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ANTI Q-FUZZY FINITE DIMENSIONAL G – MODULAR DISTRIBUTIVE LATTICES

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ABSTRACT

***W**e study the characterization of fuzzy G-modular distributive lattice such as ring sums, lattice homomorphism and upper level sets and characterization of these lattices which is similar to a well known result of lattice theory.The main goal of this paper is to study the finite groups whose lattices of fuzzy G-modular are distributive.*

***Key Words**: Fuzzy lattices, level set, G-modular, anti Q-Fuzzy G-modular lattices, fuzzy filter, fuzzy ideal, lattice homomorphism.*

SECTION-1: INTRODUCTION

The theory of fuzzy sets which was introduced by L. A. Zadeh [14] is applied to many mathematical branches. Rosenfeld [14] inspired the fuzzification of algebraic structures and introduced the notion of fuzzy sub groups. P. Das [6] characterized fuzzy sub groups by their level sub groups. A.Solairaju and R.Nagarajan introduced the notion of Q-fuzzy groups [15] .In [11] Liu applied the concept of fuzzy sets to the theory of rings and introduced and examined the notion of a fuzzy ideal of a ring. The study of fuzzy sub module was introduced by pan and Golan in [7]. Also Pan [13] studied the fuzzy finitely generated modules and fuzzy quotient modules. Later Katsaras and D.B. Liu introduced the concept of fuzzy vector spaces and fuzzy topological vector spaces. The formation of a lattice of sub modules of a module is well known features in classical algebra. However the same has not been explored in fuzzy setting. In order to initiate such studies the concept of fuzzy sub-module generated by an arbitrary fuzzy set is formulate in this note. Using this concept S.K. Bambri and Pratibakumar [5] introduced Lattice of fuzzy sub modules and established an embedding of the lattice of all sub module of a module M into the lattice of fuzzy sub module.

Q. Zhang and Meng [17] considered the sub property to be an assumption and established the more general result that “the lattice I_t (R) of all fuzzy ideals of a ring R with the same tip “t” is modular. On the other hand in [11] the authors have arrived at the false result that The lattice I(R) is distributive. In fact the lattice I(R) of ideals of a ring R is not distributive and has an obvious embedding in I(R). The corrected result has already appeared in several papers [10, 16]. I Jahan in [8], by constructing and employing the technique of strong level subsets, he proved that the lattice (R) of all ideals of a ring R is modular. This proof of the modularity of I(R) is different from the Ajmal’s proof of modularity of the lattice of fuzzy normal sub groups of group appeared in [1]. K. C. Gupta and S. Roy in introduce the indirect proof of the above result that the modularity of the quasi Hamiltonians fuzzy sub groups. Hence N. Ajmal and K.V. Thomas in [2] initiated a discussion on the aspect of modularity of the set of fuzzy normal sub groups. A special class of fuzzy normal sub groups of group has been shown to constitute a modular sub lattice of its fuzzy sub group lattice. They first established that the sub lattice (G) is modular. Moreover using the same technique in [3], they demonstrate that whenever the set (G) of all fuzzy quasi normal sub groups of a given group forms a lattice, the lattice is modular. B. Yuan and Wu. Wangning in [4], introduced fuzzy ideals on a distributive lattice. In this paper, we study the characterization of fuzzy G-modular distributive lattice such as ring sums, lattice homomorphism and upper level sets and characterization of these lattices which is similar to a well known result of lattice theory

SECTION- 2: PRELIMINARIES

Definition: 2.1 A non empty set L together with two binary operations V and Λ on L is called a lattice if it satisfies the following identities.

L1: $x \vee y \approx y \vee x$, $x \wedge y \approx y \wedge x$
L2: $x \vee (y \vee z) \approx (x \vee y) \vee z$, $x \wedge (y \wedge z) \approx (x \wedge y) \wedge z$
L3: $x \vee x \approx x$, $x \wedge x \approx x$

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L4: $x \approx x \vee (x \wedge y), x \approx x \wedge (x \vee y)$.The operation Λ is called meet and the operation V is called join. Let L be the set of proposition and let V denote the connective "or" and Λ denote the connective "and". Then L1 to L4 are well known properties from prepositional logic.

