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EFFECT OF THERMAL MODULATION AND ROTATION ON THE ONSET OF CONVECTION IN A WALTERS B FLUID SATURATED POROUS LAYER

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ABSTRACT

The combined effect of thermal modulation and rotation on the onset of convection in a Walters B viscoelastic fluid saturated porous layer is studied analytically using linear stability theory. Darcy's law that includes the Coriolis term is used to describe the fluid motion. A method based on small amplitude of the modulation is employed to compute the critical value of Rayleigh number and wavenumber. The stability of the system characterized by a correction Rayleigh number is calculated as a function of Taylor number, viscoelastic parameter, Prandtl number, Darcy number and the frequency of modulation. We have shown that it is possible to advance or delay the onset of convection by time-periodic modulation of the wall temperature. It is also shown that the large Taylor number has a strong influence on the stability of the system. The effect of viscoelastic parameter, Prandtl number, and Darcy number is discussed.

Key words: *Thermal modulation. Rotation. Porous medium. Stability. Walters B fluid.*

INTRODUCTION

With the growing importance of viscoelastic fluids in modern technology and industries, the investigations of thermal convective instability in such fluids are desirable. In the asthenosphere and the deeper mantle, it is well known now that viscoelastic behavior is an important rheological process. If we take a glance at the literature, we find that there are many models for considering fluids that have both elastic and viscous properties. One such class of fluids is Walters (Model B) elastico-viscous fluid having relevance and in chemical technology and industry. Walters (Model B) elastico-viscous fluid forms the basis for the manufacture of many important polymers and useful products.

The problem of natural convection in a viscoelastic horizontal fluid layer and a fluid-saturated porous layer has been investigated by many authors [1-2] because of its applications in applied geophysics. These investigations have been mainly concerned with finding the conditions for the onset of convection and the corresponding heat and mass transfer. However, a mechanism to control convection has not been given much attention in spite of its numerous applications in geothermal energy storage devices, polymer industries, thermal insulation, the cooling of electronic equipment, post accident cooling of nuclear reactors, and in solidifying casting processes. One of the effective mechanisms to control convection is by maintaining a nonuniform temperature gradient. Such a temperature gradient may be generated by (i) an appropriate heating or cooling at the boundaries [3-4], (ii) injection of fluid at one boundary and removal of the same at the other boundary [5-6], (iii) an appropriate distribution of heat sources [7-8], and (iv) radiative heat transfer [9]. These methods are mainly concerned with only space-dependent temperature gradients. However, in many of the practical situations cited earlier, the nonuniform temperature gradient finds its origin in transient heating or cooling at the boundaries, so the basic temperature profile depends explicitly on position and time. This has to be determined by solving the energy equation under suitable time-dependent temperature boundary conditions, called thermal modulation.

Venezian [10] investigated the stability of a horizontal viscous Newtonian fluid layer by considering thermally modulating boundaries. The equivalent problem in fluid-saturated porous media has been studied by Caltagirone [11]. It was found that the critical Rayleigh number (corresponding to the onset of convection) in these problems depends on the frequency of the imposed temperature modulation and hence it seems possible to control convection by tuning this modulation.

The problem of the flow of a viscoelastic fluid through porous media is most intriguing because it combines the complexities of viscoelastic fluids and the porous medium. Thus, there have been many attempts to find a suitable model to predict the viscoelastic effects in the flows through a porous medium.

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Nonetheless, the studies related to the effect of thermal modulation on the onset of convection in a viscoelastic fluid-saturated porous medium have not received much attention. Chung Liu [12] has examined the stability of a horizontally extended second-grade fluid layer heated from below subject to temperature modulation at walls. Rudraiah et al. [13] have investigated the effect of thermal modulation on the onset of convection in a viscoelastic fluid-saturated porous medium using Oldroyd model, and the effect of anisotropy on the problem has been analyzed by Malashetty et al. [14, 15]. Recently, Shivakumara et al. [16] have studied the effect of thermal modulation on the onset of convection in Walters B viscoelastic fluid-saturated porous medium.

While the rotation effect on the thermally driven flows is well understood by Chandrasekhar [17], Kloosterziel et al. [18], there seems to have only one short communication on the effect of rotation on the stability of thermally modulated system. A brief study of the combined effect of thermal modulation and rotation on the onset of convection in a rotating fluid layer was made by Rauscher and Kelly [19] for the case of the lower wall temperature modulation and when Prandtl number equal to unity. They have reported that high Taylor number has a destabilizing effect over a range of frequencies. For small Prandtl numbers, convection in a rotating fluid layer can begin in an oscillatory manner and the modulation might be expected to have more of a resonant effect. Malashetty and Swamy [20] studied the combined effect of thermal modulation and rotation on the onset of stationary convection in a porous layer. It was established that the convection can be advanced by the low frequency in-phase and lower-wall temperature modulation, where as delayed by the out-of-phase modulation. The effect of thermal modulation on the onset of convection in a rotating fluid layer studied by Malashetty and Swamy [21]. They have reported that the instability can be enhanced by the rotation at low frequency symmetric modulation and with moderate to high frequency lower wall temperature modulation, whereas the stability can be enhanced by the rotation in case of asymmetric modulation. Veena and Gian [22] had investigated the thermal instability of Walters (model B) elastico-viscous fluid in the presence of variable gravity field and rotation in porous medium. Thermosolutal instability of Walters (model B) visco-elastic rotating fluid permeated with suspended particles and variable gravity field in porous medium was studied by Veena and Gian [23]. More recently Malashetty and Begum [24] studied the effect of thermal/gravity modulation on the onset of convection in a Maxwell fluid saturated porous layer. It is found that the low frequency symmetric thermal modulation is destabilizing while moderate and high frequency symmetric modulation is always stabilizing. The asymmetric modulation and lower wall temperature modulations are, in general, stabilizing while the system becomes unstable for large values of Darcy-Prandtl number and for small frequencies. In general, their results indicate that the gravity modulation has stabilizing effect on the onset of convection for moderate and high frequency. The small frequency gravity modulation is found to have destabilizing effect on the stability of the system.

The aim of the present study is to analyze the combined effect of small amplitude thermal modulation and rotation on the onset of convection in a horizontal layer of porous medium saturated with Walters B liquid for a wide range of values of frequency of the modulation, Taylor number, elastic parameter, Prandtl number and Darcy number. We intend to provide a fundamental understanding of how rotation would influence natural convection arising from thermal perturbation.

