

Slightly  $pg$ -continuous; Somewhat  $pg$ -continuous and Somewhat  $pg$ -open functions

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**ABSTRACT**

In this paper we discuss new type of continuous functions called slightly  $pg$ -continuous; somewhat  $pg$ -continuous and somewhat  $pg$ -open functions; its properties and interrelation with other such functions are studied.

**Keywords:** slightly continuous functions; slightly semi-continuous functions; slightly pre-continuous; slightly  $\beta$ -continuous functions; slightly  $\gamma$ -continuous functions and slightly  $\nu$ -continuous functions; somewhat continuous functions; somewhat semi-continuous functions; somewhat pre-continuous; somewhat  $\beta$ -continuous functions; somewhat  $\gamma$ -continuous functions and somewhat  $\nu$ -continuous functions; somewhat open functions; somewhat semi-open functions; somewhat pre-open; somewhat  $\beta$ -open functions; somewhat  $\gamma$ -open functions and somewhat  $\nu$ -open functions

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**1. Introduction**

In 1995 T. M. Nour introduced slightly semi-continuous functions. After him T. Noiri and G. I. Chae further studied slightly semi-continuous functions in 2000. T. Noiri individually studied about slightly  $\beta$ -continuous functions in 2001. C. W. Baker introduced slightly precontinuous functions in 2002. Erdal Ekici and M. Caldas studied slightly  $\gamma$ -continuous functions in 2004. Arse Nagli Uresin and others studied slightly  $\delta$ -continuous functions in 2007. Recently S. Balasubramanian and P.A.S. Vyjayanthi studied slightly  $\nu$ -continuous functions in 2011.

$b$ -open sets are introduced by Andrijevic in 1996. K. R. Gentry introduced somewhat continuous functions in the year 1971. V. K. Sharma and the present authors of this paper defined and studied basic properties of  $\nu$ -open sets and  $\nu$ -continuous functions in the year 2006 and 2010 respectively. T. Noiri and N. Rajesh introduced somewhat  $b$ -continuous functions in the year 2011. Inspired with these developments we introduce in this paper slightly  $pg$ -continuous, somewhat  $pg$ -continuous functions and somewhat  $pg$ -open functions and study its basic properties and interrelation with other type of such functions. Throughout the paper  $(X, \tau)$  and  $(Y, \sigma)$  (or simply  $X$  and  $Y$ ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

**2. Preliminaries**

**Definition 2.1:**  $A \subseteq X$  is called

(i) closed if its complement is open.

(ii)  $\alpha$ -open [ $\nu$ -open] if  $\exists U \in \alpha O(X)[RO(X)]$  such that  $U \subseteq A \subseteq \alpha cl(U)[U \subseteq A \subseteq cl(U)]$ .

(iii) semi- $\theta$ -open if it is the union of semi-regular sets and its complement is semi- $\theta$ -closed.

(iv) Regular closed [ $\alpha$ -closed; pre-closed;  $\beta$ -closed] if  $A = cl\{A^\circ\}$  [resp:  $cl(A^\circ)^\circ \subseteq A$ ;  $cl(A^\circ) \subseteq A$ ;  $cl((cl\{A\}))^\circ \subseteq A$ ].

(v) Semi closed [ $\nu$ -closed] if its complement is semi open [ $\nu$ -open].

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(vi) g-closed[rg-closed] if  $cl A \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

(vii) g-closed [pg-closed] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is pre-open[open] in  $X$ .

(viii) b-open if  $A \subseteq (cl\{A\})^o \cap cl\{A^o\}$ .

**Definition 2.2:** A function  $f: X \rightarrow Y$  is said to be

(i) continuous [resp: nearly-continuous;  $\alpha$ -continuous;  $\nu$ -continuous;  $\alpha$ -continuous; semi-continuous;  $\beta$ -continuous; pre-continuous] if inverse image of each open set is open[resp: regular-open;  $\alpha$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen].

(ii) nearly-irresolute[resp:  $\alpha$ -irresolute;  $\nu$ -irresolute;  $\alpha$ -irresolute; irresolute;  $\beta$ -irresolute; pre-irresolute] if inverse image of each regular-open[resp:  $\alpha$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen] set is regular-open[resp:  $\alpha$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen].

(iii) almost continuous[resp: almost-nearly-continuous; almost  $\alpha$ -continuous; almost  $\nu$ -continuous; almost  $\alpha$ -continuous; almost semi-continuous; almost  $\beta$ -continuous; almost pre-continuous] if for each  $x$  in  $X$  and each open set  $(V, f(x))$ ,  $\exists$  an open[resp: regular-open;  $\alpha$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen] set  $(U, x)$  such that  $f(U) \subseteq (cl(V))^o$ .

(iv) weakly continuous[resp: weakly nearly-continuous; weakly  $\alpha$ -continuous; weakly  $\nu$ -continuous; weakly  $\alpha$ -continuous; weakly semi-continuous; weakly  $\beta$ -continuous; weakly pre-continuous] if for each  $x$  in  $X$  and each open set  $(V, f(x))$ ,  $\exists$  an open[resp: regular-open;  $\alpha$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen] set  $(U, x)$  such that  $f(U) \subseteq cl(V)$ .

(v) slightly continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly  $\beta$ -continuous; slightly  $\gamma$ -continuous; slightly  $\alpha$ -continuous; slightly r-continuous; slightly  $\nu$ -continuous] at  $x$  in  $X$  if for each clopen subset  $V$  in  $Y$  containing  $f(x)$ ,  $\exists U \in \tau(X)$  [ $\exists U \in SO(X)$ ;  $\exists U \in PO(X)$ ;  $\exists U \in \beta O(X)$ ;  $\exists U \in \gamma O(X)$ ;  $\exists U \in \alpha O(X)$ ;  $\exists U \in RO(X)$ ;  $\exists U \in \nu O(X)$ ] containing  $x$  such that  $f(U) \subseteq V$ .

(vi) slightly continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly  $\beta$ -continuous; slightly  $\gamma$ -continuous; slightly  $\alpha$ -continuous; slightly r-continuous; slightly  $\nu$ -continuous] if it is slightly-continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly  $\beta$ -continuous; slightly  $\gamma$ -continuous; slightly  $\alpha$ -continuous; slightly r-continuous; slightly  $\nu$ -continuous] at each  $x$  in  $X$ .

(vii) almost strongly  $\theta$ -semi-continuous[resp: strongly  $\theta$ -semi-continuous] if for each  $x$  in  $X$  and for each  $V \in \sigma(Y, f(x))$ ,  $\exists U \in SO(X, x)$  such that  $f(scl(U)) \subseteq scl(V)$ [resp:  $f(scl(U)) \subseteq V$ ].

