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NEIGHBORHOODS OF DIRECTED SIMPLEX GRAPH

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ABSTRACT

In this paper we will compute first and second neighborhood for directed simplex graph.

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1. DEFINITIONS AND BACKGROUND

(1) Abstract graph: An abstract graph G is a diagram consisting of a finite non empty set of the elements, called "vertices" denoted by V (G) together with a set of unordered pairs of these elements, called "edges" denoted by E(G). The set of vertices of the graph G is called "the vertex -set of G" and the list of edges is called "the edge -list of G"[1, 2].

(2) Simplex: Given any set $V = \{v_0, v_1, \ldots, v_n\}$ of n + 1 points in R^n; such that the differences $v_1 - v_0, v_2 - v_0, \ldots, V_n - v_0$ are linearly independent, the n-simplex with vertices V is the convex hull of V, i.e. the set of all points of the form $t_0 v_0 + t_1 v_1 + \ldots\ldots + t_n v_n$, where
$\sum_{i=0}^{n} t_i = 1$ and $t_i \geq 0$ for all i [3].

(3) Degree of vertex: Let G be an undirected graph or multi-graph. For each vertex of G, the degree of v, written deg (v), is the number of edges in G that are incident with v [5].

(4)Regular graphs: A graph G is said to be regular when the degree of its vertices are all equal [4].

(5) Path graph: A graph $G = (X; E)$ where $X = \{x_0, x_1, \ldots\ldots, x_k\}$ and $E = \{e_1, e_2, \ldots.., e_k\}$ so that $(x_0, e_1, x_1, \ldots.., e_k, x_k)$ is a path of G [2].

(6) directed and undirected graph: A directed graph or digraph is consists of two finite sets, a set V (G) of vertices and a set D(G) of directed edges, where each is associated with the pair of vertices called its endpoints, If edge e is associated with the pair (v, w) of vertices, then e is said to be the (directed) edge from v to w . Undirected graph is each graph has an associated ordinary graph which is obtained by ignoring the directions of the edges.

(7) Indegree and outdegree: Let $G = (V, E)$ be a directed graph, for each $v \in V$:

(a) The indegree of v is the number of edges in G that are incident into v, and this denoted by id(v).
(b)The outdegree of v is the number of edges in G that are incident from v, and this denoted by od(v).[6]

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2. MAIN RESULT

We will introduce the follows definitions :

Definition1: First neighborhood of vertex V_i on the n-simplex graph G denoted by $N^1(V_i)$ is the set of all vertices adjacent to V_i by one vertex where $i = 0, 1, 2,\ldots\ldots, n$.

Definition 2: Second neighborhood of vertex V_i on the n-simplex graph G denoted by $N^2(V_i)$ is the set of all vertices adjacent to V_i by path of length two , where $i = 0, 1, 2,\ldots\ldots, n$.

Definition 3: First neighborhood of edge e_i on the n-simplex graph G denoted by $N^1(e_i)$ is the set of all edges connect e by one edge , where $i = 0, 1, 2,\ldots.., n$.

Definition 4: Second neighborhood of edge e_i on the n-simplex graph G denoted by $N^2(e_i)$ is the set of all edges connect e_i by path of length two ,where $i = 0, 1, 2,\ldots., n$.

Definition 5: bi-directed: The edge which has two side directed.

Definition 6: one-directed: The edge which has one side directed.

Definition 7: bi-degree: The vertex which has both indegree and outdegree.

We will discuss first and second neighborhood for n - simplex directed graph G, where $n = 1, 2\ldots k$.

3. THE NEIGHBORHOOD OF VERTEX V_i ON N-SIMPLEX DIRECTED GRAPH

0-simplex directed graph:

For the graph shown in Fig. (1) we find:

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	non	non

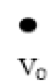

Fig.(1)

Lemma1: There is no first and second neighborhood for 0 - simplex directed graph.

