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CHARACTERISTIC FEATURES OF FLOW ENTITIES ON FLOW RATE AND FILM THICKNESS

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ABSTRACT

An unsteady state flow of a visco elastic fluid of Rivlin Ericksen type over a vertical flat plate has been examined in this case. The influence of various parameters on the flow field have been studied in detail. When the visco elasticity of the fluid is held constant and as time increases the flow rate also increases, but such an increase is found to be very minimal. Further, it is noticed that over a period of time, the flow rate does not depend much on the visco elasticity of the fluid. Also, for a fixed time, as the porosity of the bounding surface increases, the flow rate is found to be proportional. An interesting observation is that, if the porosity of the fluid bed is held constant, the flow rate remains unchanged over a period of time. Relatively for smaller values of time, as visco elasticity increases the flow rate is in tune with such a parameter. However, the dispersion is found to be not that significant for higher values of time. It is observed that, increase in pore size of the boundary contributes to marginal increase in the flow rate. It is seen that, over a period of time and as it increases, initially a backward flow is observed and thereafter a forward motion is observed.

Key Words: *Second order fluid, porous boundary, visco elasticity of the fluid, flow rate, film thickness*

INTRODUCTION:

In several problems related to demanding of efficient transfer of mass over inclined beds related to geophysical, petroleum, chemical, bio-mechanical, chemical technology and in situations the viscous drainage over an inclined porous plane is a subject of considerable interest to both theoretical and experimental investigators. Especially, in the flow of oil through porous rock, the extraction of geo-thermal energy from the deep interior of the earth to the shallow layers, the evaluation of the capability of heat removal from particulate nuclear fuel debris that may result from hypothetical accident in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion exchange beds, drug permeation through human glands, chemical reactor for economical separation or purification of mixtures flow through porous medium has been the subject of considerable research activity in recent years due to its notable applications. An important application in the petroleum industry where crude oil is trapped from natural underground reservoirs in which oil is entrapped since the flow behavior of fluids in petroleum reservoir rock depends to a large extent on the properties of the rock, techniques that yield new or additional information on the characteristics of the rock would enhance the performance of petroleum reservoirs. An important bio medical application is the flow of fluids in lungs, blood vessels, arteries and so on, where the fluid is bounded by two layers which are held together by a set of fairly regularly spaced tissues. Slurry adheres to the reactor vessel and gets consolidated in many chemical processing industries, as a result of which chemical compounds within the reactor vessels percolates through the boundaries. Thus adhered substance within the reactor vessel acts as a porous boundary. The problem assumes greater importance in all such situations. The thin film adhering to the surfaces of the container must be taken into account for the purpose of precise chemical calculations in all such situations wherein hear and mass transfer occurs. Failure to do so leads to severe experimental errors. Hence, there is a need for such an analysis in detail. A mathematical model related to such a situation has been studied in detail.

Statistical aggregate of large number of solid particles containing several capillaries such as porous rock can be considered as a porous medium. The true path of individual fluid particle cannot be estimated analytically due to the complexity of microscopic flow in the pores. In all such complex situations, one has to consider the final total gross effect of the phenomena that is being represented by a macroscopic view and applied to the masses of the fluid which are large compared to the dimensions of the pore structure of the medium.

The driving force necessary to move a specific volume of fluid at a certain speed through a porous medium must be in equilibrium with the resistive force generated by internal friction between the fluid and the pore structure. This resistive force is characterised in a more general way and philosophical way which was conformed through experimentation by Darcy's [1] semi empirical law. Darcy's law indicates that for an incompressible fluid flowing through a channel filled with a fixed, uniform and isotropic porous matrix, the flow speed varies linearly with longitudinal pressure variation. Jeffreys [2] initiated the problem of steady state profile over a vertical flat plate which was further examined by Green [3]. Later, Gutfinger and Tallmadge [4] investigated steady state drainage over a vertical cylinder. Later Bhattacharya [5] examined the problem when uniform tangential force $'F'$ acts on the upper surface for a finite interval of time. These authors examined the problem in the absence of fluid inertia.

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The problem of flow of viscous incompressible fluid moving under gravity down a fixed inclined plane with the assumption that the velocity of the fluid at the free surface has been examined earlier by Sneddon.

The constitutive relation of a second order fluid is given by:

$$S_{ij} = -P\delta_{ij} + \phi_1 E_{ij}^{(1)} + \phi_2 E_{ij}^{(2)} + \phi_3 E_{ij}^{(1)^2} \tag{1}$$

where

$$E_{ij}^{(1)} = U_{i,j} + U_{j,i} \tag{2}$$

and

$$E_{ij}^{(2)} = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \tag{3}$$

In the above equations, S_{ij} is the stress-tensor, U_i, A_i are the components of velocity and acceleration in the direction of the i th co-ordinate X_i, P is indeterminate hydrostatic pressure and the coefficients ϕ_1, ϕ_2 and ϕ_3 are material constants.

In view of several industrial and technological importances, Ramacharyulu [6] studied the problem of the exact solutions of two dimensional flows of a second order incompressible fluid by considering the rigid boundaries. The effect of small amplitude wall waviness upon the stability of the laminar boundary layer had been studied by Lessen and Gangwani [7]. Later, Lekoudis *et al* [8] presented a linear analysis of the compressible boundary layer flow over a wall. Subsequently, Shankar and Sinha [9] studied the problem of Rayleigh for wavy wall. Further, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a vertical flat wall was examined by Vajravelu and Shastri [10]. A numerical and experimental investigation of the effects of the presence of a solid boundary and initial forces on mass transfer in porous media was presented by Vafai and Tien [11]. Rao *et al* [12] examined the oscillatory flow of blood through channels of variable cross section under the influence of the magnetic field and presented the investigations in a linear analysis.