MATHEMATICAL FORMULATION

We consider a horizontal layer of Walters B viscoelastic fluid-saturated porous layer confined between the planes $z=0$ and $z=d$, with vertically downward gravity force $\mathbf{g}$ acting on it. A Cartesian frame of reference is chosen with the origin in the lower boundary and the *z*-axis vertically upwards. The porous layer is subjected to the rotation with an angular velocity $\mathbf{\Omega}$. The axis of rotation is taken along the *z*-axis. The Darcy law that includes the Coriolis term is used. The porous medium is assumed to be isotropic and is in local thermal equilibrium with fluid phase. The Boussinesq approximation, which states that the variation in density is negligible everywhere in the conservations except in the buoyancy term, is assumed to hold. With these assumptions the basic governing equations are

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\rho_0 \left(\frac{1}{\phi}\frac{\partial \mathbf{q}}{\partial t} + \frac{2}{\phi}\mathbf{\Omega}\times\mathbf{q} \right) = -\nabla p + \rho\,\mathbf{g} - \frac{1}{k}\left(\mu - \mu_v \frac{\partial}{\partial t} \right)\mathbf{q}, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + \left(\mathbf{q}\cdot\nabla\right)T = \nabla\cdot\left(\kappa\cdot\nabla T\right), \tag{3}$$

$$\rho = \rho_0\left[1-\beta_T\left(T-T_0\right)\right], \tag{4}$$

where $\mathbf{q}=(u,v,w)$ is the velocity, p the pressure, T the temperature, ϕ the porosity, $\mathbf{\Omega}=(0,0,\Omega)$ constant angular velocity, k is the permeability of the porous medium, κ the effective thermal diffusivity, μ the viscosity, μ_v the viscoelastic constant of Walters B liquid, ρ_0 the density, β_T thermal expansion coefficient. Further, $\gamma=(\rho c)_m/(\rho c_p)_f$, $(\rho c)_m=(1-\phi)(\rho c)_s+\phi(\rho c_p)_f$, c_p is the specific heat of the fluid at constant pressure, c is the specific heat of the solid, the subscripts f, s and m denote fluid, solid and porous medium values, respectively.

The external driving force is modulated harmonically in time by varying the temperatures of lower and upper horizontal boundary. Accordingly, we take

$$T(z,t)=T_0+\frac{\Delta T}{2}[1+\varepsilon\cos\,\Omega t] \qquad \text{at } z=0, \tag{5}$$

$$T(z,t)=T_0-\frac{\Delta T}{2}[1-\varepsilon\cos\,(\Omega t+\varphi)] \qquad \text{at } z=d, \tag{6}$$

where ε represents small amplitude of modulation (which is used as a perturbation parameter to solve the problem), Ω the frequency, φ the phase angle. We consider three types of thermal modulation, viz.,

Case (a): symmetric (in phase, $\varphi=0$),

Case (b): asymmetric (out of phase, $\varphi=\pi$) and

Case (c): only lower wall temperature is modulated while the upper one is held at constant temperature ($\varphi=-i\infty$).

BASIC STATE

The basic state of the fluid is quiescent and is given by

$$\mathbf{q}_b=(0,0,0),\ T=T_b(z,t),\ p=p_b(z,t),\ \rho=\rho_b(z,t) \tag{7}$$

The temperature $T=T_b(z,t)$ is a solution of

$$\gamma\frac{\partial T_b}{\partial t}=\kappa\frac{\partial^2 T_b}{\partial z^2}, \tag{8}$$

and pressure $p_b(z,t)$ balances the buoyancy force. The solution of equation (8) subject to the boundary conditions (5) and (6) is

$$T_b=T_0+\frac{\Delta T}{2}\left\{(1-\frac{2z}{d})+\varepsilon\,\mathrm{Re}\{[a(\lambda)e^{\lambda z/d}+a(-\lambda)e^{-\lambda z/d}]e^{-i\Omega t}\}\right\}, \tag{9}$$

where

$$\lambda=(1-i)\left(\frac{\gamma\Omega d^2}{2\kappa}\right)^{1/2},\ a(\lambda)=\left[\frac{e^{-i\varphi}-e^{-\lambda}}{e^{\lambda}-e^{-\lambda}}\right].$$

and Re stands for the real part. We do not record the expressions of p_b and ρ_b as these are not explicitly required in the remaining part of the paper.

We now superimpose infinitesimal perturbations on the quiescent basic state and study the stability of the system.

LINEAR STABILITY ANALYSIS

Let the basic state be disturbed by an infinitesimal perturbation. We now have

$$\mathbf{q}=\mathbf{q}_b+\mathbf{q}',\quad T=T_b+T',\quad p=p_b+p',\quad \rho=\rho_b+\rho', \tag{10}$$

where the prime indicates that the quantities are infinitesimal perturbations.

Substituting equation (10) into equations (1) - (4), and using the basic state solutions, we get the linearized equations governing the perturbations in the form

$$\rho_0\left(\frac{1}{\phi}\frac{\partial \mathbf{q}'}{\partial t}+\frac{2}{\phi}\mathbf{\Omega}\times\mathbf{q}'\right)=-\nabla p'-\rho_0\beta_T T'\mathbf{g}-\frac{\mu}{k}\left(1-\frac{\mu_v}{\mu}\frac{\partial}{\partial t}\right)\mathbf{q}', \tag{11}$$

$$\gamma\frac{\partial T'}{\partial t}+w'\left(\frac{\partial T_b}{\partial z}\right)=\kappa\nabla^2 T', \tag{12}$$

$$\rho'=-\beta_T\,\rho_0\,T'. \tag{13}$$

We eliminate ρ' between equations (11) and (13) and then eliminate p' from the resulting equation by operating curl twice. We render the resulting equation and equation (12) dimensionless by setting

$$\left(x',y',z'\right)=d\left(x^*,y^*,z^*\right),\ t'=\left(\frac{d^2\gamma}{\kappa}\right)t^*, w'=\left(\frac{\kappa}{d}\right)w^*,\ \left(T',T_b\right)=\Delta T\left(T^*,T_b\right) \tag{14}$$

to obtain (after dropping the asterisks for simplicity)

$$\left[\left\{\frac{1}{Pr}\frac{\partial}{\partial t}+Da^{-1}\left(1-\frac{\Gamma p}{Pr}\frac{\partial}{\partial t}\right)\right\}^2\nabla^2+Ta\frac{\partial^2}{\partial z^2}\right]w=Ra\left\{\frac{1}{Pr}\frac{\partial}{\partial t}+Da^{-1}\left(1-\frac{\Gamma p}{Pr}\frac{\partial}{\partial t}\right)\right\}\nabla_1^2 T, \tag{15}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T=-w\frac{\partial T_b}{\partial z}, \tag{16}$$

where $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$. The dimensionless groups, that appear are $Pr=\gamma\phi\nu/\kappa$, the Prandtl number, $Da=\frac{k}{d^2}$, the Darcy number, $Ta=\left(\frac{2\Omega d^2}{\phi\nu}\right)^2$, the Taylor number, $Ra=\beta_T g\Delta T d^3/\nu\kappa$, the Rayleigh number, $\Gamma p=\frac{\mu_v\phi}{\rho_0 d^2}$, the elastic parameter.

