (X, \tau) \to (Y, \sigma)$ is a continuous and $\delta\hat{g}$-closed, bijection map and (X, τ) is a regular space then (Y, σ) is $\delta\hat{g}$-regular.

Proof: let $y \in Y$ and B be an open set containing y of (Y, σ) Let x be a point of (X, τ) such that $y = f(x)$. Since f is continuous, $f^{-1}(V)$ is open in (X, τ). since (X, τ) is regular, there exists an open set U such that $x \in U \subset cl(U) \subset f^{-1}(V)$. Hence $y = f(x) \in f(U) \subset f(cl(U)) \subset V$. Since f is an $\delta\hat{g}$-closed map, f(cl(U)) is an $\delta\hat{g}$-closed set Contained in the open set V, which is $\hat{g}$-open. Hence we have $cl_\delta(f(cl(U))) \subset V$. Therefore $y \in f(U) \subset cl_\delta(f(U)) \subset cl_\delta(f(cl(U))) \subset (V)$ .This implies $y \in f(U) \subset cl_\delta(f(U)) \subset V$. Since f is $\delta\hat{g}$-closed map, U^c is closed in X, $f(U^c)$ is $\delta\hat{g}$-closed in (Y, σ). But $f(U^c) = [f(U)]^c$ is $\delta\hat{g}$-closed in (Y, σ). Hence f(U) is $\delta\hat{g}$-open in (Y, σ). Thus for every point y of (Y, σ) and every open set V containing y there exists an $\delta\hat{g}$-open set f(U) such that

$$y \in f(U) \subset cl_\delta(f(U)) \subset V.$$

Hence by the above theorem, (Y, σ) is $\delta\hat{g}$-regular.

Theorem: 5.6. If $f:(X, \tau) \to (Y, \sigma)$ is a continuous and weakly $\delta\hat{g}$-closed bijective map and if (X, τ) is $\hat{T}_{3/4}$-space and regular space then (Y, σ) is $\delta\hat{g}$-regular

Proof: let $y \in (Y, \sigma)$ and V be an open set containing y. Let x be a point of (X, τ), such that $y = f(x)$. Since f is continuous, $f^{-1}(V)$ is open in (X, τ). By assumptions and theorem 5.3, there exists an $\delta\hat{g}$-open set U such that $x \in U \subset cl_\delta(U) \subset f^{-1}(V)$. Then

$$y \in f(U) \subset f(cl_\delta(U) \subset V$$

We know that

$cl_\delta(U)$ is δ-closed. Since f is weakly $\delta\hat{g}$-closed, $f(cl_\delta(U))$ is $\delta\hat{g}$-closed set in (Y, σ). Every open set is $\hat{g}$-open and hence V is $\hat{g}$-open. Therefore we get $cl_\delta(f(cl_\delta(U))) \subset V$. This implies $y \in f(U) \subset cl_\delta(f(U)) \subset cl_\delta(f(cl_\delta(U))) \subset V$. That is $y \in f(U) \subset cl_\delta(f(U)) \subset V$. Now U is $\delta\hat{g}$-open implies U^c is $\delta\hat{g}$-closed in (X, τ). Since (X, τ) is $\hat{T}_{3/4}$ and f is weakly $\delta\hat{g}$-closed map $f(U^c)$ is $\delta\hat{g}$-closed in (Y, σ). But $f(U^c) = [f(U)]^c$. That is $[f(U)]^c$ is $\delta\hat{g}$-closed in (Y, σ). This implies f(U) is $\delta\hat{g}$-open in (Y, σ). Thus for every point y of (Y, σ) and every open set V containing y, there exists an $\delta\hat{g}$-open set f(U) such that $y \in f(U) \subset cl_\delta(f(U)) \subset V$. Hence by theorem 5.3, (Y, σ) is $\delta\hat{g}$-regular.

REFERENCES:

[1] Devi. R, Balachandran. K, Maki. H, "Generalized α-closed maps and α-generalized closed maps", Indian J. Pure. Appl. Math., 29(1) (1998), 37-49.

[2] Dontchev. J and Ganster.M,"On δ-generalized closed sets and $T_{3/4}$– spaces" Mem. Fac.Sci.Kochi Univ. Ser. A, Math., 17(1996), 15-31.

[3] Levine.N, "Semi-open sets and semi-continuity in topological spaces", Amer Math. Monthly, 70(1963), 36-41.

[4] Levine.N, "Generalized closed sets in topology", Rend. Circ. Mat. Palermo, 19(1970) 89-96.

[5] Lellis Thivagar.M, Meera Devi.B," Some new class of generalized continuous functions". Communicated.

[6] M. E Abd El-Monsef, S.Rose Mary and M. Lellis Thivagar "On $\alpha\hat{g}$ - closed sets in topological spaces", Assiut University Journal of Mathematics and Computer Science, Vol. 36(1), P-P. 43-51(2007).

[7] Malghan. S.R, "Generalized closed maps", J. Karnatak Univ, Sci., 27(1982), 82-88.

[8] Njastad, O., "On some classes of nearly open sets", Pacific J Math. 15(1965), 961-970.

[9] Noiri. T, " A generalization of perfect functions", J.Korean Math. Soc.17 (1978), 540-544.

[10] Noiri.T, " On δ-continuous functions", J. Korean Math.soc.18 (1980), 161-166.

[11] Stone.M, "Application of the theory of Boolian rings to general topology" Trans. Amer. Math. Soc. 41(1937), 374-481.

[12] Veera Kumar.M.K.R.S, "$\hat{g}$-closed sets in topologycal spaces", Bull.Allah. Math.Soc, 18(2003), 99-112.

[13] Velicko.N.V, "H-closed topological spaces" Amer. Math. Soc. Transl., 78(1968), 103-118.