Subsequently, Das and Ahmed [13] examined the problem of free convective MHD flow and heat transfer confined between a vertical wavy wall and a parallel flat wall . The free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls was investigated by Patidar and Purohit [14]. Rajeev Taneja and Jain [15] had examined the problem of MHD flow with slip effects and temperature dependent heat in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate.

Recently, Ramana Murthy *et al* [16] presented a detailed analysis of visco elastic fluid of second order type between two parallel plates with the lower plate possessing natural permeability. In their analysis, it has been reported that the skin friction on the upper plate is almost linear with respect to visco elasticity of the fluid. However, the situation seems to be not stated as above at the lower plate. The reason can be attributed to the fact that the lower plate possessing natural permeability. Subsequently, the disturbances due to sinusoidal motion of the bounding surface when the fluid under consideration is visco elastic in nature was examined by Ramana Murthy *et al* [17].

The case of linear analysis by considering visco elasticity of the fluid over an inclined porous plate was studied by Ramana Murthy and Kavitha [18]. Similar such analysis, but of course under the conditions that the bounding surface is rigid has been investigated by Ramana Murthy and Gowthami *et al* [19]. The local volume averaging technique has been used by these authors to establish and solve the governing equations of motion. The numerical solutions of governing equations are used to investigate the mass concentration field inside a porous media close to an impermeable boundary. In conjunction with the numerical solution, a transient mass transfer experiment has been conducted to demonstrate the boundary and inertial effects on the rate of mass transfer. This was accomplished by measuring the time and space averaged mass flux through porous medium. The results clearly indicate the presence of the inertial effects on mass transfer through porous media.

In all above investigations, much of attention has been paid on the characteristic features that influence the velocity profiles and factors affecting there on. An attempt has been made by Ramana Murthy and Kavitha [20] wherein the influence of the participating parameters influencing the skin friction has been examined in detail. Subsequently Kavitha and Ramana Murthy[21] had examined a more microscopic view of film thickness and the flow rate over a vertical flat plate in which the influence of the visco elasticity, time parameter had been examined in detail.

The aim of the present study is to examine the nature of the participating parameters in the field equations that influence the film thickness and flow rate on a vertical porous plate. The concept is of prime importance as most of the reacting compounds within the reaction chamber are mostly visco elastic. Further, many such applications are found in the polymer science/technology.

MATHEMATICAL FORMULATION OF THE PROBLEM:

The problem is examined hereunder with reference to the rectangular Cartesian co-ordinate system with the x - axis along the plate in the direction of the motion and y - axis into the fluid perpendicular to this direction. The motion is assumed to be unidirectional i.e., along x - axis and hence the components of the velocity can be regarded as $\left[u(y,t), 0, 0\right]$

The motion is now governed by the equation in the dimensional form

$$\frac{\partial u}{\partial t} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\left[\phi_1 + \phi_2 \frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial y^2} - g \sin \alpha - \frac{u}{k} \tag{4}$$

where α is the angle of inclination of the plane with the horizontal, ρ is the fluid density which is assumed to be constant throughout and g is the acceleration due to gravity.

The condition of no slip on the boundary would yield $u = 0$ when $y = 0$. Further, the condition of uniform tangential force F on the free surface for a finite interval of time is

$$\left[\phi_1 + \phi_2 \frac{\partial}{\partial t}\right]\frac{\partial u}{\partial y} = F\left[H(t) - H\left(t - t_0\right)\right], t > 0 \tag{5}$$

together, with the initial condition $u\left(y, 0\right) = 0$. (6)

Introducing the following non-dimensionalization scheme

$$\left.\begin{aligned} &X = x/H, \quad Y = y/H, \quad \phi_2 = \rho H^2 \beta \\ &t = \rho H^2 T / \phi_1, \quad u = \phi_1^2 U / (\rho H), \quad k = \rho H^3 /(\phi_1^2 K) \\ &g = \phi_1^2 G / (\rho^2 H^3), \quad P = \phi_1^2\ p /(\rho H^2) \end{aligned}\right\} \tag{7}$$

where β is the non-dimensional visco elastic parameter.

The governing equation for the fluid motion together with the required conditions reduces to

$$\frac{\partial U}{\partial T} = \frac{-\partial P}{\partial X} + \frac{\partial^2 U}{\partial Y^2} + \beta \frac{\partial^3 U}{\partial Y^2 \partial T} - G \sin \alpha - \frac{U}{K} \tag{8}$$

with the condition $U = 0$ when $Y = 0$, (9)

$$\frac{\partial U}{\partial Y} + \beta \frac{\partial^2 U}{\partial Y \partial T} = F\left[H\left(T\right) - H\left(T - T_0\right)\right] \text{ at } Y = 1. \tag{10}$$

$H\left(T\right)$ in eqn. (5) represents the Heavisides Unit step function given by

$$\left.\begin{aligned} H\left(T\right) &= 0 \text{ for } T < 0 \\ &= 1 \text{ for } T > 0 \end{aligned}\right\} \tag{11}$$