Definition: 2.2 A distributive lattice is a lattice which satisfies either of the distributive laws.D1.$x \wedge (y \vee z) \approx (x \wedge y) \vee (x \wedge z)$, D2. $x \vee (y \wedge z) \approx (x \vee y) \wedge (x \vee z)$.One can see a lattice L satisfies D1 if and only if satisfies D2.

Definition: 2.3 Let G be a group and L be a vector distributive lattice then L is called G-modular if for $a \in G$ and $l \in L$, there exist a product $a.\ l \in L$ satisfies the following axioms.

(i) $e.\ l = l$, for all $l \in L$

(ii) $(a.\ h)\ .\ l = a\ (h\ .\ l)$

(iii) $a.\ (k_1 m_1 + k_2 m_2) = k_1 (am_1) + k_2(am_2)$ for all $k_1, k_2 \in K$ and $m_1, m_2 \in M$.

Definition: 2.4 A mapping $\mu: X \times Q \rightarrow [0, 1]$ where X is an arbitrary non-empty set and is called a Q-fuzzy set in X.

Definition: 2.5 Let X be a Q-fuzzy set and $\mu: X \times Q \rightarrow G$ be a lattice ordered group of X. Then μ is called Q-fuzzy lattice ordered group (QFLG) if

(i) $\mu(x + y, q) \geq \min \{\mu(x, q), \mu (y, q)\}$

(ii) $\mu (-x, q) \geq \mu(x, q)$

(iii) $\mu(0, q) = 1$, for all $x, y \in G$.

Definition: 2.6 Let μ be a Q-fuzzy lattice ordered group of G and $\mu: X \times Q \rightarrow G$. Then μ is called Q-fuzzy lattice if

(i) $\mu(x + y, q) \geq \min \{\mu(x, q), \mu (y, q)\}$

(ii) $\mu (-x, q) \geq \mu(x, q)$

(iii) $\mu(x \vee y) \geq \min \{\mu(x), \mu (y)\}$

(iv) $\mu(x \wedge y) \geq \min \{\mu(x), \mu (y)\}$, for all $x, y \in G$.

Definition: 2.7 Let X be any group and L be a vector distributive lattice extended real valued functions on X. If μ is called a anti Q-fuzzy G-modular lattice on L then it satisfies the following conditions.

(i) $\mu (ax + by, q) \leq \max\{\mu(x), \mu (y)\}$

(ii) $\mu (gx, q) \leq \mu(x)$

(iii) $\mu (x \vee y)_q \wedge \mu(x \wedge y)_q \leq \max\{\mu(x, q), \mu (y, q)\}$, for all $x, y \in L$.

Definition: 2.8 A Q-fuzzy sub set μ is called monotonic if

(i) $\mu(x,q) \leq \mu(y,q)$ whenever $x \leq y$

Definition: 2.9 Let μ be any Q-fuzzy sub set of X. Then the set

$\mu_t = \{x \in X / \mu(x,q) \geq t, t \in [0, 1]\}$ is called a level sub set of μ.

Definition: 2.10

(i) A monotonic fuzzy G-modular lattice is called a fuzzy filter of L.

(ii) A anti-monotonic fuzzy G-modular lattice is called a fuzzy ideal of L.

(iii) A Q-fuzzy sub set μ is called a Q-fuzzy filter if and only if $\mu(x \wedge y)_q = \mu (x, q) \wedge \mu(y, q)$, for all $x, y \in L$.

(iv) A Q-fuzzy subset μ is called a Q-fuzzy ideal if and only if $\mu(x \vee y)_q = \mu(x, q) \wedge \mu(y, q)$, for all $x, y \in L$.

Definition: 2.11

(i) A Q-fuzzy filter is prime if
$\mu (x \vee y)_q \leq S\{\mu(x ,q), \mu(y, q)\}$

(ii) A Q-fuzzy ideal is prime if
$\mu (x \wedge y)_q \leq S \{ \mu(x, q), \mu(y, q)\}$, holds for all $x, y \in L$.