1-simplex directed graph:

We have two cases:

Case (1): when the edge has one-directed For the graph shown in Fig. (2), we can compute first and second neighborhood as follows:

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	V_1	non
V_1	non	non

Fig.(2)

Lemma 2: For 1 - simplex one -directed graph only vertices without degree Has first neighborhood and second neighborhood dose not exist.

Case (2): when the edge has bi-directed

Example1: In Fig. (3) we can compute first and second neighborhood as follows:

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	V_1	non
V_1	V_0	non

Fig. (3)

Lemma 3: For 1 - simplex bi-directed graph every vertex has one first neighborhood.

2-simplex directed graph:

We have three cases:

Case (1): all edges have one-directed.

Example 2: consider a graph shown in Fig.(4) which all directed have the same way, we can compute first and second neighborhood as follows:

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	V_1	V_2
V_1	V_2	V_0
V_2	V_0	V_1

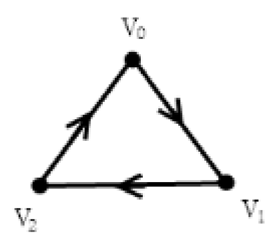

Fig.(4)

Lemma 4: For 2 - simplex directed graph which all its directions have the same way then every vertex has one first neighborhood and one second neighborhood.

But if the directions have different way as shown in Fig.(5) we can compute first and second neighborhood as follows:

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	$V_1 ; V_2$	V_2
V_1	V_2	non
V_2	non	non

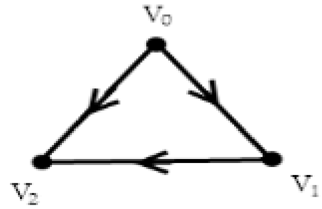

Fig.(5)

Theorem: For 2-simplex directed graph which its directions have different ways we have :

(i) each vertex without degree= 2 has two first neighborhood and one second neighborhood.

(ii) each vertex with indegree= 2 has neither first neighborhood nor second neighborhood.

(iii) each vertex with outdegree= 1 and indegree= 1 has only one first neighborhood.

Proof: The proof comes directly from the above discussion.

Case (2): some of edges have one-directed and others have bi- directed.

Example 3: Consider a graph shown in Fig.(6) which one edge has bi-directed and one vertex has only out degree.

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	V_1	non
V_1	V_0	non
V_2	$V_0; V_1$	$V_0; V_1$

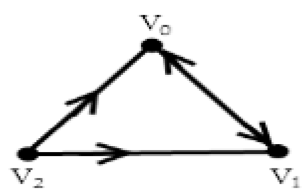

Fig.(6)

Example 4: For a graph shown in Fig.(7) which one edge has bi-directed and one vertex has only indegree, we have:

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	$V_1 ; V_2$	V_2
V_1	$V_1 ; V_2$	V_2
V_2	non	non

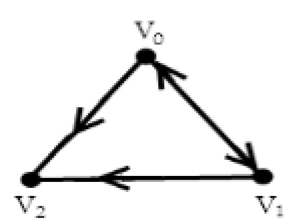

Fig.(7)

Lemma 5: For 2 - simplex directed graph every vertex with one bi-directed and one outdegree has (2 first and 1second) neighborhood and every vertex with one bi-directed and one indegree has 1first neighborhood and at most one second neighborhood.

Case (3): when all edges have bi-directed.

Example 5: For a graph shown in Fig.(8) if each edge has bi-directed we have:

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	$V_1 ; V_2$	$V_2 ; V_1$
V_1	$V_0 ; V_2$	$V_2 ; V_0$
V_2	$V_0 ; V_1$	$V_1 ; V_0$

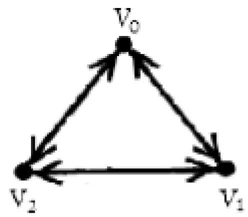

Fig.(8)

Lemma 6: For 2 - simplex directed graph if all its edges have bi-directed find that

$$N^1(V_i) = N^2(V_i)$$

3-simplex directed graph:

We will discuss it in three cases
case (1): all edges have one-directed.