Taking Laplace Transforms for eqn. (8), we now have

$$[1 + \beta s]\frac{d^2 \overline{U}}{dY^2} - \left(s + \frac{1}{K}\right)\overline{U} = \frac{1}{s}\left(\frac{\partial P}{\partial X} + G \sin \alpha\right) \tag{12}$$

together with the conditions on the boundary as $\overline{U} = 0$ when $Y = 0$, (13)

$$[1 + \beta s]\frac{d\overline{U}}{dY} = \frac{F}{s}\left[1 - \exp\left(-T_0 s\right)\right] \text{at } \ Y = 1 \tag{14}$$

The solution of eqn. (12) satisfying eqn. (13) and eqn. (14) is given by

$$\overline{U} = \left(\frac{1}{C^2}\right)\left(\frac{D + G \sin \alpha}{s\left(1 + \beta s\right)}\right)\cosh CY + \left[\frac{F\left(1 - e^{-T_0(s)}\right)}{Cs\left(1 + \beta s\right)\cosh C} - \frac{1}{C^2}\left(\frac{D + G \sin \alpha}{s\left(1 + \beta s\right)}\right)\tanh C\right]\sinh CY$$
$$- \frac{1}{C^2}\left(\frac{D + G \sin \alpha}{s\left(1 + \beta s\right)}\right) \tag{15}$$

$$\text{where} \quad C^2 = \frac{s + \left(\frac{1}{K}\right)}{1 + \beta s}, \ D = \frac{\partial P}{\partial X} \tag{16}$$

Taking inverse Laplace transform of eqn. (15) the velocity field is obtained as

$$U = K\left(D+G\sin\alpha\right)\left(\cosh\frac{Y}{\sqrt{K}}-1\right) - K\left(D+G\sin\alpha\right)\left(\tanh\frac{1}{\sqrt{K}}\cdot\sinh\frac{Y}{\sqrt{K}}\right)$$

$$-2F\sum\frac{(-1)^n\sin(2n+1)\frac{\pi}{2}Y}{(2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}}\left[\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)T}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)-\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)(T-T_0)}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)\right]$$

$$+2\left(D+G\sin\alpha\right)\cdot\left[\sum\frac{\sin(2n+1)\frac{\pi}{2}Y}{\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)\left((2n+1)\frac{\pi}{2}\right)}\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)T}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)\right] \tag{17}$$

The complete solution of velocity field is given as

$$U = K\left(D+G\sin\alpha\right)\left(2\cosh\frac{Y}{\sqrt{K}}-2-\left(\tanh\frac{1}{\sqrt{K}}-\cos ech\frac{1}{\sqrt{K}}+\coth\frac{1}{\sqrt{K}}\right)\sinh\frac{Y}{\sqrt{K}}\right)$$

$$-2F\sum\frac{(-1)^n\sin(2n+1)\frac{\pi}{2}Y}{(2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}}\left[\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)T}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)-\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)(T-T_0)}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)\right]$$

$$+2\left(D+G\sin\alpha\right)\left[\sum\frac{\sin(2n+1)\frac{\pi}{2}Y}{\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)\left((2n+1)\frac{\pi}{2}\right)}\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)T}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)\right] \tag{18}$$

Flow rate for eqn. (18) is obtained by $\mathrm{Q}=\int_0^{\mathrm{h}}\mathrm{U}(\mathrm{Y},\mathrm{T})\,\mathrm{dY}$ (19)

$$Q = K^{\frac{3}{2}}\left(D+G\sin\alpha\right)\left(2\sinh\frac{h}{\sqrt{K}}-\frac{2h}{\sqrt{K}}-\left(\tanh\frac{1}{\sqrt{K}}-\cos eh\frac{1}{\sqrt{K}}-\coth\frac{1}{\sqrt{K}}\right)\left(\cosh\frac{h}{\sqrt{K}}-1\right)\right)$$

$$-2\left(D+G\sin\alpha\right)\left[\sum\frac{\left(\cos(2n+1)\frac{\pi}{2}h-1\right)}{\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)\left((2n+1)\frac{\pi}{2}\right)^2}\sum\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)T}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)\right]$$

$$+2F\sum\frac{(-1)^n\left(\cos(2n+1)\frac{\pi}{2}h-1\right)}{\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)(2n+1)\frac{\pi}{2}}\cdot\left[\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)T}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)\right.$$

$$\left.-\exp\left(\frac{-\left((2n+1)^2\frac{\pi^2}{4}+\frac{1}{K}\right)(T-T_0)}{1+(2n+1)^2\frac{\pi^2}{4}\beta}\right)\right] \tag{20}$$

The thickness of the fluid film $Y = h(X,T)$ (21)

is related to the flow rate through the equation of continuity which in this case takes the form $\frac{\partial Q}{\partial X} + \frac{\partial h}{\partial T} = 0$ (22)

Since the eqn. (20) gives Q as a function of h, eqn. (22) can be rewritten as

$$\left(\frac{\partial Q}{\partial h}\right)\left(\frac{\partial h}{\partial X}\right) + \frac{\partial h}{\partial T} = 0 \tag{23}$$

The solution of this partial differential equation (which is of the Lagrange form) can be expressed as