Definition: 2.12 A Q-fuzzy G-modular lattice A is modular if and only if
$A (x, z) \geq 0 \Rightarrow x \vee (y \wedge z) = (x \vee y) \wedge z$. for all x, y and $z \in X$.

Definition: 2.13 Let μ be a anti Q-fuzzy G-modular lattice. If μ is distributive then

(i) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge y)$ and

(ii) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ for all x, y and $Z \in X$.

Definition: 2.14 Let L and L′ be lattices. A mapping f: L $\to$ L′ is said to be lattice homomorphism if

(i) f(x + y) = f(x) + f(y) and

(ii) f(xy) = f(x) . f(y), for all x, y $\in$ L.

Definition: 2.15 We say that a Anti Q-fuzzy G-modular distributive lattice M = (L, μ_M) in L satisfies the imaginable property if

$$I_m(\mu_M) \subseteq \Delta T$$

Definition: 2.16 Let λ and μ be two anti Q-fuzzy G-modular distributive lattice of L then the sum of λ and μ are denoted by $\lambda + \mu$ and is defined as

$$(\lambda + \mu)(z, q) = \inf_{z = x + y} \{\max\{\lambda(x, q), \mu(y, q)\}\} \quad \text{where } x, y \in L.$$

Clearly $\lambda + \mu$ is a anti Q-fuzzy G modular distributive lattice in L.

Definition: 2.17 Let λ and μ be two anti Q-fuzzy G-modular distributive lattice of L, Then the ring sum of λ and μ is defined as

$$(\lambda \oplus \mu)(z, q) = \inf_{Z \in \lambda \oplus \mu} \{(\mu_1 + \mu_2)(z,q) / x = f^{-1}(\lambda,,q), y = f^{-1}(\mu,q)\} \text{ for all } x, y \text{ and } z \in L$$

SECTION -3: CHARACATERIZATION OF ANTI Q-FUZZY G-MODULAR DISTRUBUTIVE LATTICES

Throughout this paper G be a finite groups and L = <L, +, $\cdot$ > denotes a lattice.

That is of maps from L into < [0, 1]; V, Λ >, where [0, 1] is the set of real's between 0 and 1. and

(i) x V y = max (x, y), if $y \le x$ then $\mu(y) \ge \mu(x)$ and

(ii) x Λ y = min (x, y), if $x \le y$ then $\mu(x) \ge \mu(y)$

Proposition: 3.1 Let M be a G-modular lattice and N be a anti Q-fuzzy G-sub modular lattice of M has a anti Q-fuzzy G-modular distributive lattice then M/N and N has anti Q-fuzzy G-modular distributive lattice.

Proof: Let μ be a fuzzy sub modular lattice of M then V: μ_N is anti Q-fuzzy G-modular lattice on N.

Define $\mu : \frac{M}{N} \times Q \to [0, 1]$ by $\mu(x + N, q) = \mu(x, q)$, for all $x \in G$ and $n \in \frac{M}{N}$

(1)
$$\begin{aligned}\mu(a(x + N) + b(y + N), q) &= \mu((ax + by) + N, q)\\ &= \mu(ax + by, q)\\ &\le \max\{\mu(x, q) + \mu(y, q)\}\\ &\le \max\{\mu(x + N, q) + \mu(y + N, q)\}\end{aligned}$$

(2)
$$\begin{aligned}\mu(g(x + N, q)) = \mu(gx + N, q) &= \mu(gx, q)\\ &\le \mu(x, q)\\ &\le \mu(x + N, q)\end{aligned}$$

(3)
$$\begin{aligned}(\mu(x \vee y)q \wedge \mu(x \wedge y))q + N &= \max\{\mu((x \vee y)q + N, \mu(x \wedge y)q + N\}\\ &\le \max\{\max\{(\mu(x, q), \mu(y, q)) + N\}, \{\max\{(\mu(x, q), \mu(y, q)) + N\}\}\\ &\le \max\{\mu(x, q) + N, \mu(y, q) + N\}\\ &\le \max\{\mu(x + N, q), \mu(y + N, q)\}\\ &\le \max\{\mu(x, q), \mu(y, q)\}, \text{ for all } x, y \in G\end{aligned}$$

Therefore $\frac{M}{N}$ and N are anti Q-fuzzy G-modular distributive lattice.

