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EXISTENCE OF SOLUTIONS FOR THREE POINT BVP ON TIME SCALES

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ABSTRACT

In this paper we shall be concerned with the existence of solutions for three point boundary value problems associated with a system of first order differential equations on time scales.

Key words: *Time scale, boundary value problem, dynamical equation.*

AMS Subject Classification: *34B99, 39A99.*

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1. INTRODUCTION

The theory of time scales, which has received a lot of attention, was introduced by Hilger [9] in order to unify the theory on continuous and discrete analysis. Their unifies the theories of differential equation and difference equations, and also it is able to extend these classical cases to case in between, and can be applied on different types of time scales. A time scales T is an arbitrary closed subset of the real, and the cases when this time scale is equal to the real or to the integers that represent the classical theories of differential and of difference equations. Many other interesting time scales exist, and they give rise to plenty of applications in the study of populations dynamic models. The book on the subject of time scales by Bohner and Peterson [5,6], summarizes and organizes much of the time scale calculus.

In this paper we consider the following three-point boundary value problem

$$P(\sigma(t))y^{\Delta}+Q(t)y=f(t) \tag{1}$$

Satisfying the general boundary conditions

$$My(a)+Ny(b)+Ry(\sigma(c))=0,\ (a<b<\sigma(c)) \tag{2}$$

Where $P(\sigma(t))$, is a matrix of order $m\times n$ whose components are regressive and rd-continuous on $[a,\sigma(c)],Q(t)$ is a matrix of order $m\times n$ whose components are rd-continuous on $[a,\sigma(c)],M,N$ and R being constant $m\times n$ matrices and f is a column matrix whose components $f_1, f_2\dots f_n$ are rd-continuous on $[a,\sigma(c)]$. Throughout this paper we assume that the rows of $P(\sigma(t))$ are linearly independent on $[a,\sigma(c)]$, also assume that $\sigma(c)$ is right dense.

This paper is organized as follows. In Section 2, we develop existence and uniqueness of solutions for three point boundary value problems associated with first order differential system on time scale. Finally we give an example to demonstrate our result.

Definition 1.1: If $P^TY(t)$ is a fundamental matrix for the corresponding equation $P(\sigma(t))y^{\Delta}+Q(t)y=0$ then the matrix D is defined by

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$D = MP^T Y(a) + NP^T Y(b) + RP^T Y(\sigma(c))$ is called a characteristic matrix for the boundary value problem given by (1) and (2).

2. THREE POINT BVP ON TIME SCALES

In this section we establish existence and uniqueness of solutions to three point boundary value problems associated with the system of first order matrix time scale differential equation (1) satisfying the boundary conditions (2) and also study the properties of the Green's matrix.

Lemma 2.1: If the system of equation $Ax = B$ where A is $m \times n$ matrix whose rows are linearly independent is consistent then three exists a unique solutions to $Ax = B$ and is given by $x = A^T\left(AA^T\right)^{-1} b$.

Proof: For the transformation $x = A^T y$ transforms $Ax = B$ into a system of equations $AA^T y = b$. Since AA^T is non singular, it follows that $y = \left(AA^T\right)^{-1} b$ and hence $x = A^T\left(AA^T\right)^{-1} b$.

Theorem 2.2: Any solution of the system first order time scale differential equation

$P(\sigma(t))y^\Delta + Q(t)y = 0$ is of the form $P^T Y(t)C$, where $Y(t)$ is a fundamental matrix of $z^\Delta = -\left[A^{-1}(t)B(t)\right]z$, where $A(t) = P(\sigma(t))P^T(\sigma(t))$ and $B(t) = P(\sigma(t))\left(P^T(t)\right)^\Delta + Q(t)P^T(t)$.

Proof: The transformation $y = P^T z$ transforms

$$P(\sigma(t))\left(P^T z\right)^\Delta + Q(t)\left(P^T z\right) = 0$$

$$P(\sigma(t))\left[P^{T^\Delta} z(t) + P^T(\sigma(t))z^\Delta(t)\right] + Q(t)\left(P^T z\right) = 0$$

$$\left[P(\sigma(t))P^T(\sigma(t))\right]z^\Delta(t) + \left[P(\sigma(t))\left(P^{T^\Delta} + Q(t)P^T\right)\right]z(t) = 0$$

$$Az^\Delta(t) + Bz(t) = 0$$

Where $A(t) = P(\sigma(t))P^T(\sigma(t))$ and $B(t) = P(\sigma(t))\left(P^T(t)\right)^\Delta + Q(t)P^T(t)$.