(viii) somewhat continuous[resp: somewhat b-continuous; somewhat  $\nu$ -continuous] if for  $U \in \sigma$  and  $f^{-1}(U) \neq \emptyset$ , there exists a non empty open[resp: non empty b-open; non empty  $\nu$ -open] set  $V$  in  $X$  such that  $V \neq \emptyset$  and  $V \subseteq f^{-1}(U)$ .

(ix) somewhat-open[resp: somewhat b--open; somewhat  $\nu$ -open] provided that if  $U \in \tau$  and  $U \neq \emptyset$ , then there exists a non empty b-open set[resp: non empty b-open; non empty  $\nu$ -open]  $V$  in  $Y$  such that  $V \neq \emptyset$  and  $V \subseteq f(U)$ .

(x) somewhat  $\nu$ -irresolute if for  $U \in \nu O(\sigma)$  and  $f^{-1}(U) \neq \emptyset$ , there exists a non-empty  $\nu$ -open set  $V$  in  $X$  such that  $V \subseteq f^{-1}(U)$ .

**Definition 2.3:**  $X$  is said to be a

(i) compact [resp: nearly-compact; pre-compact; mildly-compact] space if every open[resp: regular-open; preopen; clopen] cover has a finite subcover.

(ii) Lindeloff [resp: nearly-Lindeloff; pre-Lindeloff; mildly-Lindeloff] space if every open[resp: regular-open; preopen; clopen] cover has a countable subcover.

(iii) Extremely disconnected [briefly e.d] if the closure of each open set is open.

**Definition 2.4:**  $X$  is said to be a

(i)  $T_0$ [resp: r- $T_0$ ; pre- $T_0$ ; pg- $T_0$ ; Ultra  $T_0$ ] space if for each  $x \neq y \in X \exists U \in \tau(X)$ [resp:  $RO(X)$ ;  $PO(X)$ ;  $PGO(X)$ ;  $CO(X)$ ] containing either  $x$  or  $y$ .

(ii)  $T_1$  [resp:  $r-T_1$ ;  $pre-T_1$ ;  $pg-T_1$ ; Ultra  $T_1$ ] space if for each  $x \neq y \in X \exists U, V \in \tau(X)$  [resp:  $RO(X)$ ;  $PO(X)$ ;  $PGO(X)$ ;  $CO(X)$ ] such that  $x \in U - V$  and  $y \in V - U$ .

(iii)  $T_2$  [resp:  $r-T_2$ ;  $pre-T_2$ ;  $pg-T_2$ ; Ultra  $T_2$ ] space if for each  $x \neq y \in X \exists U, V \in \tau(X)$  [resp:  $RO(X)$ ;  $PO(X)$ ;  $PGO(X)$ ;  $CO(X)$ ] such that  $x \in U$ ;  $y \in V$  and  $U \cap V = \phi$ .

(iv)  $C_0$  [resp:  $r-C_0$ ;  $pre-C_0$ ;  $pg-C_0$ ; Ultra  $C_0$ ] space if for each  $x \neq y \in X \exists U \in \tau(X)$  [resp:  $RO(X)$ ;  $PO(X)$ ;  $PGO(X)$ ;  $CO(X)$ ] whose closure contains either  $x$  or  $y$

(v)  $C_1$  [resp:  $r-C_1$ ;  $pre-C_1$ ;  $pg-C_1$ ; Ultra  $C_1$ ] space if for each  $x \neq y \in X \exists U, V \in \tau(X)$  [resp:  $RO(X)$ ;  $PO(X)$ ;  $PGO(X)$ ;  $CO(X)$ ] whose closure contains  $x$  and  $y$ .

(vi)  $C_2$  [resp:  $r-C_2$ ;  $pre-C_2$ ;  $pg-C_2$ ; Ultra  $C_2$ ] space if for each  $x \neq y \in X \exists$  disjoint  $U, V \in \tau(X)$  [resp:  $RO(X)$ ;  $PO(X)$ ;  $PGO(X)$ ;  $CO(X)$ ] whose closure contains  $x$  and  $y$ .

(vii)  $D_0$  [resp:  $r-D_0$ ;  $pre-D_0$ ;  $pg-D_0$ ; Ultra  $D_0$ ] space if for each  $x \neq y \in X \exists U \in \mathcal{D}(X)$  [resp:  $RD(X)$ ;  $PD(X)$ ;  $PGD(X)$ ;  $COD(X)$ ] containing either  $x$  or  $y$ .

(viii)  $D_1$  [resp:  $r-D_1$ ;  $pre-D_1$ ;  $pg-D_1$ ; Ultra  $D_1$ ] space if for each  $x \neq y \in X \exists U, V \in \mathcal{D}(X)$  [resp:  $RD(X)$ ;  $PD(X)$ ;  $PGD(X)$ ;  $COD(X)$ ] such that  $x \in U - V$  and  $y \in V - U$ .

(ix)  $D_2$  [resp:  $r-D_2$ ;  $pre-D_2$ ;  $pg-D_2$ ; Ultra  $D_2$ ] space if for each  $x \neq y \in X \exists U, V \in \mathcal{D}(X)$  [resp:  $RD(X)$ ;  $PD(X)$ ;  $PGD(X)$ ;  $CD(X)$ ] such that  $x \in U$ ;  $y \in V$  and  $U \cap V = \phi$ .

(x)  $R_0$  [resp:  $r-R_0$ ;  $pre-R_0$ ;  $pg-R_0$ ] space if for each  $x$  in  $X \exists U \in \tau(X)$  [resp:  $RO(X)$ ;  $PO(X)$ ;  $PGO(X)$ ;  $CO(X)$ ]  $cl\{x\} \subseteq U$  [resp:  $rcl\{x\} \subseteq U$ ;  $pcl\{x\} \subseteq U$ ;  $pgcl\{x\} \subseteq U$ ] whenever  $x \in U \in \tau(X)$  [resp:  $x \in U \in RO(X)$ ;  $x \in U \in PO(X)$ ;  $x \in U \in PGO(X)$ ]

(xi)  $R_1$  [resp:  $r-R_1$ ;  $pre-R_1$ ;  $pg-R_1$ ] space if for  $x, y \in X$  such that  $cl\{x\} \neq cl\{y\}$  [resp: such that  $rcl\{x\} \neq rcl\{y\}$ ; such that  $pcl\{x\} \neq pcl\{y\}$ ; such that  $pgcl\{x\} \neq pgcl\{y\}$ ]  $\exists$  disjoint  $U, V \in \tau(X)$  such that  $cl\{x\} \subseteq U$  [resp:  $RO(X)$  such that  $rcl\{x\} \subseteq U$ ;  $PO(X)$  such that  $pcl\{x\} \subseteq U$ ;  $PGO(X)$  such that  $pgcl\{x\} \subseteq U$ ] and  $cl\{y\} \subseteq V$  [resp:  $RO(X)$  such that  $rcl\{y\} \subseteq V$ ;  $PO(X)$  such that  $pcl\{y\} \subseteq V$ ;  $PGO(X)$  such that  $pgcl\{y\} \subseteq V$ ]

**Lemma 2.1:**

(i) Let  $A$  and  $B$  be subsets of a space  $X$ , if  $A \in PGO(X)$  and  $B \in RO(X)$ , then  $A \cap B \in PGO(B)$ .