Equations (15) and (16) are to be solved subject to the impermeable and isothermal boundary conditions

$$w=T=0,\ \text{at}\ z=0,\ 1. \tag{17}$$

We now combine equations (15) and (16) to obtain a single differential equation for the vertical component of velocity w as

$$\left[\left\{\frac{1}{Pr}\frac{\partial}{\partial t}+Da^{-1}\left(1-\frac{\Gamma p}{Pr}\frac{\partial}{\partial t}\right)\right\}^2\nabla^2+Ta\frac{\partial^2}{\partial z^2}\right]\left(\frac{\partial}{\partial t}-\nabla^2\right)w=-Ra\left\{\frac{1}{Pr}\frac{\partial}{\partial t}+Da^{-1}\left(1-\frac{\Gamma p}{Pr}\frac{\partial}{\partial t}\right)\right\}\nabla_1^2\frac{\partial T_b}{\partial z}w \tag{18}$$

The boundary conditions (17) can also be expressed in terms of w in the form

$$w=\frac{\partial^2 w}{\partial z^2}=0,\ \text{at}\ z=0,\ 1. \tag{19}$$

Using equation (9), the dimensionless temperature gradient appearing in equation (18) may be written as

$$\frac{\partial T_b}{\partial z} = -1 + \varepsilon\, f\,, \tag{20}$$

where

$$f = \operatorname{Re}\left\{ \left[A(\lambda)\, e^{\lambda z} + A(-\lambda)\, e^{-\lambda z} \right] e^{-i\omega t} \right\},$$

$$\mathrm{A}(\lambda) = \frac{\lambda}{2}\left[\frac{\mathrm{e}^{-i\varphi} - \mathrm{e}^{-\lambda}}{\mathrm{e}^{\lambda} - \mathrm{e}^{-\lambda}}\right] \text{ and } \lambda = (1-i)\left(\frac{\omega}{2}\right)^{1/2}.$$

METHOD OF SOLUTION

We now seek the eigenfunctions w and eigenvalues Ra of equation (18) for the basic temperature gradient given by equation (20) that departs from the linear profile $\left(\partial T_b / \partial z\right) = -1$ by quantities of order ε. We therefore assume the solution of equation (18) in the form:

$$\left(w, Ra\right) = \left(w_0, Ra_0\right) + \varepsilon\left(w_1, Ra_1\right) + \varepsilon^2\left(w_2, Ra_2\right) + \cdots \tag{21}$$

Substituting equation (21) into equation (18) and equating the coefficients of various powers of ε on either side of the resulting equation, we obtain the following system of equations up to the order of ε^2:

$$Lw_0 = 0\,, \tag{22}$$

$$Lw_1 = Ra_1\, \nabla_1^2\, w_0 - Ra_0\, f\, \nabla_1^2\, w_0\,, \tag{23}$$

$$Lw_2 = Ra_2\, \nabla_1^2\, w_0 + Ra_1\, \nabla_1^2\, w_1 - Ra_1\, f\, \nabla_1^2\, w_0 - Ra_0\, f\, \nabla_1^2\, w_1\,, \tag{24}$$

where

$$L = \left[\left\{\frac{1}{Pr}\frac{\partial}{\partial t} + Da^{-1}\left(1 - \frac{\Gamma p}{Pr}\frac{\partial}{\partial t}\right)\right\}\nabla^2 + \frac{Ta\dfrac{\partial^2}{\partial z^2}}{\dfrac{1}{Pr}\dfrac{\partial}{\partial t} + Da^{-1}\left(1 - \dfrac{\Gamma p}{Pr}\dfrac{\partial}{\partial t}\right)}\right]\left(\frac{\partial}{\partial t} - \nabla^2\right) - Ra_0\, \nabla_1^2$$

and w_0, w_1, w_2 are required to satisfy the boundary conditions of equation (19).

We now assume the marginally stable solutions for equation (22) in the form $w_0 = W_0(z)\exp[i(lx + my)]$, where $W_0(z) = W_0^n(z) = \sin(n\pi z)$, n = 1, 2, 3… and l, m are the wave numbers in the x, y-plane such that $l^2 + m^2 = a^2$. The corresponding eigenvalues $Ra_0 = Ra_0^{(n)}$ are given by

$$Ra_0^{(n)} = \frac{1}{a^2}\left[\left(n^2a^2 + \pi^2\right)^2 Da^{-1} + n^2\pi^2\left(n^2a^2 + \pi^2\right) Da\, Ta\right]. \tag{25}$$

For a fixed value of the wave number a the least eigenvalue occurs for n =1, and is given by

$$Ra_0 = \frac{1}{a^2}\left[\left(a^2 + \pi^2\right)^2 Da^{-1} + \pi^2\left(a^2 + \pi^2\right) Da\, Ta\right]. \tag{26}$$

When both sides of equation (26) are multiplied by Da one can obtain the expression for Darcy-Rayleigh number

$$R_{D0} = \frac{1}{a^2}\left(\pi^2 + a^2\right)\left[a^2 + \pi^2\left(1 + Ta_D\right)\right], \tag{C1}$$

with modified Taylor number, $Ta_D = \left(2\rho_0 \Omega k / \phi\mu\right)^2$ for the Darcy porous layer. The corresponding critical values of Darcy-Rayleigh number and wavenumber are, respectively, given by

$$R_{D0c} = \pi^2\left[1+\left(1+Ta_D\right)^{1/2}\right]^2 \text{ and } a_c = \pi\left(1+Ta_D\right)^{1/4}. \tag{C2}$$

These are the results obtained by Palm and Tyvand [25] for the problem of thermal instability in a rotating porous layer in the absence of modulation. Further, in the absence of rotation one can obtain $R_{Doc} = 4\pi^2$ and $a_c = \pi$, which are the classical results of Horton and Rogers [26] and Lapwood [27] for convection in a porous layer.