Example 6: when 3 edges have the same way directed and one vertex has only indegree, We can compute first and second neighborhood for the graph shown in Fig.(9) as follows:

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	$V_1 ; V_3$	$V_2 ; V_3$
V_1	$V_2 ; V_3$	$V_0 ; V_3$
V_2	$V_0 ; V_3$	$V_3 ; V_1$
V_3	non	non

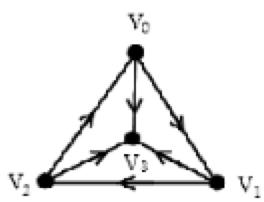

Fig.(9)

Example 7: When 3 edges have the same way directed and one vertex has outdegree, as shown in Fig.(10) we have:

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	V_1	V_2
V_1	V_2	V_0
V_2	V_0	V_1
V_3	$V_0 ; V_1 ; V_2$	$V_0 ; V_1 ; V_2$

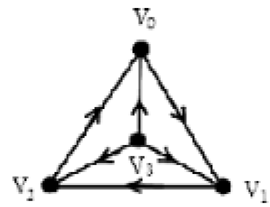

Fig.(10)

Case (2): some of edges have one-directed and others have bi- directed (different directed).

Example 8: Consider a graph shown in Fig.(11) which 2 edges have bi-directed and one vertex has outdegree.

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	V_1	V_2
V_1	V_1 ; V_2	V_0
V_2	V_1	V_0
V_3	V_0 ; V_1 ; V_2	V_0 ; V_1 ; V_2

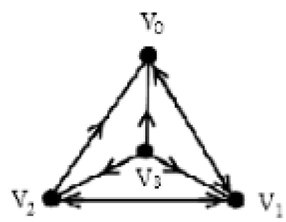

Fig.(11)

Lemma 7: For 3 - smplex directed graph which its edges have (differen directed) and one vertex has outdegree we have the first neighborhood equal outdegree and second neighborhood at less= 1.

Example 9: onsider a graph shown in Fig.(12) which 3 edges have bi-directed and one vertex has only outdegree.

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	V_1 ; V_2	V_1 ; V_2
V_1	V_0 ; V_2	V_0 ; V_2
V_2	V_1 ; V_2	V_1 ; V_2
V_3	V_0 ; V_1 ; V_2	V_0 ; V_1 ; V_2

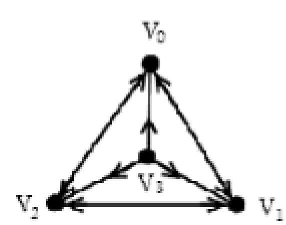

Fig.(12)

Lemma 8: For 3 - simplex directed graph if 3 edges have bi-directed and one vertex has outdegree then $N^1(V_i) = N^2(V_i)$.

Case (3): when all edges have bi-directed

Example 10: Consider a graph in shown Fig.(13) which all edges have bi-directed and one vertex has indegree.

vertex	$N^1(V_i)$	$N^2(V_i)$
V_0	$V_1; V_2; V_3$	$V_1; V_2; V_3$
V_1	$V_0; V_2; V_3$	$V_0; V_2; V_3$
V_2	$V_0; V_1; V_3$	$V_0; V_1; V_3$
V_3	$V_0; V_1; V_2$	$V_0; V_1; V_2$

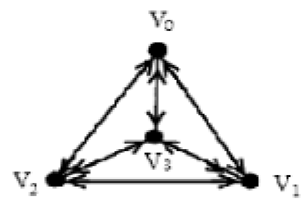

Fig.(13)

Lemma 9: For 3 - simplex directed which all edges have bi-directed we have: $N^1(V_i) = N^2(V_i)$

Theorem: For n - simplex directed graph we find:

(i) the vertex which has only indegree , it does not has first or second neighborhood .

(ii) the vertex which has only outdegree , it has n first neighborhood and second neighborhood at less n - 1.

(iii) the vertex which has both outdegree and indegree, it has first neighborhood = outdegree and at less one second neighborhood.