(ii) Let  $A \subset B \subset X$ , if  $A \in PGO(B)$  and  $B \in RO(X)$ , then  $A \in PGO(X)$ .

**3. Slightly pg-continuous functions:**

**Definition 3.1:** A function  $f: X \rightarrow Y$  is said to be

- (i) slightly  $pg$ -continuous function at  $x$  in  $X$  if for each clopen subset  $V$  in  $Y$  containing  $f(x)$ ,  $\exists U \in PGO(X)$  containing  $x$  such that  $f(U) \subseteq V$ .
- (ii) slightly  $pg$ -continuous function if it is slightly  $pg$ -continuous at each  $x$  in  $X$ .

**Note 2:** Here after we call slightly  $pg$ -continuous function as  $sl.pg.c$  function shortly.

**Example 3.1:**  $X = Y = \{a, b, c\}$ ;  $\tau = \{\phi, X\}$  and  $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$ . Let  $f: X \rightarrow Y$  defined as  $f(a) = b$ ;  $f(b) = c$  and  $f(c) = a$ , then  $f$  is  $sl.pg.c$ .

**Example 3.2:**  $X = Y = \{a, b, c\}$ ;  $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$ . Let  $f: X \rightarrow Y$  defined as follows:

- (i)  $f(a) = b$ ;  $f(b) = c$  and  $f(c) = a$ , then  $f$  is not  $sl.pg.c$ .
- (ii)  $f(a) = b$ ;  $f(b) = a$  and  $f(c) = c$ , then  $f$  is not  $sl.pg.c$ .

**Theorem 3.1:** The following are equivalent:

- (i)  $f: X \rightarrow Y$  is  $sl.pg.c$ .
- (ii)  $f^{-1}(V)$  is  $pg$ -open for every clopen set  $V$  in  $Y$ .
- (iii)  $f^{-1}(V)$  is  $pg$ -closed for every clopen set  $V$  in  $Y$ .
- (iv)  $f(pgcl(A)) \subseteq pgcl(f(A))$ .

**Corollary 3.1:** The following are equivalent.

- (i)  $f: X \rightarrow Y$  is sl.pg.c.
- (ii) For each  $x$  in  $X$  and each clopen subset  $V \in (Y, f(x)) \exists U \in PGO(X, x)$  such that  $f(U) \subseteq V$ .

**Theorem 3.2:** Let  $\Sigma = \{U_i; i \in I\}$  be any cover of  $X$  by regular open sets in  $X$ . A function  $f$  is sl.pg.c. iff  $f|_{U_i}$  is sl.pg.c., for each  $i \in I$ .

**Proof:** Let  $i \in I$  be an arbitrarily fixed index and  $U_i \in RO(X)$ . Let  $x \in U_i$  and  $V \in CO(Y, f_{U_i}(x))$ . Since  $f$  is sl.pg.c.,  $\exists U \in PGO(X, x)$  such that  $f(U) \subset V$ . Since  $U_i \in RO(X)$ , by Lemma 2.1  $x \in U \cap U_i \in PGO(U_i)$  and  $(f|_{U_i})U \cap U_i = f(U \cap U_i) \subset V$ . Hence  $f|_{U_i}$  is sl.pg.c.

Conversely Let  $x$  in  $X$  and  $V \in CO(Y, f(x))$ ,  $\exists i \in I$  such that  $x \in U_i$ . Since  $f|_{U_i}$  is sl.pg.c.,  $\exists U \in PGO(U_i, x)$  such that  $f|_{U_i}(U) \subset V$ . By Lemma 2.1,  $U \in PGO(X)$  and  $f(U) \subset V$ . Hence  $f$  is sl.pg.c.

**Theorem 3.3:**

- (i) If  $f: X \rightarrow Y$  is pg-irresolute and  $g: Y \rightarrow Z$  is sl.pg.c.[slightly-continuous], then  $g \circ f$  is sl.pg.c.
- (ii) If  $f: X \rightarrow Y$  is pg-irresolute and  $g: Y \rightarrow Z$  is g.-continuous, then  $g \circ f$  is sl.pg.c.
- (iii) If  $f: X \rightarrow Y$  is pg-continuous and  $g: Y \rightarrow Z$  is slightly-continuous, then  $g \circ f$  is sl.pg.c.

**Theorem 3.4:** If  $f: X \rightarrow Y$  is pg-irresolute, pg-open and  $PGO(X) = \tau$  and  $g: Y \rightarrow Z$  be any function, then  $g \circ f: X \rightarrow Z$  is sl.pg.c iff  $g: Y \rightarrow Z$  is sl.pg.c.

**Proof:** If part: Theorem 3.3(i) Only if part: Let  $A$  be clopen subset of  $Z$ . Then  $(g \circ f)^{-1}(A)$  is a pg-open subset of  $X$  and hence open in  $X$  [by assumption]. Since  $f$  is pg-open  $f((g \circ f)^{-1}(A))$  is pg-open  $Y \Rightarrow g^{-1}(A)$  is pg-open in  $Y$ . Thus  $g: Y \rightarrow Z$  is sl.pg.c.

**Corollary 3.2:** If  $f: X \rightarrow Y$  is pg-irresolute, pg-open and bijective,  $g: Y \rightarrow Z$  is a function. Then  $g: Y \rightarrow Z$  is sl.pg.c. iff  $g \circ f$  is sl.pg.c.

**Theorem 3.5:** If  $g: X \rightarrow X \times Y$ , defined by  $g(x) = (x, f(x))$  for all  $x$  in  $X$  be the graph function of  $f: X \rightarrow Y$ . Then  $g: X \rightarrow X \times Y$  is sl.pg.c iff  $f$  is sl.pg.c.

**Proof:** Let  $V \in CO(Y)$ , then  $X \times V$  is clopen in  $X \times Y$ . Since  $g: X \rightarrow Y$  is sl.pg.c.,  $f^{-1}(V) = f^{-1}(X \times V) \in PGO(X)$ . Thus  $f$  is sl.pg.c.