Since A is nonsingular the above equation becomes $z^\Delta(t) = -\left[A^{-1}(t)B(t)\right]z(t)$. Any solution of $z^\Delta(t) = -\left[A^{-1}(t)B(t)\right]z(t)$ is of the form $Y(t)C$, where C is constant vector and $Y(t)$ is a fundamental matrix. Hence, any solution of (1) is of the form $P^T Y(t)C$.

Theorem 2.3 Any solution of the system first order time scale differential equation (1) is of the form $y(t) = P^T Y(t)C + \overline{y}(t)$, where $\overline{y}(t)$ is a particular solution of and C is an n-vector that belongs to vector space V_n (R).

Proof: It can be easily verified that $y(t) = P^T Y(t)C + \overline{y}(t)$ is a solution of (1). Now we prove that every solution is of that form. Let $y(t)$ be any solution of (1) and $\overline{y}(t)$ be a particular solution of (1). Then it can easily be verified that $y(t) - \overline{y}(t)$ is a solution of $P(\sigma(t))y^\Delta + Q(t)y = 0$. Hence, we have $y(t) - \overline{y}(t) = P^T Y(t)C$ or $y(t) = P^T Y(t)C + \overline{y}(t)$.

Theorem 2.4: A particular solution $\overline{y}(t)$ of (1) is of the form

$$\overline{y}(t)=P^TY(t)\int_a^t Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1} f(s)\Delta s\cdot$$

Proof: Using the well known method of variation of parameters formula, the particular solution $\overline{y}(t)$ of (1) is of the form $\overline{y}(t)=P^TY(t)k(t)$, where $k(t)$ is an $n-$vector whose components are rd-continuous on T.

Then,

$$P(\sigma(t))\left[P^TY(t)k(t)\right]^{\Delta}+Q(t)P^TY(t)k(t)=f(t)$$

$$P(\sigma(t))\left[P^{T^{\Delta}}Y(t)k(t)+P^T(\sigma(t))Y^{\Delta}(t)k(t)+P^T(\sigma(t))Y(\sigma(t))k^{\Delta}(t)\right]+Q(t)P^TY(t)k(t)=f(t)$$

$$\left[P(\sigma(t))P^{T^{\Delta}}Y(t)+P(\sigma(t))P^T(\sigma(t))Y^{\Delta}(t)+Q(t)P^TY(t)k(t)\right]k(t)$$
$$+P(\sigma(t))P^T(\sigma(t))Y(\sigma(t))k^{\Delta}(t)=f(t)$$

$$k^{\Delta}(t)=\left[P(\sigma(t))P^T(\sigma(t))Y(\sigma(t))\right]^{-1} f(t)\cdot$$

Hence,

$$k^{\Delta}(t)=Y^{-1}(\sigma(t))\left[P(\sigma(t))P^T(\sigma(t))\right]^{-1} f(t)$$

$$k(t)=\int_a^t Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1} f(s)\Delta s$$

$$\overline{y}(t)=P^TY(t)\int_a^t Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1} f(s)\Delta s$$

the general solution of (1) is given by

$$y(t)=P^TY(t)C+P^TY(t)\int_a^t Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1} f(s)\Delta s$$

Theorem 2.5: Suppose that the homogeneous boundary value problem corresponding to (1) is incompatible. Then there exists a unique solution to the boundary value problem (1) satisfying (2) is given by

$$y(t)=P^TY(t)D^{-1}\alpha+\int_a^{\sigma(c)} G(t,s)f(s)\Delta s \tag{3}$$

Where $G(t,s)$ is the Green's matrix for the corresponding homogeneous boundary value problem.