Equation (23) is inhomogeneous and poses a problem because of the presence of the resonance terms. The solvability condition requires that the time independent part of the right-hand side must be orthogonal to w_0. The term independent of time on the right-hand side is $Ra_1\nabla_1^2 w_0$ so that Ra_1 must be zero. It follows that all the odd coefficients, i.e., Ra_1, Ra_3, … in equation (21) must vanish. The equation for w_1 then takes the form

$$Lw_1 = Ra_0\, a^2\, \mathrm{Re}\left\{ \left[A(\lambda)\, e^{\lambda z} + A(-\lambda)\, e^{-\lambda z}\right] e^{-i\omega t} \right\}\sin(\pi z). \tag{27}$$

We solve equation (27) for w_1 by expanding the right-hand side in a Fourier series and inverting the operator L term by term. So we take

$$e^{\lambda z}\sin(m\pi z) = \sum_{n=1}^{\infty} g_{nm}(\lambda)\sin(n\pi z)$$

where $g_{nm}(\lambda) = 2\int_0^1 e^{\lambda z}\sin(n\pi z)\sin(m\pi z)\,dz = -\dfrac{4nm\pi^2\lambda\left[1+(-1)^{n+m+1}e^{\lambda}\right]}{\left[(n-m)^2\pi^2+\lambda^2\right]\left[(n+m)^2\pi^2+\lambda^2\right]}$

We now define

$$M(\omega, n) = B_1 + i\omega B_2,$$

where

$$B_1 = Ra_0 a^2 + \frac{\omega^2}{Pr}\left(a^2+n^2\pi^2\right) - \frac{\left(a^2+n^2\pi^2\right)^2}{Da} - \frac{\omega^2\Gamma p}{Pr\,Da}\left(a^2+n^2\pi^2\right) - \frac{Da\, n^2\pi^2\, Pr\, Ta\left(n^2\pi^2 Pr + Pr\, a^2 + \left(Da-\Gamma p\right)\omega^2\right)}{Pr^2+\left(Da-\Gamma p\right)^2\omega^2},$$

$$B_2 = \frac{\left(a^2+n^2\pi^2\right)^2}{Pr} + \frac{\left(a^2+n^2\pi^2\right)}{Da} - \frac{\Gamma p}{Pr\,Da}\left(a^2+n^2\pi^2\right)^2 + \frac{Da\, n^2\pi^2\, Pr\, Ta\left(Pr - \left(a^2+n^2\pi^2\right)\left(Da-\Gamma p\right)\right)}{Pr^2+\left(Da-\Gamma p\right)^2\omega^2}.$$

So that

$$w_1 = Ra_0\, a^2\, \mathrm{Re}\left\{\sum_{n=1}^{\infty}\frac{B_n(\lambda)}{M(\omega,n)}e^{-i\omega t}\sin(n\pi z)\right\}. \tag{28}$$

where

$$B_n(\lambda) = A(\lambda)g_{n1}(\lambda) + A(-\lambda)g_{n1}(-\lambda).$$

The equation for w_2 becomes

$$Lw_2 = -Ra_2\, a^2\, w_0 + Ra_0\, a^2\, f\, w_1. \tag{29}$$

We do not require the solution of equation (29) but shall use it to determine the correction Rayleigh number Ra_2.

The solvability condition requires that the time-independent part of right-hand side of equation (29) be orthogonal to $w_0 = \sin(\pi z)$. To that end we multiply the right-hand side of equation (29) by $\sin(\pi z)$ and integrate between the limits 0 and 1 to obtain

$$Ra_2 = 2\, Ra_0\, \mathrm{Re}\int_0^1 \overline{f\, w_1}\sin(\pi z)\,dz, \tag{30}$$

where the over bar indicates time average. Now from equation (27) we obtain,

$$\overline{f\, w_1}\sin(\pi z) = \frac{Ra_0 a^2}{2}\operatorname{Re}\left\{\sum_{n=1}^{\infty}\frac{|B_n(\lambda)|^2}{M(\omega,n)}\sin^2(n\pi z)\right\}, \text{ so that}$$

$$\int_0^1 \overline{f\, w_1}\sin(\pi z)\,dz = \frac{Ra_0 a^2}{4}\operatorname{Re}\left\{\sum_{n=1}^{\infty}\frac{|B_n(\lambda)|^2}{M(\omega,n)}\right\}. \tag{31}$$

Substituting equation (31) into equation (30), we get

$$Ra_2 = \frac{Ra_0^2\, a^2}{2}\sum_{n=1}^{\infty}\frac{|B_n(\lambda)|^2}{|M(\omega,n)|^2}\operatorname{Re}\left\{ M^*(\omega,n)\right\}. \tag{32}$$

Now,

$\operatorname{Re}\left\{ M^*(\omega,n)\right\} = B_1$ and $|M(\omega,n)|^2 = B_1^2 + (\omega B_2)^2$ Thus, we may write

$$Ra_2 = \frac{Ra_0^2 a^2}{2}\sum_{n=1}^{\infty}\left|B_n(\lambda)\right|^2 C_n, \tag{33}$$

where

$$\left|B_n(\lambda)\right|^2 = \frac{16\pi^4 n^2\omega^2}{\left[\omega^2+(n+1)^4\pi^4\right]\left[\omega^2+(n-1)^4\pi^4\right]}, \quad C_n = \frac{B_1}{B_1^2+(\omega B_2)^2}.$$

In the equation (33) the summation extends over even values of n for case (a), odd values of n for case (b) and all integer values of n for case (c).

Equation (29) could now be solved for w_2 if desired, and the procedure may be continued to obtain further corrections to w and *Ra*. The value of Rayleigh number Ra obtained by this procedure is the eigenvalue corresponding to the eigenfunctions w, which, though oscillating, remains bounded in time. Ra is a function of horizontal wavenumber a and the amplitude of modulation ε, accordingly we expand

$$Ra(a,\,\varepsilon) = Ra_0(a) + \varepsilon^2 Ra_2(a) + \varepsilon^4 Ra_4(a) + \cdots, \tag{34}$$

$$a = a_0 + \varepsilon^2 a_2 + \cdots, \tag{35}$$

where Ra_0 and a_0 are the Rayleigh number and wavenumber, respectively for the unmodulated system. The Rayleigh number Ra as a function of wavenumber a has a least value Ra_c which occurs at $a = a_c$

The critical value of the Rayleigh number Ra is computed up to $O\left(\varepsilon^2\right)$ by evaluating Ra_0 and Ra_2 at $a = a_0$. It is only when one wishes to evaluate Ra_4, a_2 must be taken into account (see also Venezian (1969)). If Ra_{2c} is positive, the effect of modulation is to stabilize the system as compared to the unmodulated system. When Ra_{2c} is negative, the effect of modulation is to destabilize the system as compared to the unmodulated system.
To the order of $O\left(\varepsilon^2\right)$, Ra_{2c} is obtained for the three cases (a) in-phase temperature modulation, (b) out-phase temperature modulation and (c) only lower wall temperature modulation.