Proposition: 3.2 Let f: L $\rightarrow$ L* be a G-modular distributive lattice homomorphism. Where L and L* are G-modular distributive lattices. If V is anti Q-fuzzy G –modular distributive lattice on L* then $f^{-1}(v)$ is anti Q- fuzzy G-modular distributive lattice on L.

Proof: Since V is anti Q-fuzzy G-modular distributive lattice on L* and f: L $\rightarrow$ L* be a G-modular lattice homomorphism. For any a, b$\in$ K and x, y $\in$ L, we have

(1) $f^{-1}(V)\,(ax + by, q) = V\,(f(ax + by, q))$

$= V\,(f(ax, q) + f(by, q)$

$\leq V\,\{(f(x, q) + f(y, q)\}$

$\leq \max\,\{V\,f(x, q), V\,f(y, q)\}$

$\leq \max\,\{f^{-1}(V)(x, q), f^{-1}(V)\,(y, q)\}$

(2) $f^{-1}(V)\,(gm, q) = V\,f\,(gm, q)$

$\leq V\,f(m, q)$

$\leq f^{-1}\,(V)\,(m, q)$

(3) $f^{-1}(V)\,(x \vee y)q \wedge f^{-1}(V)\,(x \wedge y)q = \max\,\{f^{-1}(V)\,((x \vee y)q, f^{-1}(V)\,(x \wedge y)q\}$

$= \max\,\{V\,f(x \vee y, q), V\,f(x \wedge y, q)\}$

$= \max\,\{V\,(f(x, q) + f(y, q)), V\,(f(x, q). f(y, q))\}$(since V is homomorphism

$\leq \max\,\{\max\,\{V\,f(x, q), V\,f(y, q)\}, \{\max\,\{V\,f(x, q), V\,f(y, q)\}\}$

$\leq \max\,\{V\,f\,(x, q), V\,f\,(y, q)\,\}$

$\leq \max\,\{f^{-1}\,(v)(x, q), f^{-1}\,(v)(y, q)\}$

Therefore f^{-1} (V) is anti Q- fuzzy G-modular distributive lattice on L.

Preposition: 3.3 Let S be a t-norm, then every imaginable anti Q-fuzzy G-modular distributive lattice of L is anti Q-fuzzy G-modular distributive lattice.

Proof: Let M = (L, μ_M) be an imaginable anti Q-fuzzy G-modular distributive lattice of L under S-norms. Then

(i) $\mu_M\,(ax + by, q) \leq S\,\{\mu_M\,(x, q), \mu_M\,(y, q)\}$
(ii) $\mu_M\,(gm, q) \leq \mu_M\,(m, q)$,
(iii) $\mu_M\,(x \vee y)q \wedge \mu_M\,(x \wedge y)q \leq S\{\mu_M\,(x, q), \mu_M\,(y, q)\}$for all a, b $\in$ G, x, y and m $\in$ M.

We have
$\max\,\{\mu_M(x, q), \mu_M(y, q)\}= S\,\{\max\{\mu_M\,(x, q), \mu_M\,(y, q)\}, \max\,\{\mu_M\,(x, q), \mu_M\,(y, q)\}$

$\geq S\,\{\mu_M\,(x, q), \mu_M\,(y, q)\}$

$\geq \max\,\{\mu_M\,(x, q), \mu_M\,(y, q)\}$

It follows that $\mu_M\,(ax + by, q) \leq S\,\{\mu_M\,(x, q), \mu_M\,(y, q)\}$

Therefore M = (L, μ_M) is an imaginable anti Q-fuzzy G-modular distributive lattice of L.

Proposition: 3.4 If L is a complete lattice then the intersection of a family of anti Q- fuzzy G-modular distributive lattice is a fuzzy G-modular distributive lattice.