Film thickness $X = \int \frac{\partial Q}{\partial h} dT + \phi(T)$ where $\phi(T) = 0$ (24)

$$X = K\left(D + G\sin\alpha\right)\left(2\cosh\frac{h}{\sqrt{K}} - 2 - \left(\tanh\frac{1}{\sqrt{K}} - \cos ech\frac{1}{\sqrt{K}} + \coth\frac{1}{\sqrt{K}}\right)\sinh\frac{h}{\sqrt{K}}\right)T$$

$$-2\left(D + G\sin\alpha\right)\left[\sum \frac{\left(\sin\left(2n+1\right)\frac{\pi}{2}h\right)\left(1 + \left(2n+1\right)^2\frac{\pi^2}{4}\beta\right)}{\left(\left(2n+1\right)^2\frac{\pi^2}{4} + \frac{1}{K}\right)^2\left(\left(2n+1\right)\frac{\pi}{2}\right)} \exp\left(\frac{-\left(\left(2n+1\right)^2\frac{\pi^2}{4} + \frac{1}{K}\right)T}{1 + \left(2n+1\right)^2\frac{\pi^2}{4}\beta}\right)\right]$$

$$+2F\sum\left(\frac{(-1)^n\left(\sin\left(2n+1\right)\frac{\pi}{2}h\right)\left(1 + \left(2n+1\right)^2\frac{\pi^2}{4}\beta\right)}{\left(\left(2n+1\right)^2\frac{\pi^2}{4} + \frac{1}{K}\right)^2}\right)$$

$$\left[\exp\left(\frac{-\left(\left(2n+1\right)^2\frac{\pi^2}{4} + \frac{1}{K}\right)T}{1 + \left(2n+1\right)^2\frac{\pi^2}{4}\beta}\right) - \exp\left(\frac{-\left(\left(2n+1\right)^2\frac{\pi^2}{4} + \frac{1}{K}\right)\left(T - T_0\right)}{1 + \left(2n+1\right)^2\frac{\pi^2}{4}\beta}\right)\right] \tag{25}$$

RESULTS AND CONCLUSIONS:

1. The influence of visco elasticity on the flow rate has been illustrated in Fig. 1. It is observed that for a constant visco elasticity parameter, as time increases the flow rate increases. But such an increase is found to be very minimal. Also it is observed that over a period of time, the flow rate does not depend much on the visco elastic nature of the fluid. From the illustrations in Fig. 1 and Fig. 2 increase in porosity of the bed is found to be proportional in the case of flow rate.