Proof: From Theorem 3.3, any solution of (1) is of the form.

$$y(t)=P^TY(t)C+P^TY(t)\int_a^t Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1} f(s)\Delta s$$

Where $P^TY(t)$ is a fundamental matrix for the equation and C is is a constant $n-$vector and will be determined uniquely from the fact that the solution $y(t)$ must satisfy the boundary conditions substituting the general form of $y(t)$ in the boundary condition matrix we get.

$$\left[MP^TY(a)+NP^TY(b)+RP^TY(\sigma(c))\right]C+NP^TY(b)\int_a^b Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1} f(s)\Delta s$$
$$+RP^TY(\sigma(c))\int_a^{\sigma(c)} Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1} f(s)\Delta s=\alpha,$$

and thus

$$C = D^{-1}\alpha - D^{-1}NP^T Y(b)\int_a^b Y^{-1}(\sigma(s))\Big[P(\sigma(s))P^T(\sigma(s))\Big]^{-1} f(s)\Delta s\cdot$$
$$-D^{-1}RP^T Y(\sigma(c))\int_a^{\sigma(c)} Y^{-1}(\sigma(s))\Big[P\big(\sigma(s)P^T(\sigma(s))\big)\Big]^{-1} f(s)\Delta s\Big],$$

where D is a characteristic matrix for the boundary value problem.

Using the value of C then the solution of the three point boundary value problem is given by

$$y(t) = P^T Y(t) D^{-1}\alpha + \int_a^{\sigma(c)} G(t,s)\, f(s)\Delta s$$

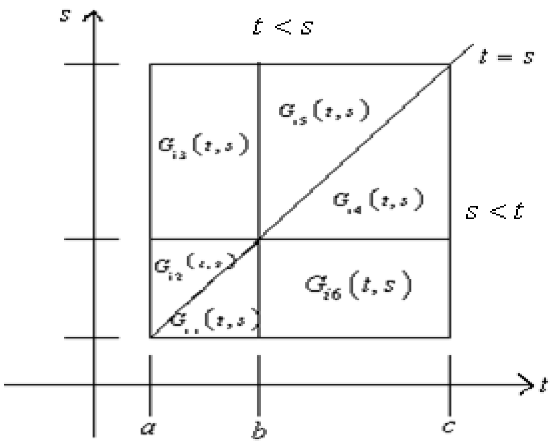
Where

$$\begin{matrix} G(t,s) \\ t\in[a,b] \end{matrix} = \begin{cases} G_{11}(t,s), & a<\sigma(s)<t\le b<\sigma(c) \\ G_{12}(t,s), & a\le t<s<b<\sigma(c) \\ G_{13}(t,s), & a<t<b<s<\sigma(c) \end{cases} \tag{4}$$

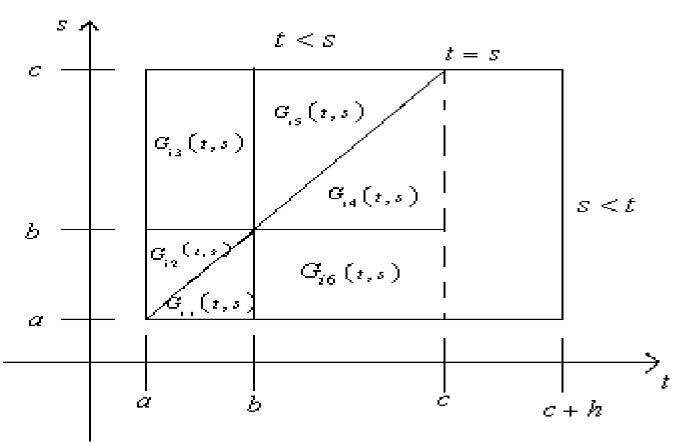
$$\begin{matrix} G(t,s) \\ t\in[b,\sigma(c)] \end{matrix} = \begin{cases} G_{21}(t,s), & a<b<\sigma(s)<t\le\sigma(c) \\ G_{22}(t,s), & a<b\le t<s<\sigma(c) \\ G_{23}(t,s), & a<\sigma(s)<b<t<\sigma(c) \end{cases} \tag{5}$$