Proof: The proof comes directly from the above discussion.

Theorem: For n -simplex directed graph if all edges have bi-directed then $N^1(V_i) = N^2(V_i)$, where

$i = 0, 1, 2, \ldots, n$.

Proof: The proof comes directly from the above discussion.

4.The neighborhood of edge e_i on n-simplex directed graph:

there is no first and second neighborhood on(0,1) simplex directed graph , see Fig.(1) and Fig.(14), But we note that first and second neighborhood appeared at the first time on 2 - simplex directed graph .

Example11: Consider a graph shown in Fig.(15) which is a closed path its all edges have the same way directed we have:

edge	path	$N^1_\Delta(e_i)$	$N^2_\Delta(e_i)$
$\overrightarrow{e_1}$	$\overrightarrow{e_1} \rightarrow e_2 \rightarrow e_3$	e_2	e_3
$\overrightarrow{e_2}$	$\overrightarrow{e_2} \rightarrow e_3 \rightarrow e_1$	e_3	e_1
$\overrightarrow{e_3}$	$\overrightarrow{e_3} \rightarrow e_1 \rightarrow e_2$	e_1	e_2

Where $(\underset{e_i}{\rightarrow})$ refer to the edge which is the first one on the bath directed.

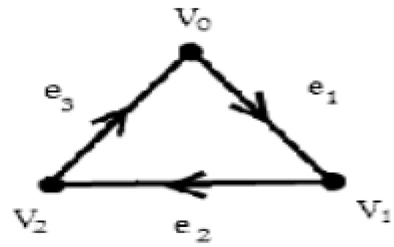

Fig.(15)

Lemma10: For 2 - simplex directed graph if all edges have the same way directed then every edge has first and second neighborhood.

Example 12: For the graph shown in Fig.(16) which two edges have the same way directed and one vertex has indegree , we have :

$edge$	$path$	$N_{\Delta}^{1}(e_i)$	$N_{\Delta}^{2}(e_i)$
$\overrightarrow{e_1}$	$\overrightarrow{e_1} \to e_2$	e_2	non
e_2	non	non	non
e_3	non	non	non

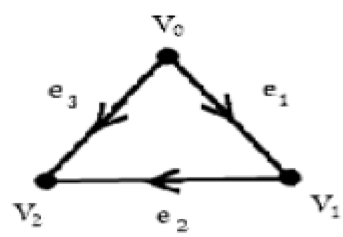

Fig.(16)

Lemma 11: For 2 - simplex directed graph if 2 edges have the same way directed , only $\overrightarrow{e_1}$ has first neighborhood.

3-simplex directed graph:

To compute first and second neighborhood for any edge e_i belongs to 3 - simplex directed graph we will take all paths directed Containing this edge.

Example 13: For the graph shown in Fig. (17) which all vertices have bi-degree we have:

$edge$	$path$	$N_{\Delta}^{1}(e_i)$	$N_{\Delta}^{2}(e_i)$
$\overrightarrow{e_1}$	$\overrightarrow{e_1} \to e_2 \to e_3$ $\overrightarrow{e_1} \to e_5 \to e_6$	e_2, e_5	e_3, e_6
$\overrightarrow{e_2}$	$\overrightarrow{e_2} \to e_3 \to e_1$	e_3	e_1
$\overrightarrow{e_3}$	$\overrightarrow{e_3} \to e_4 \to e_6$ $\overrightarrow{e_3} \to e_1 \to e_5$ $\overrightarrow{e_3} \to e_1 \to e_2$	e_4, e_1	e_2, e_5, e_6
$\overrightarrow{e_4}$	$\overrightarrow{e_4} \to e_6 \to e_3$	e_6	e_3
$\overrightarrow{e_5}$	$\overrightarrow{e_5} \to e_6 \to e_3$	e_6	e_3
$\overrightarrow{e_6}$	$\overrightarrow{e_6} \to e_3 \to e_4$ $\overrightarrow{e_6} \to e_3 \to e_1$	e_3	e_1, e_4