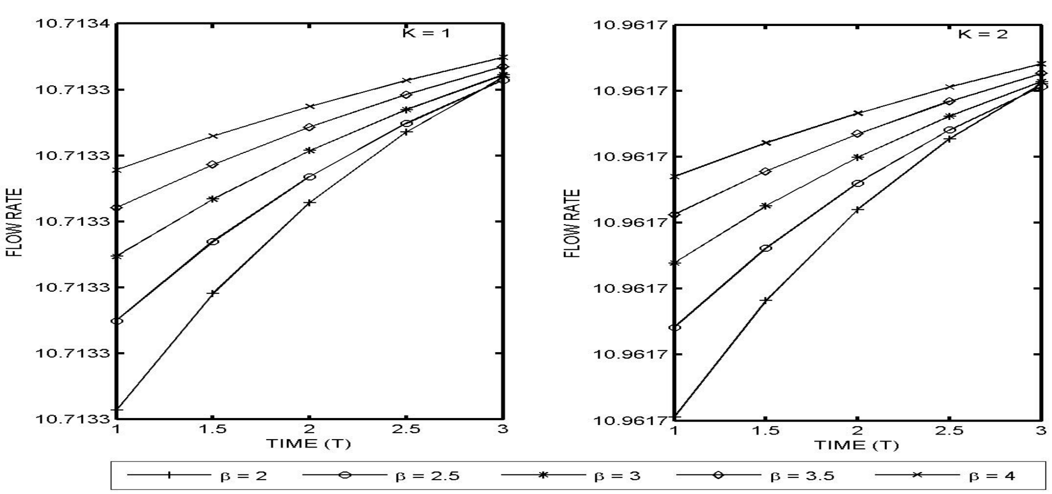

Fig. 1: Influence of Visco elasticity on Flow rate

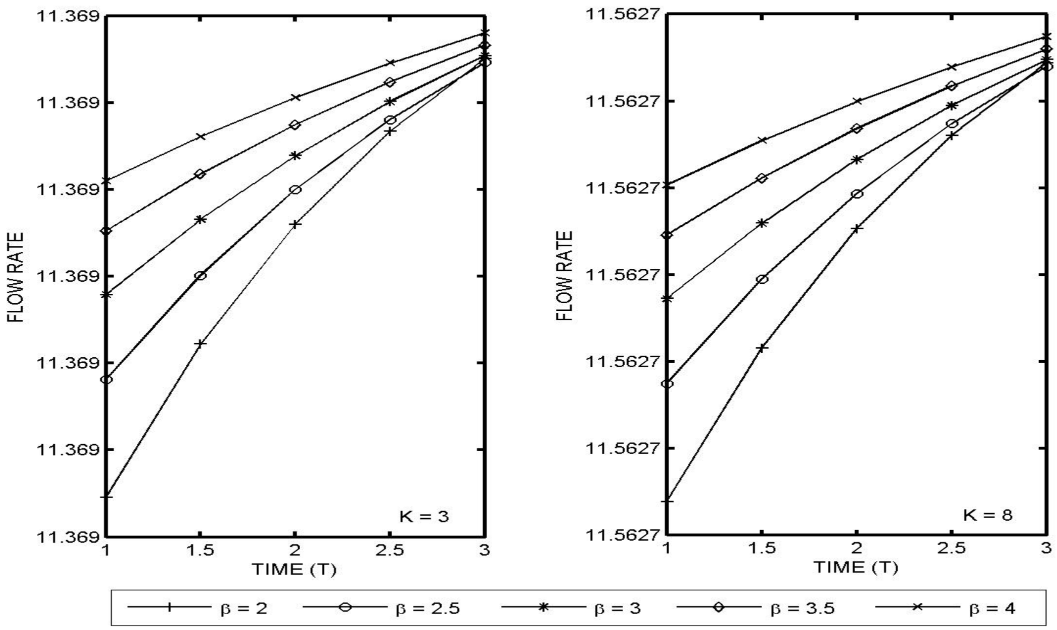

Fig. 2: Effect of visco elasticity on the Flow rate

2. Fig. 3 illustrates the consolidated effect of visco elasticity and time on the nature of flow rate. When the visco elastic nature of the fluid is held constant and as time increases the flow rate increases. Also, as was seen in earlier cases, the effect of visco elasticity does not have profound influence over a period of observation. However, all other features of the flow rate appear to be identical as was stated earlier.

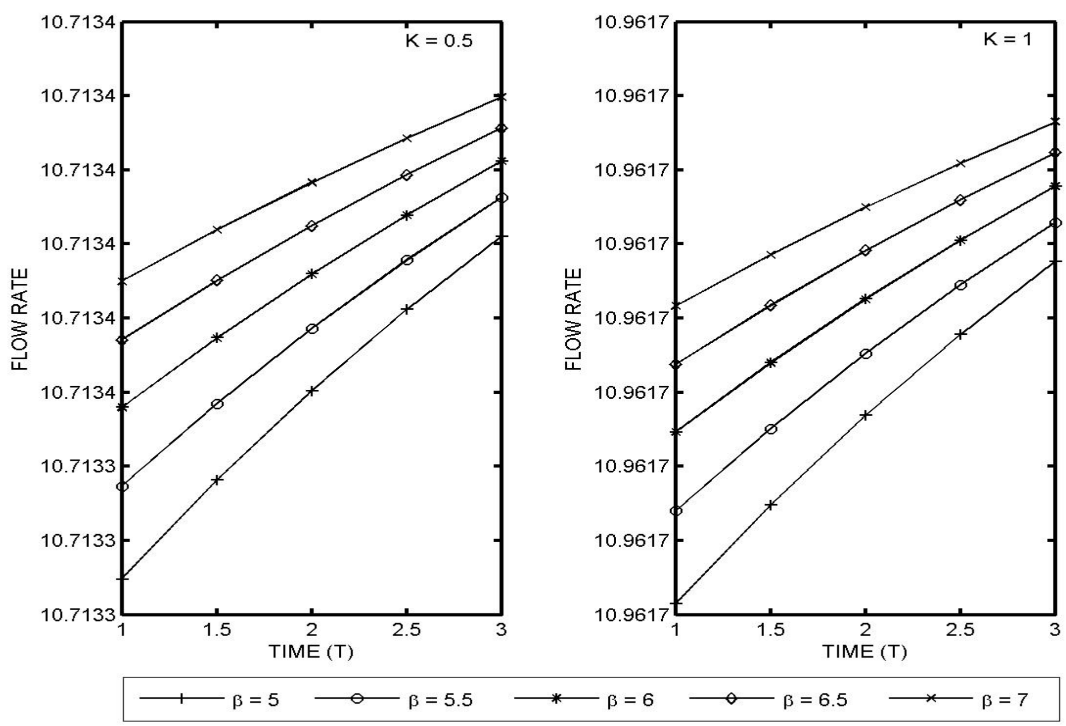

Fig. 3: Combined effect of time and Visco elasticity on flow rate

3 The influence of porosity on flow rate is shown in Fig. 4. It is seen that for a fixed time, as the porosity of the bounding surface increases, the flow rate is found to be increasing. Further, it is noted that for a fixed porosity of the fluid bed, the flow rate remains constant over a period of time. From the illustrations in Fig. 4, it is seen that the marginal increase in the visco elasticity of the fluid medium does not qualitatively alter the nature of the flow rate.