Proof: Let $\{ M_j : j \in J\}$ be a family of anti Q-fuzzy G-modular distributive lattice and let M = $\underset{.}{\cup} M_j$

We have $\mu_M (x, q) = \sup \mu_{M_j} (x, q)$ and $\mu_M (y, q) = \sup \mu_{M_j} (y, q)$

(1) $\mu_M (ax + by, q) = \sup \mu_{M_j} (ax + by, q)$

$\leq \text{Sup} \{\max \{ \mu_{M_j} (x, q), \mu_{M_j} (y, q)\}\}$

$\leq \text{Max} \{\sup \mu_{M_j} (x, q), \sup \mu_{M_j} (y, q)\}$

$\leq \text{Max} \{ \mu_M (x, q), \mu_M (y, q)\}$

(2) $\mu_M (gx, q) = \sup \mu_{M_j} (gx, q)$

$\leq \text{Sup} \mu_{M_j} (x, q)$

$\leq \mu_M (x, q)$

(3) $\mu_M (x \vee y)q \wedge \mu_M (x \wedge y)q = \max \{\mu_M ((x \vee y, q), \mu_M (x \wedge y, q) \}$

$= \max \{\sup \mu_{M_j} (x \vee y, q), \sup \mu_{M_j} (x \wedge y, q) \}$

$= \max \{\sup \max\{ \mu_{M_j} (x, q) , \mu_{M_j} (y, q)\}, \text{Sup} \max\{ \mu_{M_j} (x, q) , \mu_{M_j} (y, q)\}\}$

$\leq \max\{\sup \mu_{M_j} (x, q), \sup \mu_{M_j} (y, q)\}$

$\leq T \{\mu_M(x, q), \mu_M(y, q)\}$, for all $x, y \in L$.

Therefore $\underset{.}{\cup} \mu_{M_j}$ is a anti Q-fuzzy G-modular distributive lattice.

Proposition: 3.5 Any finite dimensional G-modular distributive lattice over a sub lattice L is anti Q- fuzzy G-modular distributive lattice over L.

Proof: Let μ: M $\to$ [0, 1] be a map and M be a finite n-dimensional anti Q-fuzzy G-modular distributive lattice over L.

For all $\alpha, \beta \in L$ and $x, y \in M$

Such that $\mu^n (\alpha x, q) = \mu(n(\alpha x, q) = \mu^n(x, q)$

(1) $\mu^n (\alpha x + \beta y, q) = \mu(n(\alpha x + \beta y, q))$

$= n \mu (\alpha x + \beta y, q)$

$\leq n \{\{\max\{\mu(\alpha x, q) + \mu(\beta y, q)\}$

$\leq \max \{ n\mu(\alpha x, q) , n\mu(\beta y, q)\}$

$\leq \max \{\mu^n(\alpha x, q) , \mu^n (\beta y, q)\}$

$\leq \max \{\mu^n(x, q) , \mu^n(y, q)\}$

(2) $\mu^n (gx, q) = \mu(n(gx, q))$

$= n\mu (gx, q)$

$\leq n\mu(x, q)$

$\leq \mu^n (x, q)$

(3) $\mu^n (x \vee y)q \wedge \mu^n (x \wedge y)q = \max \{\mu^n ((x \vee y, q), \mu^n (x \wedge y, q)\}$

$$= \max \{\mu (n (x \vee y, q), \mu(n (x \wedge y, q)\}$$

$$= \max \{n\mu\{(x \vee y, q) , n\mu (x \wedge y, q)\}$$

$$\leq \max\{n\{\max \{\mu (x, q), \mu (y, q)\}, n\{\max \{\mu (x, q), \mu (y, q)\}\}$$

$$\leq \max \{n\mu(x, q), n\mu(y, q)\}$$

$$\leq \max \{\mu^n(x, q), \mu^n(y, q)\}, \text{ for all } x, y \in M.$$

Therefore, n-dimensional anti Q-fuzzy G-modular distributive lattice is anti Q- fuzzy G-modular distributive lattice.

Proposition: 3.6 If M and N are anti Q-fuzzy G-modular distributive lattices over a fuzzy sub-lattice L then L = M $\oplus$ N is anti Q- fuzzy G-modular distributive lattice on L. Here $\oplus$ is referred as a ring sum.