Conversely, let  $x$  in  $X$  and  $F$  be a clopen subset of  $X \times Y$  containing  $g(x)$ . Then  $F \cap (\{x\} \times Y)$  is clopen in  $\{x\} \times Y$  containing  $g(x)$ . Also  $\{x\} \times Y$  is homeomorphic to  $Y$ . Hence  $\{y \in Y : (x, y) \in F\}$  is clopen subset of  $Y$ .

Since  $f$  is sl.pg.c.  $\cup \{f^{-1}(y) : (x, y) \in F\}$  is pg-open in  $X$ . Further  $x \in \cup \{f^{-1}(y) : (x, y) \in F\} \subseteq g^{-1}(F)$ . Hence  $g^{-1}(F)$  is pg-open. Thus  $g: X \rightarrow Y$  is sl.pg.c.

**Theorem 3.6:**

- (i) If  $f: X \rightarrow \prod Y_\lambda$  is sl.pg.c, then  $P_\lambda \circ f: X \rightarrow Y_\lambda$  is sl.pg.c for each  $\lambda \in \Gamma$ , where  $P_\lambda$  is the projection of  $\prod Y_\lambda$  onto  $Y_\lambda$ .
- (ii)  $f: \prod X_\lambda \rightarrow \prod Y_\lambda$  is sl.pg.c, iff  $f_\lambda: X_\lambda \rightarrow Y_\lambda$  is sl.pg.c for each  $\lambda \in \Gamma$ .

**Remark 1:**

- (i) Composition of two sl.pg.c functions is not in general sl.pg.c.
- (ii) Algebraic sum and product of sl.pg.c functions is not in general sl.pg.c.
- (iii) The pointwise limit of a sequence of sl.pg.c functions is not in general sl.pg.c.

However we can prove the following:

**Theorem 3.7:** The uniform limit of a sequence of sl.pg.c functions is sl.pg.c.

**Note 3:** Pasting Lemma is not true for sl.pg.c functions. However we have the following weaker versions.

**Theorem 3.8:** Let  $X$  and  $Y$  be topological spaces such that  $X = A \cup B$  and let  $f|_A: A \rightarrow Y$  and  $g|_B: B \rightarrow Y$  are sl.r.c maps such that  $f(x) = g(x)$  for all  $x \in A \cap B$ . Suppose  $A$  and  $B$  are r-open sets in  $X$  and  $RO(X)$  is closed under finite unions, then the combination  $\alpha: X \rightarrow Y$  is sl.pg.c continuous.

**Theorem 3.9: Pasting Lemma** Let  $X$  and  $Y$  be spaces such that  $X = A \cup B$  and let  $f_A: A \rightarrow Y$  and  $g_B: B \rightarrow Y$  are sl.pg.c maps such that  $f(x) = g(x)$  for all  $x \in A \cap B$ . Suppose  $A, B$  are r-open sets in  $X$  and  $PGO(X)$  is closed under finite unions, then the combination  $\alpha: X \rightarrow Y$  is sl.pg.c.

**Proof:** Let  $F \in CO(Y)$ , then  $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$ , where  $f^{-1}(F) \in PGO(A)$  and  $g^{-1}(F) \in PGO(B) \Rightarrow f^{-1}(F); g^{-1}(F) \in PGO(X) \Rightarrow f^{-1}(F) \cup g^{-1}(F) \in PGO(X)$  [by assumption]. Therefore  $\alpha^{-1}(F) \in PGO(X)$ . Hence  $\alpha: X \rightarrow Y$  is sl.pg.c.

#### 4. Somewhat pg-continuous function:

**Definition 4.1:** A function  $f$  is said to be somewhat pg-continuous if for  $U \in \sigma$  and  $f^{-1}(U) \neq \emptyset$ , there exists a non-empty pg-open set  $V$  in  $X$  such that  $V \subset f^{-1}(U)$ .

It is clear that every continuous function is somewhat continuous and every somewhat continuous is somewhat pg-continuous. But the converses are not true by Example 1 of [8] and the following example.

**Example 4.1:** Let  $X = \{a, b, c\}$ ,  $\tau = \wp(X)$  and  $\sigma = \{\emptyset, \{a\}, \{b, c\}, X\}$ . The function  $f: (X, \tau) \rightarrow (X, \sigma)$  defined by  $f(a) = a, f(b) = c$  and  $f(c) = b$  is somewhat pg-continuous.

**Note 4:** Every somewhat pg-continuous function is slightly pg-continuous

**Theorem 4.1:** If  $f$  is somewhat pg-continuous and  $g$  is continuous, then  $g \circ f$  is somewhat pg-continuous.

#### Corollary 4.1:

- (i) If  $f$  is somewhat pg-continuous and  $g$  is r-continuous, then  $g \circ f$  is somewhat pg-continuous.
- (ii) If  $f$  is somewhat pg-continuous and  $g$  is r-irresolute, then  $g \circ f$  is somewhat pg-continuous.
- (iii) If  $f$  is somewhat p-continuous and  $g$  is r-continuous, then  $g \circ f$  is somewhat pg-continuous.

**Theorem 4.2:** For a surjective function  $f$ , the following statements are equivalent:

- (i)  $f$  is somewhat pg-continuous.
- (ii) If  $C$  is a closed subset of  $Y$  such that  $f^{-1}(C) \neq X$ , then there is a proper pg-closed subset  $D$  of  $X$  such that  $f^{-1}(C) \subset D$ .
- (iii) If  $M$  is a pg-dense subset of  $X$ , then  $f(M)$  is a dense subset of  $Y$ .

**Proof: (i)  $\Rightarrow$  (ii):** Let  $C$  be a closed subset of  $Y$  such that  $f^{-1}(C) \neq X$ . Then  $Y - C$  is an open set in  $Y$  such that  $f^{-1}(Y - C) = X - f^{-1}(C) \neq \emptyset$ . By (i), there exists a pg-open set  $V \in PGO(X)$  such that  $V \neq \emptyset$  and  $V \subset f^{-1}(Y - C) = X - f^{-1}(C)$ . This means that  $X - V \supset f^{-1}(C)$  and  $X - V = D$  is a proper pg-closed set in  $X$ .

**(ii)  $\Rightarrow$  (i):** Let  $U \in \sigma$  and  $f^{-1}(U) \neq \emptyset$ . Then  $Y - U$  is closed and  $f^{-1}(Y - U) = X - f^{-1}(U) \neq X$ . By (ii), there exists a proper pg-closed set  $D$  such that  $D \supset f^{-1}(Y - U)$ . This implies that  $X - D \subset f^{-1}(U)$  and  $X - D$  is pg-open and  $X - D \neq \emptyset$ .