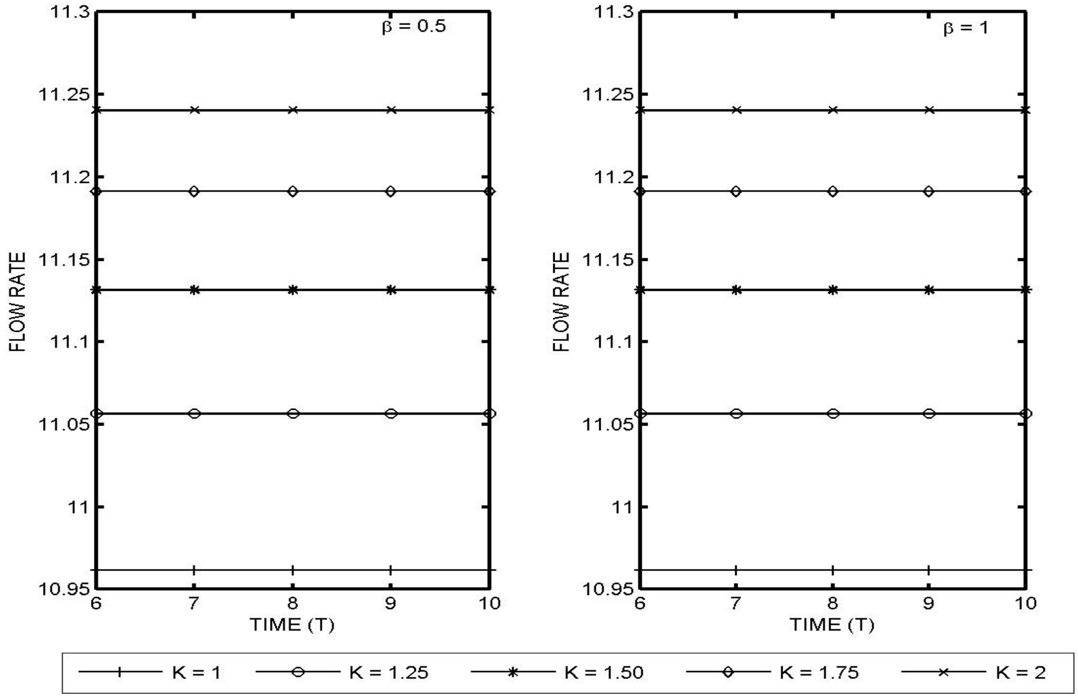

Fig. 4: Influence of porosity on the flow rate

4 Fig. 5 shows the influence of visco elasticity, with respect to time on flow rate. Relatively for smaller values of time, as visco elasticity increases the flow rate also increases. However, the dispersion is found to be not that significant for higher values of time. In each of the illustrations, it is seen that the porosity of the boundary has influence on the flow rate. It is noticed that, increase in pore size of the boundary contributes to marginal increase in the flow rate. Such an effect is anticipated due to the fact that partly the fluid is trapped and drained through the pores while the rest of the fluid flows along the boundary.

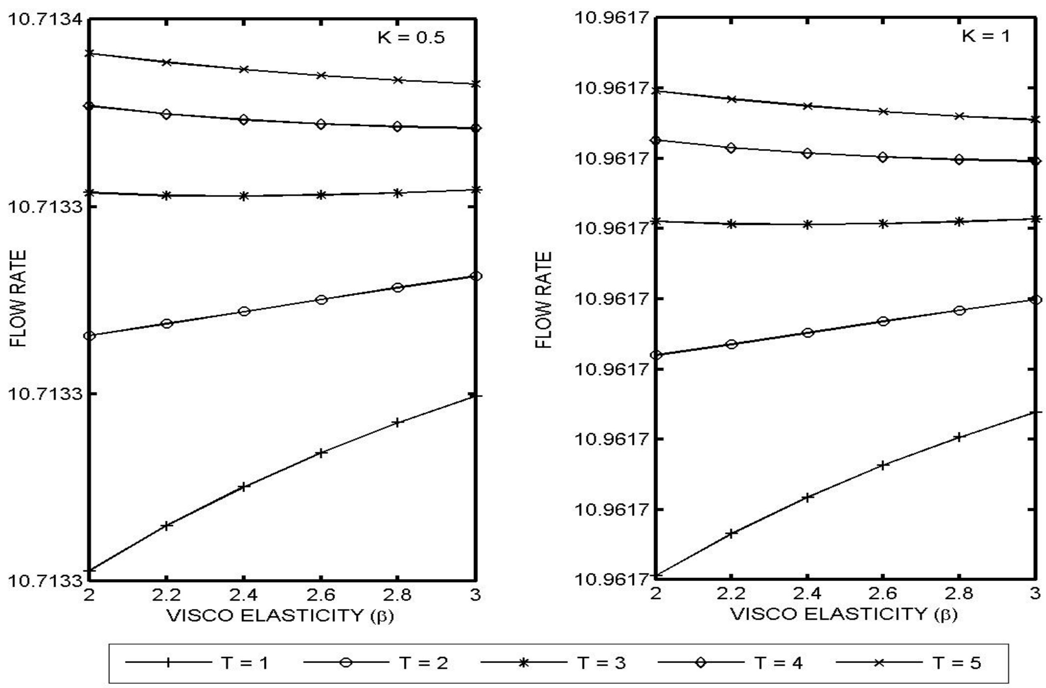

Fig. 5: Influence of Visco elasticity with respect to time on flow rate.

5. The consolidated effect of smaller values of visco elasticity of the fluid and relatively higher values of time are illustrated in Fig. 6. When the visco elasticity of the fluid is held constant and as time increases, the flow rate appears to be increasing. However, for a fixed value of time, as visco elasticity of the fluid is increased, the flow rate appears to be decreasing. Such an effect is anticipated due to the strong intra molecular forces that prevail in the fluid as viscous and elastic forces are increased. From both the illustrations, it is observed that in general a marginal increase in pore size of the boundary contributes to decrease in the flow rate.