Proof: Let M and N be two anti Q-fuzzy G-modular distributive lattices of μ_1 and μ_2 respectively. Then we define the ring sum of M. and N as

$$(M \oplus N) (z, q) = \inf_{z \in M \oplus N} \{(\mu_1 + \mu_2) (z, q) / x = f^{-1}(\mu_1,q), y = f^{-1}(\mu_2,q)\}$$

Now

(1) $(M \oplus N) (\alpha x + \beta y, q) = (\mu_1 + \mu_2) (\alpha x + \beta y, q)$

$$\leq \max \{((\mu_1 + \mu_2) (x, q), (\mu_1 + \mu_2) (y, q)\}$$

$$\leq \max \{\inf_{z \in M \oplus N} \mu_1 (z, q), \inf_{z \in M \oplus N} \mu_2 (z, q) \}$$

$$\leq \max \{(M \oplus N) (z, q), M \oplus N (z, q)\}$$

$$\leq \max \{(M \oplus N) (x, q), M \oplus N (y, q)\}$$

(2) $(M \oplus N) (gz, q) = (\mu_1 + \mu_2) (gz, q)$

$$\leq (\mu_1 + \mu_2) (z, q)$$

$$\leq \inf\{\mu_1 + \mu_2 \} (z, q)$$

$$z \in M \oplus N$$

$$\leq M \oplus N (z, q)$$

(3) $(M \oplus N)(x \vee y)q\wedge(M \oplus N)(x \wedge y)q = \max\{(M \oplus N) (x \vee y, q),(M \oplus N)(x\wedge y, q)\}$

$$= \max \{(\mu_1 + \mu_2) (x \vee y, q), (\mu_1 + \mu_2) (x \wedge y, q)\}$$

$$\leq \max \{\max\{(\mu_1 + \mu_2) (x, q), (\mu_1 + \mu_2) (y, q)\},\max\{(\mu_1 + \mu_2) (x, q), (\mu_1+\mu_2) (y, q)\}\}$$

$$\leq \max \{(\mu_1 + \mu_2) (x, q), (\mu_1 + \mu_2) (y, q)\}$$

$$\leq \max \{\inf_{z \in M \oplus N} \mu_1 (z, q), \inf_{z \in M \oplus N} \mu_2 (z, q)$$

$$\leq \max \{(M \oplus N) (x, q), M \oplus N (y, q)\}$$

Therefore, M $\oplus$ N is anti Q- fuzzy G-modular distributive lattices.

Proposition: 3.7 Let M be a G-modular distributive lattices over a sub lattice L and $M = \sum_{i=1}^{n} M_i$, where M_i 's are G sub modular distributive lattice of M. If V_i are anti Q-fuzzy G-modular distributive lattice of M_i then $V : M \to [0, 1]$ is anti Q-fuzzy G-modular distributive lattice in L.

Proof: Since V_i is anti Q- fuzzy G-modular distributive lattice on M_i for every $x, y \in M$;
$g \in L$ and $\alpha, \beta \in L$. We have

(1) $V(\alpha x + \beta y, q) = V(\Sigma(\alpha M_i + \beta M_i', q))$

$= \Lambda (V_i(\alpha M_i + \beta M_i', q))$, where i = 1, 2 … n.

$= V_j(\alpha M_j + \beta M_j', q))$, for some j

$\leq \max (V_j(M_j, q), V_j(M_j', q))$

$\leq \max (V(x, q), V(y, q)\}$

(2) $V(gx, q) = V(\Sigma g M_i, q)$

$= \Lambda (V_i(gM_i, q)$, where i = 1, 2 … n.

$= V_j(gM_j, q)$, for some j

$\leq V_j(M_j, q)$

$\leq V(x, q)$

(3) $V(x \vee y)q \;\Lambda\; V(x \Lambda y)q = \max \{V((x \vee y, q), V(x \Lambda y, q$

$= \max \{V \Sigma (x \vee y, q)M_i, V \Sigma (x \Lambda y, q) M_i'\}$ for i = 1, 2, … n

$= \max \{V(x \vee y, q)M_i, V(x \Lambda y, q) M_i'\}$

$\leq \max\{\max \{V(x, q), V(y, q)\} M_i, \{\max \{V(x, q), V(y, q)\} M_i'\} \max \{V(x, q)Mi, V(y, q)M_i'\}$

$\leq \max \{V_i M_j, V_j M_j'\}$, for some j.