$$G_{11}(t,s) = \Big[P^T Y(t) D^{-1} M P^T Y(a) Y^{-1}(\sigma(s))\Big]\Big[P(\sigma(s))P^T(\sigma(s))\Big]^{-1}$$
$$G_{12}(t,s) = \Big[-P^T Y(t) D^{-1} N P^T Y(b) Y^{-1}(\sigma(s)) - P^T Y(t) D^{-1} R P^T Y(\sigma(c)) Y^{-1}(\sigma(s))\Big]\Big[P(\sigma(s))P^T(\sigma(s))\Big]^{-1}$$
$$G_{13}(t,s) = \Big[-P^T Y(t) D^{-1} R Y^{-1}(\sigma(c)) Y^{-1}(\sigma(s))\Big]\Big[P(\sigma(s))P^T(\sigma(s))\Big]^{-1}$$
$$G_{21}(t,s) = \Big[P^T Y(t) Y^{-1}(\sigma(s)) - P^T Y(t) D^{-1} R P^T Y(\sigma(c)) Y^{-1}(\sigma(s))\Big]\Big[P(\sigma(s))P^T(\sigma(s))\Big]^{-1}$$
$$G_{22}(t,s) = \Big[-P^T Y(t) D^{-1} R P^T Y(\sigma(c)) Y^{-1}(\sigma(s))\Big]\Big[P(\sigma(s))P^T(\sigma(s))\Big]^{-1}$$
$$G_{23}(t,s) = \Big[P^T Y(t) D^{-1} M P^T Y(a) Y^{-1}(\sigma(s))\Big]\Big[P(\sigma(s))P^T(\sigma(s))\Big]^{-1}$$

The following graph demonstates that the Green's matrix for (1)-(2) should be taken in the form of (4)-(5). Here $s\in[a,\sigma(c)]$. In the graph we take $\sigma(s)=s_1$ and $\sigma(c)=c_1$.



$T = R$



$T = hZ, h > 0$

Theorem 2.6: The Green's matrix has the following properties.

(1). $G(t,s)$ as a function of t with fixed s and has continuous delta derivatives everywhere except at $t=\sigma(s)$. At the point $t=\sigma(s)$, $G(t,s)$ has an upward jump discontinuity $P^T\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1}$

i.e. $G\left((\sigma(s))^+,s\right)-G\left(\sigma(s)^-,s\right)=P^T\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1}$.

(2). $G(t,s)$ is a formal solution of the homogeneous boundary value problem satisfying. G fails to be a true solution because of the discontinuity at $t=\sigma(s)$.

(3). $G(t,s)$ satisfying the properties (1) and (2) is unique.

Proof:

$$
\begin{aligned}
G\left((\sigma(s))^+,s\right)-G\left((\sigma(s))^-,s\right) &= \Big[P^TY((s))D^{-1}MP^TY(a)+P^TY(\sigma(s))D^{-1}NP^TY(b)\\
&\quad +P^TY(\sigma(s))D^{-1}RP^TY(\sigma(c))\Big]Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1}\\
&= P^TY(\sigma(s))D^{-1}\left[MP^TY(a)+NP^TY(b)+RP^TY(\sigma(c))\right]\\
&\quad Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1}\\
&= P^T\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1}
\end{aligned}
$$

The representation of $G(t,s)$ by (4) and (5) shows that $G(t,s)$ is a formal solution of $P(\sigma(t))y^{\Delta}+Q(t)y(t)=0$. It fails to be a true solution because of discontinuity at $t=\sigma(s)$. $G(t,s)$ satisfies the boundary conditions. For

$$
\begin{aligned}
&MG(a,s)+NG(b,s)+RG(\sigma(c),s),\left[s\in[a,b]\right]\\
&= \Big[-MP^TY(a)D^{-1}NP^TY(b)-MP^TY(a)D^{-1}RP^TY(\sigma(c))\\
&\qquad +NP^TY(b)D^{-1}MP^TY(a)+RP^TY(\sigma(c))D^{-1}MP^TY(a)\Big]\\
&\qquad Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1}\\
&= \Big[-MP^TY(a)D^{-1}\left\{NP^TY(b)+RP^TY(\sigma(c))\right\}\\
&\qquad +\left\{NP^TY(b)+RP^TY(\sigma(c))\right\}D^{-1}MP^TY(a)\Big]Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1}\\
&= \Big[-MP^TY(a)D^{-1}\left\{D-MP^TY(a)\right\}+\left\{D-MP^TY(a)\right\}D^{-1}MP^TY(a)\Big]\\
&\qquad Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1}\\
&= \Big[-MP^TY(a)+MP^TY(a)D^{-1}MP^T(a)+MP^TY(a)-MP^TY(a)D^{-1}MY(a)\Big]\\
&\qquad Y^{-1}(\sigma(s))\left[P(\sigma(s))P^T(\sigma(s))\right]^{-1}\\
&= 0
\end{aligned}
$$

Similarly when $s \in [b, \sigma(c)], MG(a,s) + NG(b,s) + RG(\sigma(c), s) = 0$. For $t \in [a,b]$, we can prove that G is unique with properties (1) and (2). Let $G_1(t,s)$ be another Green's matrix with properties (1) and (2). Let $X(t,s) = G(t,s) - G_1(t,s)$. At $t = \sigma(s)$, we have

$$X\left(\sigma(s)^+, s\right) - X\left(\sigma(s)^-, s\right) = G\left(\sigma(s)^+, s\right) - G_1\left(\sigma(s)^-, s\right) - G\left(\sigma(s)^+, s\right) + G_1\left(\sigma(s)^-, s\right) = 0$$

Thus $X(t,s)$ has a removable discontinuity at $t = \sigma(s)$.