**(ii)  $\Rightarrow$  (iii):** Let  $M$  be a pg-dense set in  $X$ . Suppose that  $f(M)$  is not dense in  $Y$ . Then there exists a proper closed set  $C$  in  $Y$  such that  $f(M) \subset C \subset Y$ . Clearly  $f^{-1}(C) \neq X$ . By (ii), there exists a proper pg-closed set  $D$  such that  $M \subset f^{-1}(C) \subset D \subset X$ . This is a contradiction to the fact that  $M$  is pg-dense in  $X$ .

**(iii)  $\Rightarrow$  (ii):** Suppose (ii) is not true. there exists a closed set  $C$  in  $Y$  such that  $f^{-1}(C) \neq X$  but there is no proper pg-closed set  $D$  in  $X$  such that  $f^{-1}(C) \subset D$ . This means that  $f^{-1}(C)$  is pg-dense in  $X$ . But by (iii),  $f(f^{-1}(C)) = C$  must be dense in  $Y$ , which is a contradiction to the choice of  $C$ .

**Theorem 4.3:** Let  $f$  be a function and  $X = A \cup B$ , where  $A, B \in \tau(X)$ . If the restriction functions  $f_A: (A; \tau_A) \rightarrow (Y, \sigma)$  and  $f_B: (B; \tau_B) \rightarrow (Y, \sigma)$  are somewhat pg-continuous, then  $f$  is somewhat pg-continuous.

**Proof:** Let  $U \in \sigma$  such that  $f^{-1}(U) \neq \emptyset$ . Then  $(f_A)^{-1}(U) \neq \emptyset$  or  $(f_B)^{-1}(U) \neq \emptyset$  or both  $(f_A)^{-1}(U) \neq \emptyset$  and  $(f_B)^{-1}(U) \neq \emptyset$ . Suppose  $(f_A)^{-1}(U) \neq \emptyset$ , Since  $f_A$  is somewhat pg-continuous, there exists a pg-open set  $V$  in  $A$  such that  $V \neq \emptyset$  and  $V \subset (f_A)^{-1}(U) \subset f^{-1}(U)$ . Since  $V$  is pg-open in  $A$  and  $A$  is r-open in  $X$ ,  $V$  is pg-open in  $X$ . Thus  $f$  is somewhat pg-continuous.

The proof of other cases are similar.

**Definition 4.2:** If  $X$  is a set and  $\tau$  and  $\sigma$  are topologies on  $X$ , then  $\tau$  is said to be equivalent[resp: pg- equivalent] to  $\sigma$  provided if  $U \in \tau$  and  $U \neq \emptyset$ , then there is an open[resp:pg-open] set  $V$  in  $X$  such that  $V \neq \emptyset$  and  $V \subset U$  and if  $U \in \sigma$  and  $U \neq \emptyset$ , then there is an open[resp:pg-open] set  $V$  in  $(X, \tau)$  such that  $V \neq \emptyset$  and  $U \supset V$ .

**Definition 4.3:**  $A \subset X$  is said to be pg-dense in  $X$  if there is no proper pg-closed set  $C$  in  $X$  such that  $M \subset C \subset X$ .

Now, consider the identity function  $f$  and assume that  $\tau$  and  $\sigma$  are pg-equivalent. Then  $f$  and  $f^{-1}$  are somewhat pg-continuous. Conversely, if the identity function  $f$  is somewhat pg-continuous in both directions, then  $\tau$  and  $\sigma$  are pg-equivalent.

**Theorem 4.4:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a somewhat pg-continuous surjection and  $\tau^*$  be a topology for  $X$ , which is pg-equivalent to  $\tau$ . Then  $f: (X, \tau^*) \rightarrow (Y, \sigma)$  is somewhat pg-continuous.

**Proof:** Let  $V \in \sigma$  such that  $f^{-1}(V) \neq \emptyset$ . Since  $f$  is somewhat pg-continuous, there exists a nonempty pg-open set  $U$  in  $(X, \tau)$  such that  $U \subset f^{-1}(V)$ . But by hypothesis  $\tau^*$  is pg-equivalent to  $\tau$ . Therefore, there exists a pg-open set  $U^* \in (X, \tau^*)$  such that  $U^* \subset U$ . But  $U \subset f^{-1}(V)$ . Then  $U^* \subset f^{-1}(V)$ ; hence  $f: (X, \tau^*) \rightarrow (Y, \sigma)$  is somewhat pg-continuous.

**Theorem 4.5:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a somewhat pg-continuous surjection and  $\sigma^*$  be a topology for  $Y$ , which is equivalent to  $\sigma$ . Then  $f: (X, \tau) \rightarrow (Y, \sigma^*)$  is somewhat pg-continuous.

**Proof:** Let  $V^* \in \sigma^*$  such that  $f^{-1}(V^*) \neq \emptyset$ . Since  $\sigma^*$  is equivalent to  $\sigma$ , there exists a nonempty open set  $V$  in  $(Y, \sigma)$  such that  $V \subset V^*$ . Now  $\emptyset \neq f^{-1}(V) \subset f^{-1}(V^*)$ . Since  $f$  is somewhat pg-continuous, there exists a nonempty pg-open set  $U$  in  $(X, \tau)$  such that  $U \subset f^{-1}(V)$ . Then  $U \subset f^{-1}(V^*)$ ; hence  $f: (X, \tau) \rightarrow (Y, \sigma^*)$  is somewhat pg-continuous.

## 5. Somewhat pg-open function:

**Definition 5.1:** A function  $f$  is said to be somewhat pg-open provided that if  $U \in \tau$  and  $U \neq \emptyset$ , then there exists a non-empty pg-open set  $V$  in  $Y$  such that  $V \subset f(U)$ .

**Example 5.1:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, \{a\}, X\}$  and  $\sigma = \{\emptyset, \{a\}, \{b, c\}, X\}$ . The function  $f: (X, \tau) \rightarrow (X, \sigma)$  defined by  $f(a) = a, f(b) = c$  and  $f(c) = b$  is somewhat pg-open, somewhat pg-open and somewhat open.

**Example 5.2:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$  and  $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ . The function  $f: (X, \tau) \rightarrow (X, \sigma)$  defined by  $f(a) = c, f(b) = a$  and  $f(c) = b$  is not somewhat pg-open.

**Theorem 5.1:** Let  $f$  be an  $r$ -open function and  $g$  somewhat pg-open. Then  $g \circ f$  is somewhat pg-open.

**Theorem 5.2:** For a bijective function  $f$ , the following are equivalent:

(i)  $f$  is somewhat pg-open.

(ii) If  $C$  is a closed subset of  $X$ , such that  $f(C) \neq Y$ , then there is a pg-closed subset  $D$  of  $Y$  such that  $D \neq Y$  and  $D \supset f(C)$ .

**Proof: (i)  $\Rightarrow$  (ii):** Let  $C$  be any closed subset of  $X$  such that  $f(C) \neq Y$ . Then  $X-C$  is open in  $X$  and  $X-C \neq \emptyset$ .