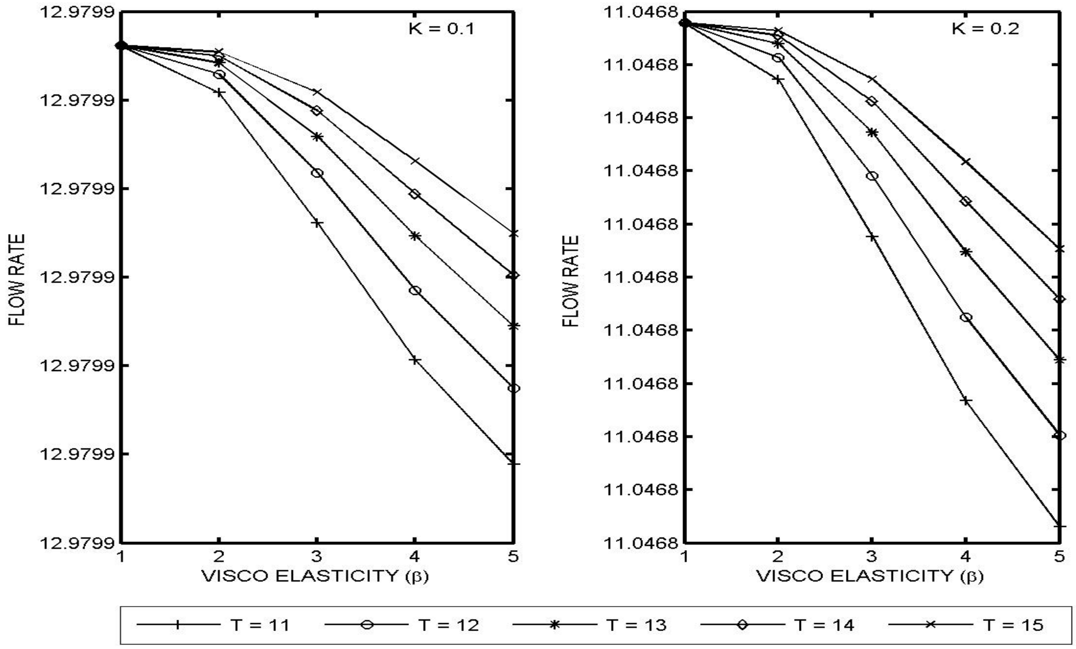

Fig. 6: The effect of time Vs. Visco elasticity on flow rate

6. Fig. 7 and fig. 8 illustrates the influence of time on the film thickness. It is seen that as T increases, initially a backward flow is observed and thereafter a forward motion is noticed. The film adhering to the walls of the boundary is found to be increasing as T increases. The phenomenon represented in fig. 7 appears to be identical even when the pore size of the bounding surface is changed. The flow pattern represented in fig. 7 which is for smaller values of time and in fig. 8 for larger values of time appears to be identical.