RESULT AND DISCUSSION

In the present paper we make an analytical study of the combined effect of rotation and time-periodic temperature modulation on the onset of thermal convection in a Walters B viscoelastic fluid-saturated porous layer using linear stability theory. The regular perturbation method based on small amplitude of modulation is employed to compute the critical value of Rayleigh number and wavenumber. The expression for the critical correction Rayleigh number Ra_{2c} is computed as a function of frequency of modulation, Taylor number, Prandtl number, Darcy number and elastic parameter and the effect of these parameters on the stability of the system is discussed. The sign of Ra_{2c} characterizes the stabilizing or destabilizing effect of modulation. A positive Ra_{2c} means the modulation effect is stabilizing while a negative Ra_{2c} means the modulation effect is destabilizing compared to the system in which the modulation is absent.

The variation of the critical correction Rayleigh number Ra_{2c} with the frequency of modulation ω is depicted in Figs. 1-4 when the oscillating temperature field is symmetric. It is reported from these figures that Ra_{2c} is negative over entire range of values of ω indicating the effective reduction in the value of critical Rayleigh number. Therefore the effect of time-periodically varying temperature field with respect to symmetric case is to destabilize the onset of thermal convection in a rotating fluid-saturated porous layer as compared to the unmodulated system. Fig. 1 depicts the effect of rotation on Ra_{2c}, for the fixed value of Prandtl number, Darcy number and elastic parameter. It is observed from this figure that the critical value of Ra_{2c} increases negatively with Ta, indicating the destabilizing effect of rotation on the onset of convection.

In Fig. 2 the effect of elastic parameter Γp on the stability of the system for fixed values of Ta, Pr and Da is displayed. It is found from this figure that Ra_{2c} decreases negatively with the increase in Γp. Therefore, Γp stabilizes the thermally modulated rotating fluid-saturated porous layer.

The variation of critical correction Rayleigh number Ra_{2c} with frequency ω for the different values of Prandtl number Pr is displayed in Fig. 3 for the fixed value of Ta, Da and Γp. We observe from this figure that Ra_{2c} is negative for the whole range of the frequency, indicating that the symmetric modulation advances the convection.

Fig. 4 presents the effect of Darcy number Da on Ra_{2c}, for the fixed value of Prandtl number Pr, Taylor number Ta and elastic parameter Γp. We find from this figure that the peak value of Ra_{2c} increases negatively with Da indicating the destabilizing effect of Darcy number on the onset of convection in a thermally modulated rotating fluid-saturated porous layer.

The variation of the critical correction Rayleigh number Ra_{2c} with frequency of modulation ω is exhibited through Figs. 5-8 when bounding walls are modulated out of phase. The effect of Taylor number Ta on Ra_{2c} for the fixed value of Prandtl number, Darcy number and elastic parameter is presented in Fig. 5. We observe from this figure that an increase in the value of Taylor number Ta increases the magnitude of Ra_{2c} indicating the stabilizing effect on the onset of convection. It is important to note from figure that the correction Rayleigh number Ra_{2c} becomes positive when frequency ω is small i.e. $\omega \leq 11$ and Taylor number is large i.e. $Ta \geq 500$. Thus, the thermal modulation with large Taylor number and low frequencies is stabilizing in case of thermally modulated rotating fluid-saturated porous medium.

The effect of elastic parameter Γp on the stability of the system for the fixed value of Pr, Ta and Da is shown in Fig. 6. We observe that the effect is destabilizing over the whole range of the frequency. We can also observe that an increase in the value of Γp increases the magnitude of Ra_{2c} indicating the stabilizing effect of elastic parameter on the onset of convection.

The effect of Prandtl number Pr on Ra_{2c} for the fixed values of Ta, Da, and Γp is presented in Fig. 7. It is evident from this figure that as Prandtl number increases the magnitude of Ra_{2c} increases negatively over the entire range of frequency making the system more unstable.

For the fixed values of Ta, Pr and Γp the effect of Darcy number Da on the stability of the system is shown in Fig. 8. We observe from this figure that the effect is destabilizing over the whole range of the frequency. We can also observe from this figure that an increase in the value of Da reduces the destabilizing effect of modulation.

The result of only lower wall temperature modulation is found to be qualitatively similar to the case of asymmetric modulation and we therefore omit the discussion of the same (Figs. 9-12).

CONCLUSION

The effect of thermal modulation on the onset of convection in a Walters B viscoelastic horizontal rotating fluid-saturated porous layer is studied using a linear stability analysis and the following conclusions are drawn:
(i) The symmetric modulation destabilizes the system over entire range of frequencies.

(ii) The asymmetric and bottom wall temperature modulation destabilizes the system while, it has a stabilizing effect for moderate and small values of frequency and at high rotation.

(iii) In case of symmetric modulation, rotation has destabilizing effect over entire range of frequencies. In case of asymmetric and lower wall temperature modulation, rotation stabilizes the system.

(iv) The effect of elastic parameter on all three cases is to make the system most stable, i.e., the elastic parameter has the stabilizing effect on symmetric, asymmetric and bottom wall temperature modulation.

(v) The effect of Prandtl number Pr on symmetric, asymmetric and bottom wall temperature modulation is to reduce the stabilizing effect over entire range of frequencies.

(vi) Increase of Darcy number enhances the destabilizing effect of thermal modulation in case of symmetric modulation while, in case of asymmetric and bottom wall temperature modulation it reduces the destabilizing effect.

(vii) The effect of thermal modulation and rotation disappear for large frequency irrespective of the type of thermal modulation.

As a summary of the results, thermal modulation in the presence of rotation can delay convection, induce convection, in a horizontal layer of fluid saturating a Darcy porous medium. Therefore, thermal modulation can be used to control convective instability in a horizontal fluid saturating a porous medium.

ACKNOWLEDGEMENT

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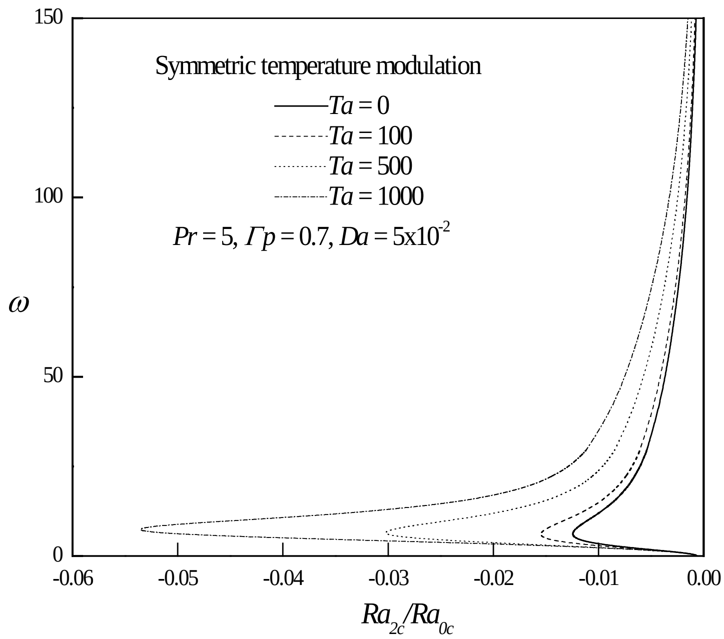

Fig. 1. Variation of Ra_{2c}/Ra_{0c} with ω for different values of Taylor number Ta.