Since  $f$  is somewhat pg-open, there exists a pg-open set  $V \neq \emptyset$  in  $Y$  such that  $V \subset f(X-C)$ . Put  $D = Y-V$ .

Clearly  $D$  is pg-closed in  $Y$  and we claim  $D \neq Y$ . If  $D = Y$ , then  $V = \emptyset$ , which is a contradiction.

Since  $V \subset f(X-C)$ ,  $D = Y-V \supset (Y-f(X-C)) = f(C)$ .

**(ii)  $\Rightarrow$  (i):** Let  $U$  be any nonempty open subset of  $X$ . Then  $C = X-U$  is a closed set in  $X$  and  $f(X-U) = f(C) = Y-f(U)$  implies  $f(C) \neq Y$ . Therefore, by (ii), there is a pg-closed set  $D$  of  $Y$  such that  $D \neq Y$  and  $f(C) \subset D$ .

Clearly  $V = Y-D$  is a pg-open set and  $V \neq \emptyset$ . Also,  $V = Y-D \subset Y-f(C) = Y-f(X-U) = f(U)$ .

**Theorem 5.3:** The following statements are equivalent:

(i)  $f$  is somewhat pg-open.

(ii) If  $A$  is a pg-dense subset of  $Y$ , then  $f^{-1}(A)$  is a dense subset of  $X$ .

**Proof: (i)  $\Rightarrow$  (ii):** Suppose A is a pg-dense set in Y. If  $f^{-1}(A)$  is not dense in X, then there exists a closed set B in X such that  $f^{-1}(A) \subset B \subset X$ . Since f is somewhat pg-open and X-B is open, there exists a nonempty pg-open set C in Y such that  $C \subset f(X-B)$ . Therefore,  $C \subset f(X-B) \subset f(f^{-1}(Y-A)) \subset Y-A$ . That is,  $A \subset Y-C \subset Y$ . Now, Y-C is a pg-closed set and  $A \subset Y-C \subset Y$ . This implies that A is not a pg-dense set in Y, which is a contradiction. Therefore,  $f^{-1}(A)$  is a dense set in X.

**(ii)  $\Rightarrow$  (i):** Suppose A is a nonempty open subset of X. We want to show that  $\text{pg}(f(A)) \neq \emptyset$ . Suppose  $\text{pg}(f(A)) = \emptyset$ . Then,  $\text{pgcl}\{f(A)\} = Y$ . Therefore, by (ii),  $f^{-1}(Y - f(A))$  is dense in X. But  $f^{-1}(Y - f(A)) \subset X - A$ . Now, X-A is closed. Therefore,  $f^{-1}(Y - f(A)) \subset X - A$  gives  $X = \text{cl}\{f^{-1}(Y - f(A))\} \subset X - A$ . This implies that  $A = \emptyset$ , which is contrary to  $A \neq \emptyset$ . Therefore,  $\text{pg}(f(A)) \neq \emptyset$ . Hence f is somewhat pg-open.

**Theorem 5.4:** Let f be somewhat pg-open and A be any r-open subset of X. Then  $f|_A: (A; \tau|_A) \rightarrow (Y, \sigma)$  is somewhat pg-open.

**Proof:** Let  $U \in \tau|_A$  such that  $U \neq \emptyset$ . Since U is r-open in A and A is r-open in X, U is r-open in X and since by hypothesis f is somewhat pg-open function, there exists a pg-open set V in Y, such that  $V \subset f(U)$ . Thus, for any open set U of A with  $U \neq \emptyset$ , there exists a pg-open set V in Y such that  $V \subset f(U)$  which implies  $f|_A$  is a somewhat pg-open function.

**Theorem 5.5:** Let f be a function and  $X = A \cup B$ , where  $A, B \in \tau(X)$ . If the restriction functions  $f|_A$  and  $f|_B$  are somewhat pg-open, then f is somewhat pg-open.

**Proof:** Let U be any open subset of X such that  $U \neq \emptyset$ . Since  $X = A \cup B$ , either  $A \cap U \neq \emptyset$  or  $B \cap U \neq \emptyset$  or both  $A \cap U \neq \emptyset$  and  $B \cap U \neq \emptyset$ . Since U is open in X, U is open in both A and B.

**Case (i):** Suppose that  $A \cap U \neq \emptyset$ , where  $U \cap A$  is open in A. Since  $f|_A$  is somewhat pg-open function, there exists a pg-open set V of Y such that  $V \subset f(U \cap A) \subset f(U)$ , which implies that f is a somewhat pg-open function.

**Case (ii):** Suppose that  $B \cap U \neq \emptyset$ , where  $U \cap B$  is r-open in B. Since  $f|_B$  is somewhat pg-open function, there exists a pg-open set V in Y such that  $V \subset f(U \cap B) \subset f(U)$ , which implies that f is also a somewhat pg-open function.

**Case (iii):** Suppose that both  $A \cap U \neq \emptyset$  and  $B \cap U \neq \emptyset$ . Then by case (i) and (ii) f is a somewhat pg-open function.

**Remark 3:** Two topologies  $\tau$  and  $\sigma$  for X are said to be pg-equivalent if and only if the identity function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is somewhat pg-open in both directions.

**Theorem 5.6:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a somewhat almost open function. Let  $\tau^*$  and  $\sigma^*$  be topologies for X and Y, respectively such that  $\tau^*$  is equivalent to  $\tau$  and  $\sigma^*$  is pg-equivalent to  $\sigma$ . Then  $f: (X; \tau^*) \rightarrow (Y; \sigma^*)$  is somewhat pg-open.

## 6. Covering and Separation properties of sl.pg.c. Functions:

**Theorem 6.1:** If  $f: X \rightarrow Y$  is sl.pg.c.[resp: sl.r.c] surjection and X is pg-compact, then Y is compact.

**Proof:** Let  $\{G_i; i \in I\}$  be any open cover for Y. Then each  $G_i$  is open in Y and hence each  $G_i$  is clopen in Y.

Since  $f: X \rightarrow Y$  is sl.pg.c.,  $f^{-1}(G_i)$  is pg-open in X. Thus  $\{f^{-1}(G_i)\}$  forms a pg-open cover for X and hence have a finite subcover, since X is pg-compact. Since f is surjection,  $Y = f(X) = \cup_{i=1}^n G_i$ . Therefore Y is compact.

**Corollary 6.1:** If  $f: X \rightarrow Y$  is sl.p.c.[resp: sl.r.c] surjection and X is pg-compact, then Y is compact.

**Theorem 6.2:** If  $f: X \rightarrow Y$  is sl.pg.c., surjection and X is pg-compact[pg-lindeloff] then Y is mildly compact[mildly lindeloff].