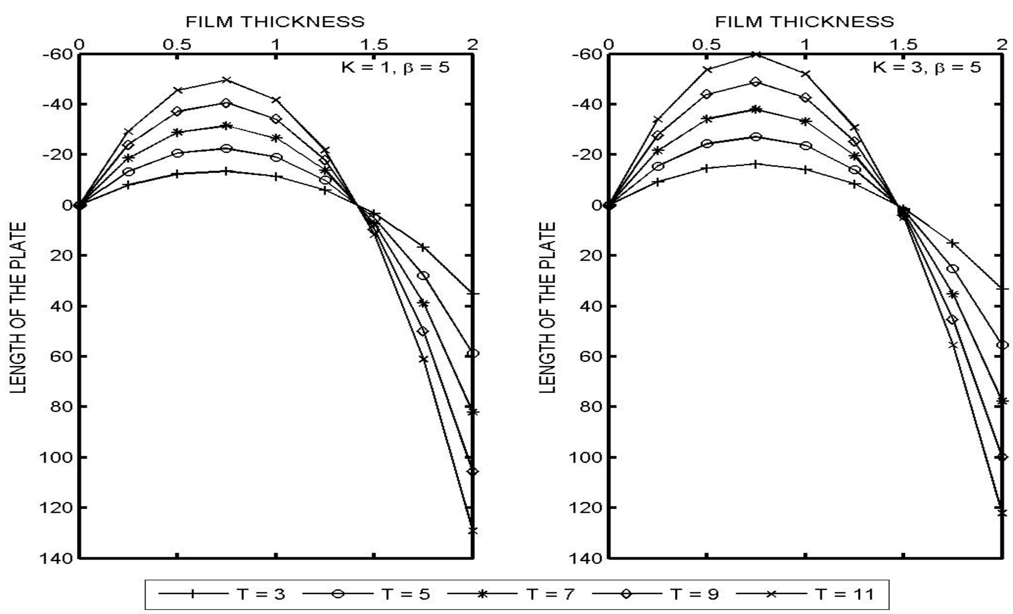

Fig. 7: The influence of time on film thickness

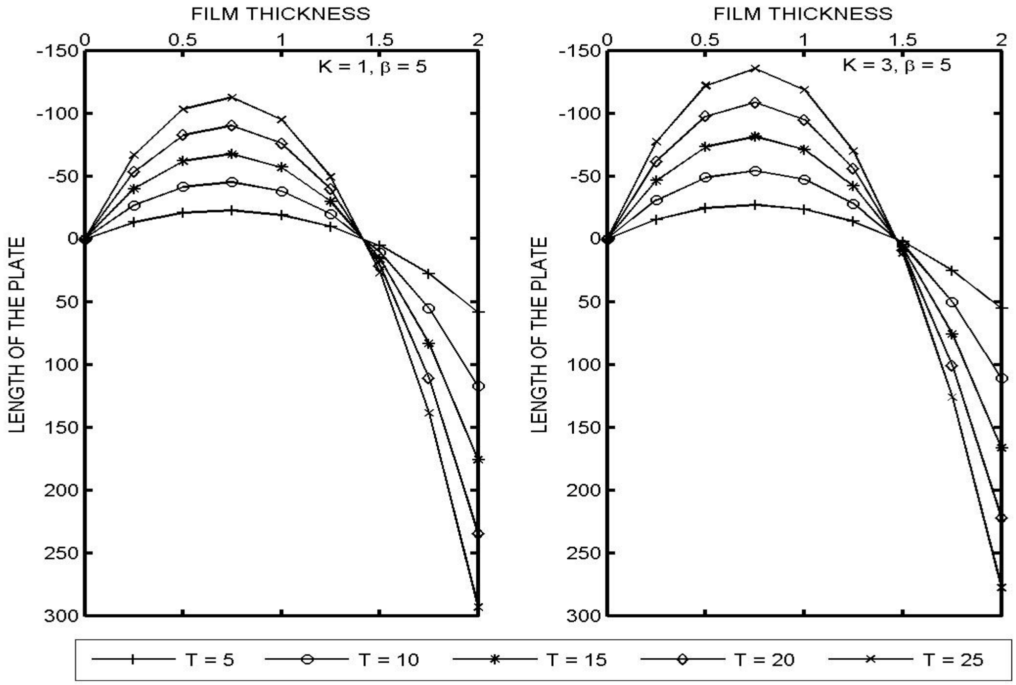

Fig.8: The influence of time on film thickness

7. The consolidated effect of visco elasticity and smaller values of T (fig. 9) and for larger values of T (fig. 10) is examined. It is noticed that as T increases, a backward flow is seen and thereafter, the forward motion is observed. The backward flow is due to the pores on the bounding surface and as the pores are fully saturated, the forward motion is observed. In the illustrations cited above, the influence of visco elasticity is not found much but, the contribution of the porosity of the boundary is noticed. In fig. 9 and fig. 10 it is seen that as the pore size of the medium is increased, the flow appears to be decreasing which is in tune with the earlier observations.

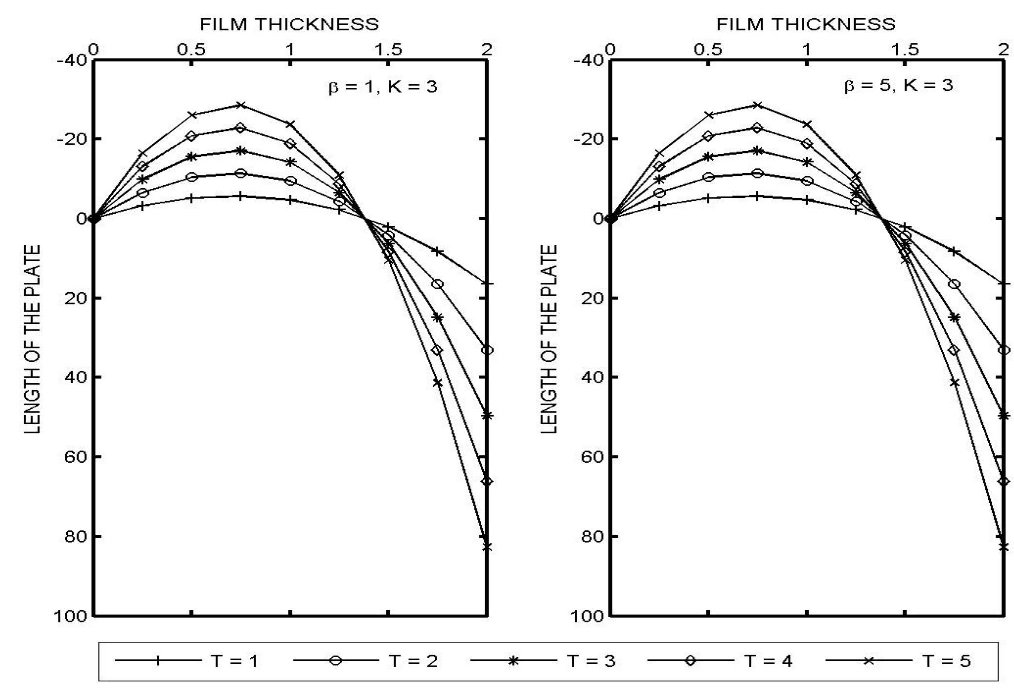

Fig. 9: The consolidated effect of visco elasticity and time on film thickness

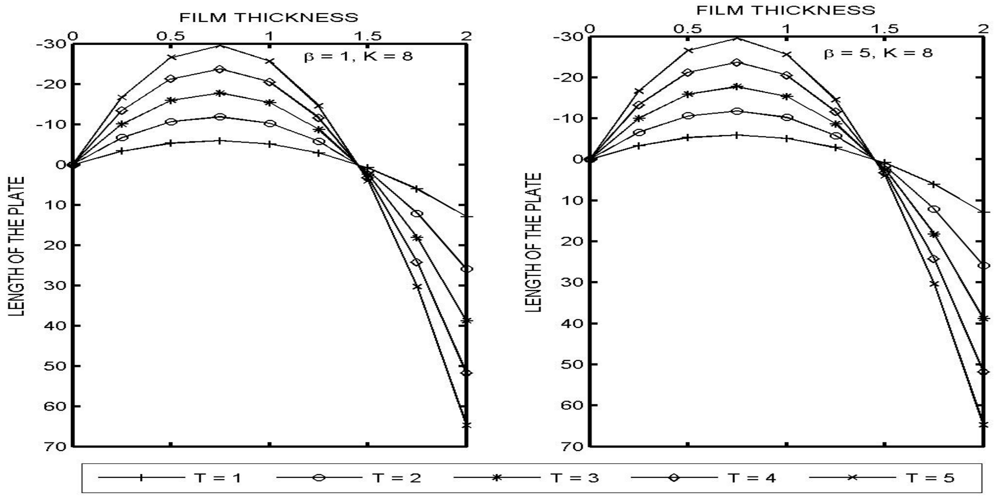

Fig. 10: The consolidated effect of visco elasticity and time on film thickness

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