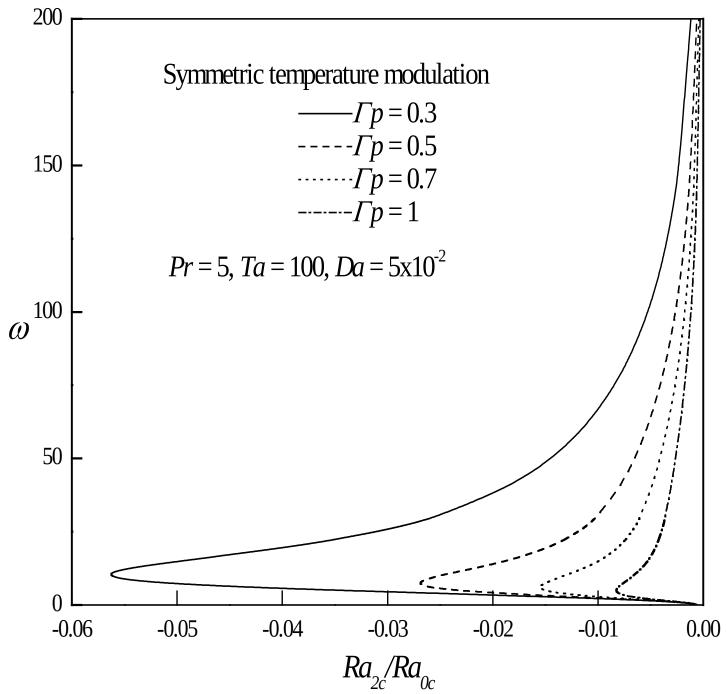

Fig. 2. Variation of Ra_{2c}/Ra_{0c} with ω for different values of elastic parameter Γp.

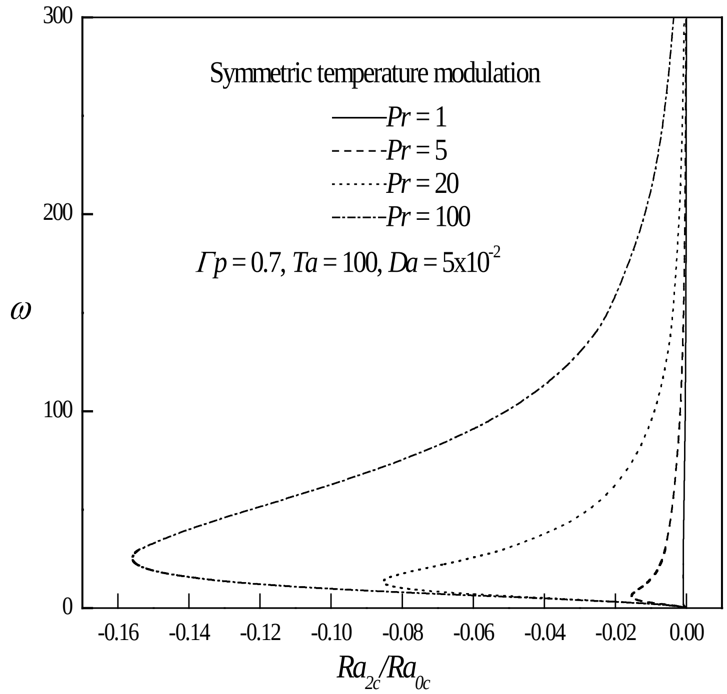

Fig. 3. Variation of Ra_{2c}/Ra_{0c} with ω for different values of Prandtl number Pr.

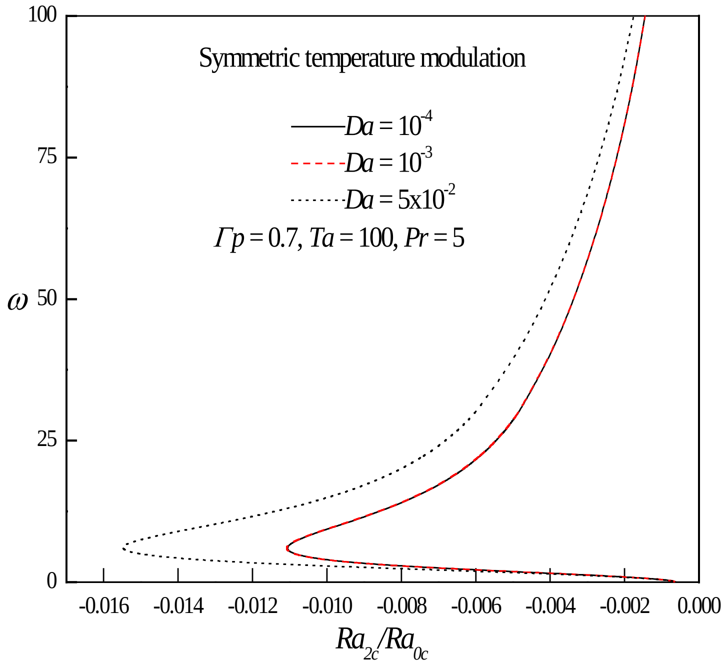

Fig. 4. Variation of Ra_{2c}/Ra_{0c} with ω for different values of Darcy number Da.