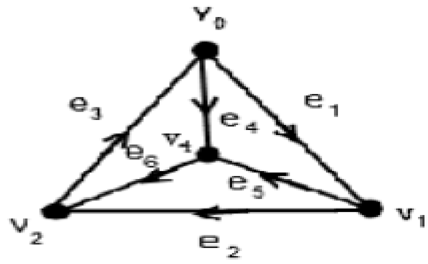

Fig.(17)

Example 13: Cosider a graph shown in Fig. (18) which one vertex has only outdegree, we have:

$edge$	$path$	$N^1_\Delta(e_i)$	$N^2_\Delta(e_i)$
$\overrightarrow{e_1}$	$\overrightarrow{e_1} \rightarrow e_2 \rightarrow e_3$	e_2	e_3
$\overrightarrow{e_2}$	$\overrightarrow{e_2} \rightarrow e_3 \rightarrow e_1$	e_3	e_1
$\overrightarrow{e_3}$	$\overrightarrow{e_3} \rightarrow e_1 \rightarrow e_2$	e_1	e_2
$\overrightarrow{e_4}$	$\overrightarrow{e_4} \rightarrow e_1 \rightarrow e_2$	e_1	e_2
$\overrightarrow{e_5}$	$\overrightarrow{e_5} \rightarrow e_2 \rightarrow e_3$	e_2	e_3
$\overrightarrow{e_6}$	$\overrightarrow{e_6} \rightarrow e_3 \rightarrow e_1$	e_3	e_1

We note that 3 edges ($e_4; e_5; e_6$) which are incident from V_4 were not neighborhood for the rest of edges.

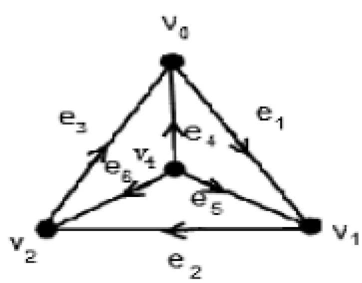

Fig.(18)

Example 14: For the graph shown in Fig. (19) which one vertex has only indegree, we have:

$edge$	$path$	$N^1_\Delta(e_i)$	$N^2_\Delta(e_i)$
$\overrightarrow{e_1}$	$\overrightarrow{e_1} \rightarrow e_2 \rightarrow e_3$ $\overrightarrow{e_1} \rightarrow e_5$	e_2, e_5	e_3
$\overrightarrow{e_2}$	$\overrightarrow{e_2} \rightarrow e_3 \rightarrow e_1$ $\overrightarrow{e_2} \rightarrow e_6$	e_3, e_6	e_1
$\overrightarrow{e_3}$	$\overrightarrow{e_3} \rightarrow e_1 \rightarrow e_2$ $\overrightarrow{e_3} \rightarrow e_4$	e_1, e_4	e_2
$\overrightarrow{e_4}$	non	non	non
$\overrightarrow{e_5}$	non	non	non
$\overrightarrow{e_6}$	non	non	non

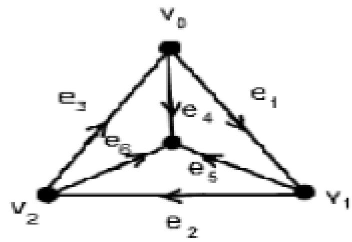

Fig.(19)

We note that 3edges (e_4, e_5, e_6) which are incident into V_4 were not have neither first neighborhood nor second neighborhood.

Theorem: For n simplex directed graph we have:

[1] If all vertices do not have indegree , every edge have at less 1 ferst neighborhood and 1 second neighborhood.

[2] If one vertex V_i has only indegree, there are n edges which are incident into V_i do not have neither first neighborhood nor second neighborhood

[3] If one vertex V_i has only outdegree , there are n edges which are incident from V_i do not be first or second neighborhood for the rest of edges.

Proof: The proof comes directly from the above discussion.

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