**Proof:** Let  $\{U_i; i \in I\}$  be clopen cover for Y. For each x in X,  $\exists \alpha_x \in I$  such that  $f(x) \in U_{\alpha_x}$  and  $\exists V_x \in PGO(X, x)$  such that  $f(V_x) \subset U_{\alpha_x}$ . Since the family  $\{V_i; i \in I\}$  is a cover of X by pg-open sets of X,  $\exists$  a finite subset  $I_0$  of I such that  $X \subset \{V_x; x \in I_0\}$ . Therefore  $Y \subset \cup \{f(V_x); x \in I_0\} \subset \cup \{U_{\alpha_x}; x \in I_0\}$ . Hence Y is mildly compact.

**Corollary 6.2:**

(i) If  $f: X \rightarrow Y$  is sl.r.c.[resp: sl.p.c.; sl.c] surjection and  $X$  is *pg*-compact[*pg*-lindeloff] then  $Y$  is mildly compact[mildly lindeloff].

(ii) If  $f: X \rightarrow Y$  is sl.pg.c.[resp: sl.r.c; sl.p.c.; sl.c] surjection and  $X$  is locally *pg*-compact{resp:*pg*-Lindeloff; locally *pg*-lindeloff}, then  $Y$  is locally compact{resp: Lindeloff; locally lindeloff}.

(iii) If  $f: X \rightarrow Y$  is sl.pg.c.[sl.r.c.], surjection and  $X$  is locally *pg*-compact{resp: *pg*-lindeloff; locally *pg*-lindeloff} then  $Y$  is locally mildly compact{resp: locally mildly lindeloff}.

**Theorem 6.3:** If  $f: X \rightarrow Y$  is sl.pg.c., surjection and  $X$  is *s*-closed then  $Y$  is mildly compact[mildly lindeloff].

**Proof:** Let  $\{V_i : V_i \in CO(Y); i \in I\}$  be a cover of  $Y$ , then  $\{f^{-1}(V_i) : i \in I\}$  is *pg*-open cover of  $X$ [by Thm 3.1] and so there is finite subset  $I_0$  of  $I$ , such that  $\{f^{-1}(V_i); i \in I_0\}$  covers  $X$ . Therefore  $\{V_i; i \in I_0\}$  covers  $Y$  since  $f$  is surjection. Hence  $Y$  is mildly compact.

**Corollary 6.3:** If  $f: X \rightarrow Y$  is sl.r.c.[resp: sl.p.c.; sl.c] surjection and  $X$  is *s*-closed then  $Y$  is mildly compact[mildly lindeloff].

**Theorem 6.4:** If  $f: X \rightarrow Y$  is sl.pg.c.,[resp: sl.r.c.; sl.p.c.; sl.c] surjection and  $X$  is *pg*-connected, then  $Y$  is connected.

**Proof:** If  $Y$  is disconnected, then  $Y = A \cup B$  where  $A$  and  $B$  are disjoint clopen sets in  $Y$ . Since  $f$  is sl.pg.c. surjection,  $X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B)$  where  $f^{-1}(A) f^{-1}(B)$  are disjoint *pg*-open sets in  $X$ , which is a contradiction for  $X$  is *pg*-connected. Hence  $Y$  is connected.

**Corollary 6.4:** The inverse image of a disconnected space under a sl.pg.c.,[resp: sl.r.c.; sl.p.c.; sl.c] surjection is *pg*-disconnected.

**Theorem 6.5:** If  $f: X \rightarrow Y$  is sl.pg.c.,[resp: sl.r.c.; sl.p.c.], injection and  $Y$  is  $UT_i$ , then  $X$  is  $pg_i; i = 0, 1, 2$ .

**Proof:** Let  $x_1 \neq x_2 \in X$ . Then  $f(x_1) \neq f(x_2) \in Y$  since  $f$  is injective. For  $Y$  is  $UT_2 \exists V_j \in CO(Y)$  such that  $f(x_j) \in V_j$  and  $\cap V_j = \phi$  for  $j = 1, 2$ . By Theorem 3.1,  $x_j \in f^{-1}(V_j) \in PGO(X)$  for  $j = 1, 2$  and  $\cap f^{-1}(V_j) = \phi$  for  $j = 1, 2$ . Thus  $X$  is  $pg_2$ .

**Theorem 6.6:** If  $f: X \rightarrow Y$  is sl.pg.c.[resp: sl.r.c.; sl.p.c.], injection; closed and  $Y$  is  $UT_i$ , then  $X$  is  $pgg_i; i = 3, 4$ .

**Proof:**(i) Let  $x$  in  $X$  and  $F$  be disjoint closed subset of  $X$  not containing  $x$ , then  $f(x)$  and  $f(F)$  be disjoint closed subset of  $Y$  not containing  $f(x)$ , since  $f$  is closed and injection. Since  $Y$  is ultraregular,  $f(x)$  and  $f(F)$  are separated by disjoint clopen sets  $U$  and  $V$  respectively. Hence  $x \in f^{-1}(U); F \subseteq f^{-1}(V), f^{-1}(U); f^{-1}(V) \in PGO(X)$  and  $f^{-1}(U) \cap f^{-1}(V) = \phi$ . Thus  $X$  is  $pgg_3$ .

(ii) Let  $F_j$  and  $f(F_j)$  are disjoint closed subsets of  $X$  and  $Y$  respectively for  $j = 1, 2$ , since  $f$  is closed and injection. For  $Y$  is ultranormal,  $f(F_j)$  are separated by disjoint clopen sets  $V_j$  respectively for  $j = 1, 2$ .

Hence  $F_j \subseteq f^{-1}(V_j)$  and  $f^{-1}(V_j) \in PGO(X)$  and  $\cap f^{-1}(V_j) = \phi$  for  $j = 1, 2$ . Thus  $X$  is  $pgg_4$ .

**Theorem 6.7:** If  $f: X \rightarrow Y$  is sl.pg.c.[resp: sl.r.c.; sl.p.c.], injection and

(i)  $Y$  is  $UC_i$ [resp:  $UD_i$ ] then  $X$  is  $pgC_i$ [resp:  $pgD_i$ ]  $i = 0, 1, 2$ .

(ii)  $Y$  is  $R_i$ , then  $X$  is  $pg-R_i; i = 0, 1$ .

**Theorem 6.8:** If  $f: X \rightarrow Y$  is sl.pg.c.[resp: sl.p.c.; sl.r.c; sl.c] and  $Y$  is  $UT_2$ , then the graph  $G(f)$  of  $f$  is *pg*-closed in the product space  $X \times Y$ .