Also

$$\begin{aligned} &MX(a,s) + NX(b,s) + RX(\sigma(c), s) \\ &\quad = M\left[G(a,s) - G(a,s)\right] + N\left[G(b,s) - G_1(b,s)\right] + R\left[G(\sigma(c), s) - G_1(\sigma(c), s)\right] \\ &\quad = 0 \end{aligned}$$

Thus for each fixed $s, X(t,s)$ is solution of the homogeneous boundary value problem. Therefore $X(t,s) = P^T Y(t) C$. By our assumption the homogenous boundary value problem is incompatible, hence D is nonsingular. Therefore $C = 0 \Rightarrow X = 0$. Hence $G(t,s) = G_1(t,s)$

Similarly for $t \in [b, \sigma(c)]$, we can prove that G is unique with properties (1) and (2).

Example: The following example illustrates the above results.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} y^{\Delta} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} y = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \forall t \in [0, \sigma(1)] \tag{6}$$

with the boundary conditions

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} y(0) + \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} y(1/2) + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} y(\sigma(1)) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

Here

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Hence the transfer matrix $y = P^T z$ transforms the problem into

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} z^{\Delta} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} z = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Now consider homogeneous problem

$$z^{\Delta} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} z$$

The fundamental matrix of the above problem is

$$Y(t) = \begin{pmatrix} 1 & 0 & e_{-1}(t,0) \\ e_1(t,0) & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\left[P(\sigma(t)) P^T(\sigma(t)) \right]^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The characteristic matrix D is

$$\begin{pmatrix} e_1(1/2,0) & 0 & 1 \\ 1 & -1 & 0 \\ e_1(\sigma(1),0) & 0 & 0 \end{pmatrix}$$

Hence the solution of the above problem is given by

$$\int_0^{\sigma(1)} G(t,s) f(s) \Delta s$$

$$\begin{matrix} G(t,s) \\ t \in [0,1/2] \end{matrix} = \begin{cases} \begin{pmatrix} A_1 & A_2 & 0 \\ 0 & 0 & 0 \\ 0 & A_3 & 0 \end{pmatrix}, & 0 < \sigma(s) < t \le 1/2 < \sigma(1) \\ \begin{pmatrix} 0 & A_2 & 0 \\ 0 & A_4 & 0 \\ 0 & A_3 & 1 \end{pmatrix}, & 0 \le t < s < 1/2 < \sigma(1) \\ \begin{pmatrix} 0 & A_5 & 0 \\ 0 & A_4 & 0 \\ 0 & A_3 & 1 \end{pmatrix}, & 0 < t < 1/2 < s < \sigma(1) \end{cases}$$

$$G(t,s) \quad t\in\left[1/2,\sigma(1)\right] = \begin{cases} \begin{pmatrix} A_1 & A_5 & 0 \\ 0 & 0 & 0 \\ 0 & A_3 & 0 \end{pmatrix}, & 0<1/2<\sigma(s)<t<\sigma(1) \\ \begin{pmatrix} 0 & A_5 & 0 \\ 0 & A_6 & 0 \\ 0 & A_3 & 1 \end{pmatrix}, & 0<1/2\le t<s<\sigma(1) \\ \begin{pmatrix} A_1 & A_2 & 0 \\ 0 & 0 & 0 \\ 0 & A_3 & 0 \end{pmatrix}, & 0<s<t\le 1/2<\sigma(1) \end{cases}$$

Where $A_1 = e_{-1}(t,\sigma(s)), A_2 = -e_1(t,0)e_1(1/2,\sigma(s)) + e_{-1}(t,0)e_1(1/2,\sigma(s))$

$A_3 = e_1(0,\sigma(s)), A_4 = -e_1(t,\sigma(s)), A_5 = e_{-1}(t,0)e_1(1/2,\sigma(s)), A_6 = -e_1(t,\sigma(s)).$

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