$\leq \max \{V(x, q), V(y, q)\}$

Therefore V is anti Q- fuzzy G-modular distributive lattice on anti Q- fuzzy G-modular distributive lattice in L.

Proposition 3.8: Let λ and μ be anti Q-fuzzy G-modular distributive lattices of L, then $\lambda + \mu$ is the smallest anti Q-fuzzy G-modular distributive lattice of L.

Proof: For any $x, y, h, f \in L$, we have

(1) $\max\{(\lambda + \mu)q\, x, (\lambda + \mu)q\, y\} = \max \{\inf_{x = a + b} \max \{(\lambda(a, q), \mu(b, q)\}, \inf_{y = c + d} \max \{(\lambda(c, q), \mu(d, q)\}\}$

$= \max \{\inf_{x = a + b} (\max \{(\lambda(a, q), \mu(b, q)\}, \max_{y = c + d} \{(\lambda(c, q), \mu(d, q))\}$

$= \max\{\inf_{x = a + b,\; y = c + d} \max \{(\lambda(a, q), \mu(b, q), \lambda(c, q), \mu(d, q)\}\}$

$\geq \max\{\inf_{x = a + b,\; y = c + d} \max (\lambda(a+(b–c–b),q), \mu(b+(c–d–c),q)\}\}$

$$\geq \max\{\inf_{x+y=h+f} \max\{(\lambda(h, q), \lambda(f, q)\}\}$$

$$\geq (\lambda + \mu)q(x + y)$$

Therefore $(\lambda + \mu)(x + y) \leq S\{(\lambda + \mu)q(x), (\lambda + \mu)q(y)\}$

(2) $(\lambda + \mu)(gx, q) = \inf_{x = a + b} \max\{(\lambda(a, q), \mu(b, q)\}$

$$= \inf_{g_x = g_a + g_b} \max\{(\lambda(a, q), \mu(b, q)\}$$

$$\leq \inf_{x = G_a + G_b} \max\{(\lambda(a, q), \mu(b, q)\}$$

$$\leq (\lambda + \mu)(x, q)$$

(3) $(\lambda + \mu)(x \vee y)q \wedge (\lambda + \mu)(x \wedge y)q) = \max\{(\lambda + \mu)(x \vee y)q, (\lambda + \mu)(x \wedge y)q\}$

$$= S\{\max\{(\lambda + \mu)(x, q), (\lambda + \mu)(y, q)\}, \max\{(\lambda + \mu)(x, q), (\lambda + \mu)(y, q)\}\}$$

$$= S\{\max\{\inf \max\{\lambda(x, q), \mu(x, q)\ x = a + b \inf_{y = c + d} \max\{\lambda(y, q), \mu(y,q)\}\},$$

$$= \max\{\inf_{x = a + b} \max\{\lambda(x, q), \mu(x, q)\} \inf_{y = c + d} \max\{\lambda(y, q), \mu(y, q)\}\}\}$$

$$\leq S\{\inf_{y = c + d} \max\{\lambda(y, q), \mu(y, q)\}, \inf_{x = a + b} \max\{\lambda(x, q), \mu(x, q)\}\}$$

$$\leq S\{\inf_{x = a + b} \max\{\lambda(x, q), \mu(x, q)\}, \inf_{y = c + d} \max\{\lambda(y, q), \mu(y, q)\}\}$$

$$\leq S\{(\lambda + \mu)(x, q), (\lambda + \mu)(y, q)\}$$

Therefore $(\lambda + \mu)(x \vee y)q \wedge (\lambda + \mu)(x \wedge y)q \leq S\{(\lambda + \mu)(x, q), (\lambda + \mu)(y, q)\}$

Hence $\lambda + \mu$ is anti Q- fuzzy G-modular distributive lattice on L.

CONCLUSIONS:

I. Jahan introduce the concept of modularity of Ajmal for the lattices of fuzzy ideals of a ring. In this paper we study the characterization of anti Q-fuzzy G-modular distributive lattices. One can obtain the similar results in soft modules and soft lattices by using a suitable mathematical tool.

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