**Proof:** Let  $(x_1, x_2) \notin G(f)$  implies  $y \neq f(x)$  implies  $\exists$  disjoint  $V; W \in CO(Y)$  such that  $f(x) \in V$  and  $y \in W$ . Since  $f$  is sl.pg.c.,  $\exists U \in PGO(X)$  such that  $x \in U$  and  $f(U) \subseteq W$  and  $(x, y) \in U \times V \subseteq X \times Y - G(f)$ . Hence  $G(f)$  is *pg*-closed in  $X \times Y$ .

**Theorem 6.9:** If  $f: X \rightarrow Y$  is sl.pg.c.[resp: sl.p.c.; sl.r.c; sl.c] and  $Y$  is  $UT_2$ , then  $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$  is *pg*-closed in the product space  $X \times X$ .

**Proof:** If  $(x_1, x_2) \in X \times X - A$ , then  $f(x_1) \neq f(x_2)$  implies  $\exists$  disjoint  $V_j \in CO(Y)$  such that  $f(x_j) \in V_j$ , and since  $f$  is sl. pg. c.,  $f^{-1}(V_j) \in PGO(X, x_j)$  for  $j = 1, 2$ . Thus  $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in PGO(X \times X)$  and  $f^{-1}(V_1) \times f^{-1}(V_2) \subseteq X \times X - A$ . Hence  $A$  is *pg*-closed.

**Theorem 6.10:** If  $f: X \rightarrow Y$  is  $sl.r.c.$ [resp:  $sl.c.$ ];  $g: X \rightarrow Y$  is  $sl.pg.c.$ [resp:  $sl.r.c.$ ;  $sl.p.c.$ ]; and  $Y$  is  $UT_2$ , then  $E = \{x \text{ in } X : f(x) = g(x)\}$  is  $pg$ -closed in  $X$ .

Following definitions 3.1; 4.1 and Note 4, we have the following consequences of theorems 6.1 to 6.10:

**Theorem 6.11:** If  $f: X \rightarrow Y$  is  $swt.pg.c.$ [resp:  $swt.p.c.$ ;  $swt.r.c.$ ] surjection and  $X$  is  $pg$ -compact, then  $Y$  is compact.

**Corollary 6.5:** If  $f: X \rightarrow Y$  is  $swt.p.c.$ [resp:  $swt.r.c.$ ] surjection and  $X$  is  $pg$ -compact, then  $Y$  is compact.

**Theorem 6.12:** If  $f: X \rightarrow Y$  is  $swt.pg.c.$ , surjection and  $X$  is  $pg$ -compact[ $pg$ -lindeloff] then  $Y$  is mildly compact[mildly lindeloff].

**Corollary 6.6:**

(i) If  $f: X \rightarrow Y$  is  $swt.r.c.$ [resp:  $swt.p.c.$ ;  $swt.c.$ ] surjection and  $X$  is  $pg$ -compact[ $pg$ -lindeloff] then  $Y$  is mildly compact[mildly lindeloff].

(ii) If  $f: X \rightarrow Y$  is  $swt.pg.c.$ [resp:  $swt.r.c.$ ;  $swt.p.c.$ ;  $swt.c.$ ] surjection and  $X$  is locally  $pg$ -compact{resp: $pg$ -Lindeloff; locally  $pg$ -lindeloff}, then  $Y$  is locally compact{resp: Lindeloff; locally lindeloff}.

(iii) If  $f: X \rightarrow Y$  is  $swt.pg.c.$ [ $swt.r.c.$ ], surjection and  $X$  is locally  $pg$ -compact{resp:  $pg$ -lindeloff; locally  $pg$ -lindeloff} then  $Y$  is locally mildly compact{resp: locally mildly lindeloff}.

**Theorem 6.13:** If  $f: X \rightarrow Y$  is  $swt.pg.c.$ , surjection and  $X$  is  $s$ -closed then  $Y$  is mildly compact[mildly lindeloff].

**Corollary 6.7:** If  $f: X \rightarrow Y$  is  $swt.r.c.$ [resp:  $swt.p.c.$ ;  $swt.c.$ ] surjection and  $X$  is  $s$ -closed then  $Y$  is mildly compact[mildly lindeloff].

**Theorem 6.14:** If  $f: X \rightarrow Y$  is  $swt.pg.c.$ ,[resp:  $swt.r.c.$ ;  $swt.p.c.$ ;  $swt.c.$ ] surjection and  $X$  is  $pg$ -connected, then  $Y$  is connected.

**Corollary 6.8:** The inverse image of a disconnected space under a  $swt.pg.c.$ ,[resp:  $swt.r.c.$ ;  $swt.p.c.$ ;  $swt.c.$ ] surjection is  $pg$ -disconnected.

**Theorem 6.15:** If  $f: X \rightarrow Y$  is  $swt.pg.c.$ ,[resp:  $swt.r.c.$ ;  $swt.p.c.$ ], injection and  $Y$  is  $UT_i$ , then  $X$  is  $pg_i$   $i = 0, 1, 2$ .

**Theorem 6.16:** If  $f: X \rightarrow Y$  is  $swt.pg.c.$ ,[resp:  $swt.r.c.$ ;  $swt.p.c.$ ], injection; closed and  $Y$  is  $UT_i$ , then  $X$  is  $pgg_i$   $i = 3, 4$ .

**Theorem 6.17:** If  $f: X \rightarrow Y$  is  $swt.pg.c.$ ,[resp:  $swt.r.c.$ ;  $swt.p.c.$ ], injection and

(i)  $Y$  is  $UC_i$ [resp:  $UD_i$ ] then  $X$  is  $pgC_i$ [resp:  $pgD_i$ ]  $i = 0, 1, 2$ . (ii)  $Y$  is  $R_i$ , then  $X$  is  $pg-R_i$   $i = 0, 1$ .

**Theorem 6.18:** If  $f: X \rightarrow Y$  is  $swt.pg.c.$ ,[resp:  $swt.p.c.$ ;  $swt.r.c.$ ;  $swt.c.$ ] and  $Y$  is  $UT_2$ , then

(i) the graph  $G(f)$  of  $f$  is  $pg$ -closed in the product space  $X \times Y$ .

(ii)  $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$  is  $pg$ -closed in the product space  $X \times X$ .

**Theorem 6.19:** If  $f: X \rightarrow Y$  is  $swt.r.c.$ ,[resp:  $swt.c.$ ];  $g: X \rightarrow Y$  is  $swt.pg.c.$ ,[resp:  $swt.r.c.$ ;  $swt.p.c.$ ]; and  $Y$  is  $UT_2$ , then  $E = \{x \text{ in } X : f(x) = g(x)\}$  is  $pg$ -closed in  $X$ .

**CONCLUSION**

In this paper we defined slightly- $pg$ - continuous; somewhat- $pg$ -continuous and somewhat- $pg$ -open functions, studied its properties and their interrelations with other types of such continuous functions.

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