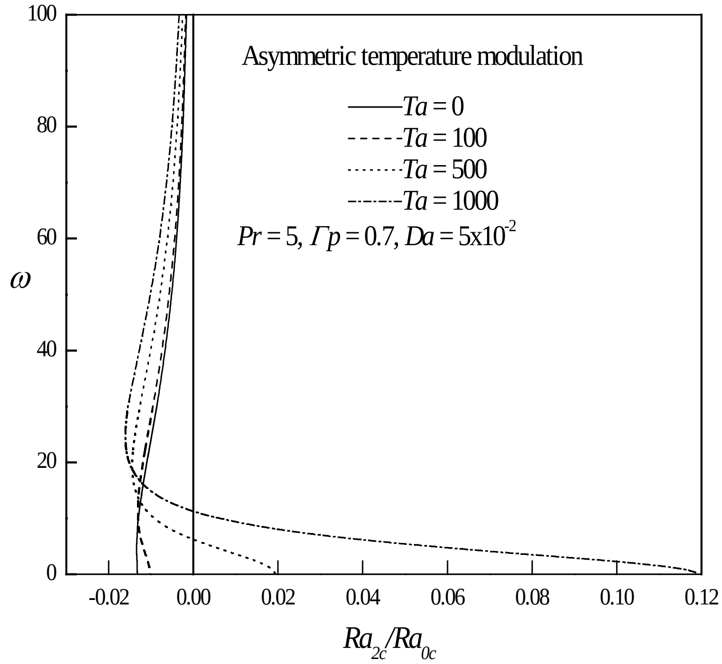

Fig. 5. Variation of Ra_{2c}/Ra_{0c} with ω for different values of Taylor number Ta.

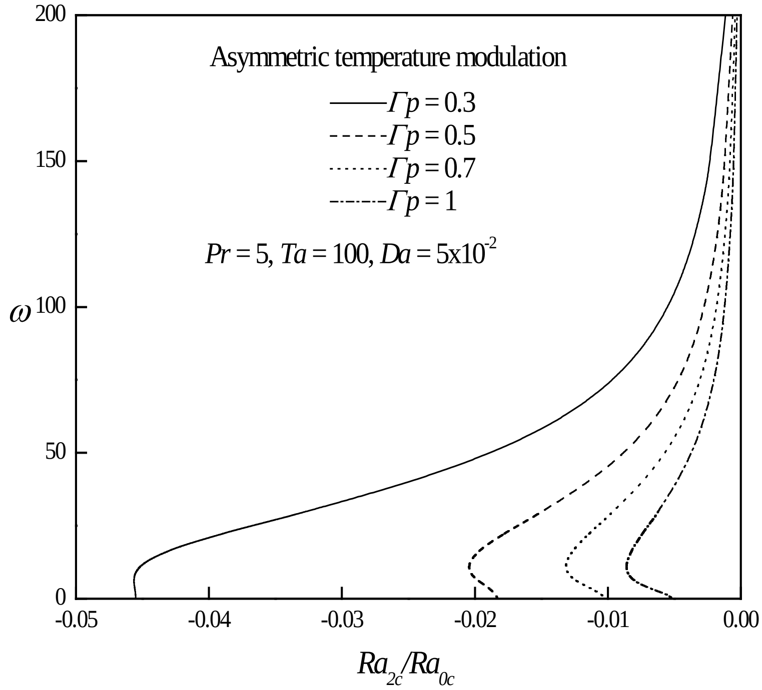

Fig. 6. Variation of Ra_{2c}/Ra_{0c} with ω for different values of elastic parameter Γp.

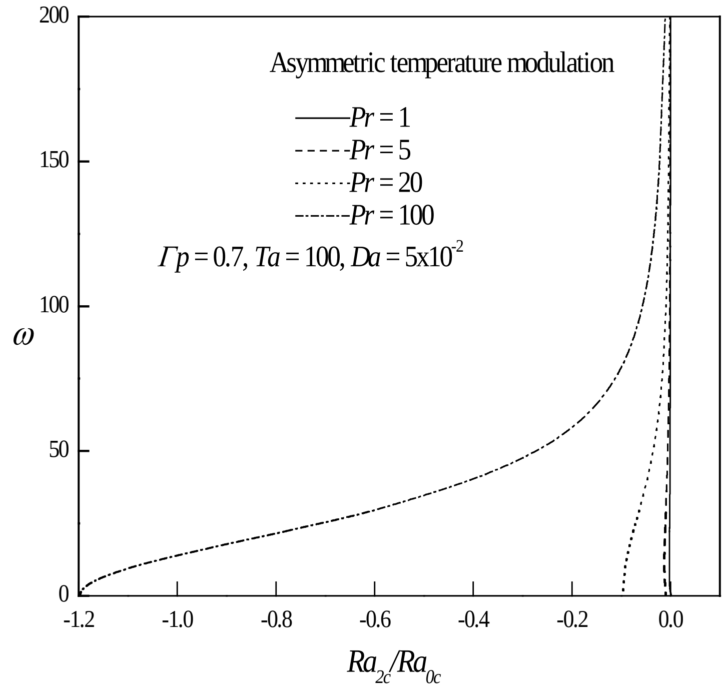

Fig. 7. Variation of Ra_{2c}/Ra_{0c} with ω for different values of Prandtl number Pr.

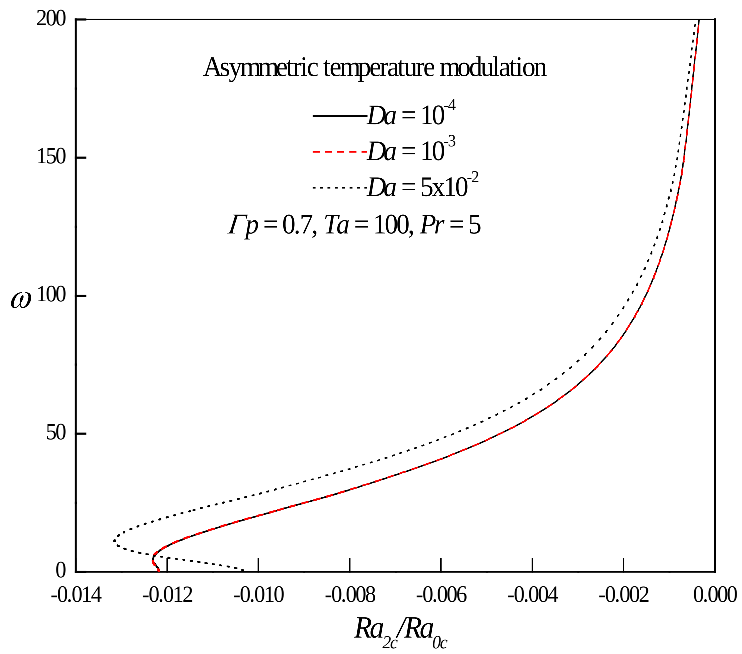

Fig. 8. Variation of Ra_{2c}/Ra_{0c} with ω for different values of Darcy number Da.

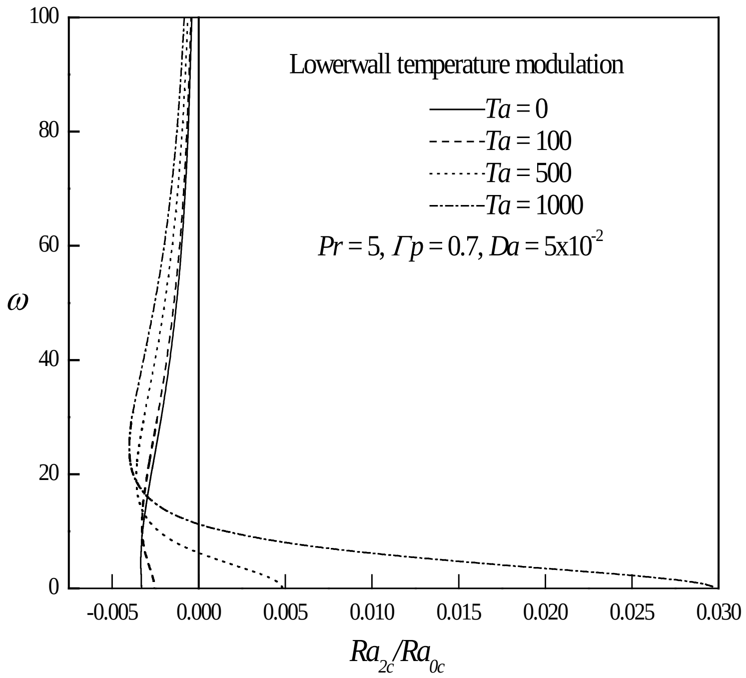

Fig. 9. Variation of Ra_{2c}/Ra_{0c} with ω for different values of Taylor number Ta.

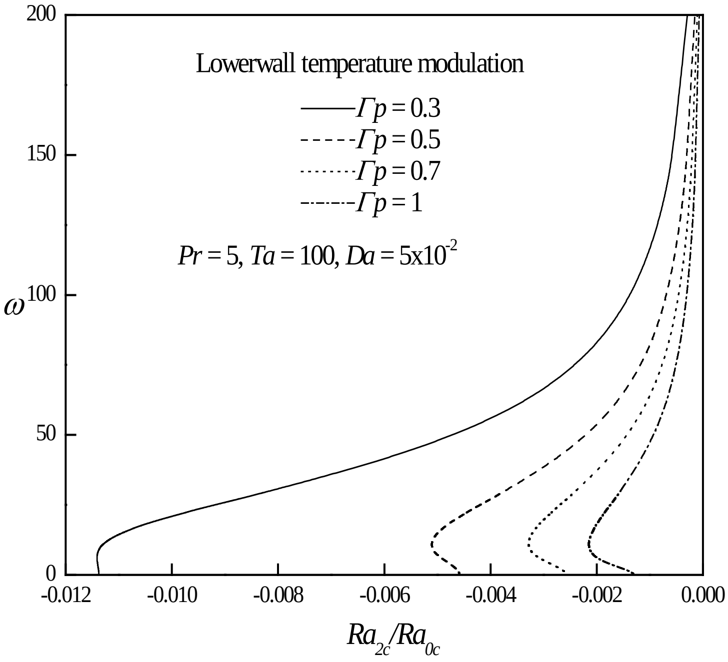

Fig. 10. Variation of Ra_{2c}/Ra_{0c} with ω for different values of elestic parameter Γp.

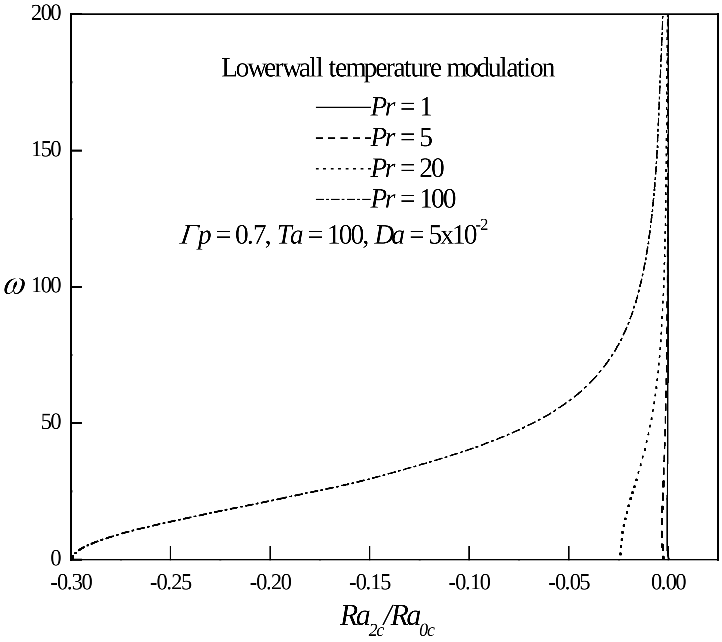

Fig. 11. Variation of Ra_{2c}/Ra_{0c} with ω for different values of Prandtl number Pr.

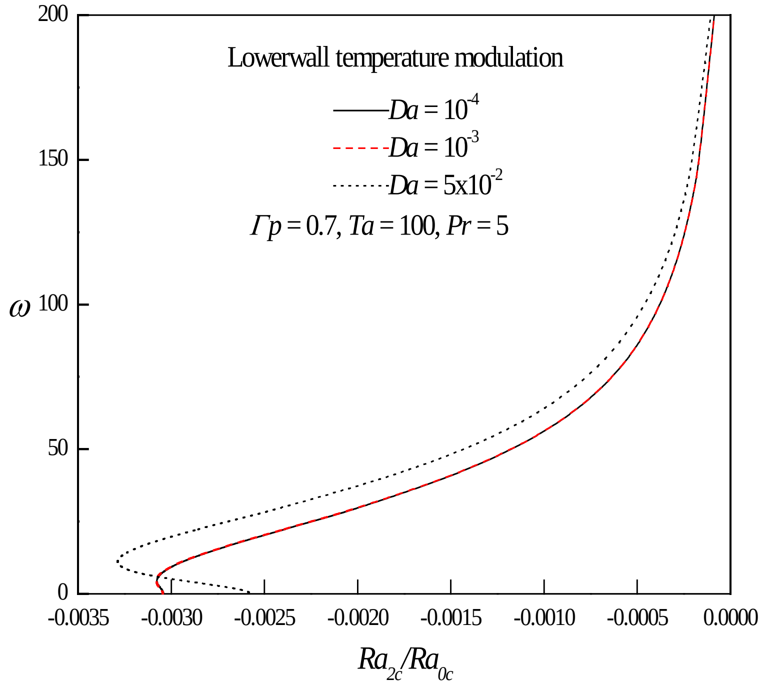

Fig. 12. Variation of Ra_{2c}/Ra_{0c} with ω for different values of Darcy number Da.

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