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2 - EQUITABLE DOMINATION IN GRAPHS

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ABSTRACT

Let $G=(V,E)$ be a graph. A subset D of $V(G)$ is called an equitable dominating set of a graph G if for every $v\in(V-D)$, there exists a vertex $v\in D$ such that $uv\in E(G)$ and $|deg(u)-deg(v)|\le 1$. An equitable dominating set D is said to be a connected equitable dominating set if the subgraph $\langle D\rangle$ induced by D is connected. In this paper we introduce the 2- equitable domination and 2-connected equitable domination in a graph, bounds and exact values for some standard graphs are found.

Keywords*:equitable domination number, 2-equitable dominating set, 2-connected equitable dominating.*

Mathematics Subject Classification: *05C69.*

1. INTRODUCTION

Introduction: By a graph $G=(V,E)$ we mean a finite, undirected with neither loops nor multiple edges the order and size of G are denoted by p and q respectively for graph theoretic terminology we refer to Chartrand and Lesnaik [1] A subset S of V is called a dominating set if $N[S]=V$ the minimum (maximum) cardinality of a minimal dominating set of G is called the domination number (upper domination number) of G and is denoted by $\gamma(G)$, $(\Gamma(G))$. An excellent treatment of the fundamentals of domination is given in the book by Haynes etal [4] A survey of several advanced topics in domination is given in the book edited by Haynes et al. [5]. Various types of domination have been defined and studied by several authors and more than 75 models of domination are listed in the appendix of Haynes et al. [4]. Sampathkumar and Walikar [7] introduced the concept of connected domination in graphs. Let $G=(V,E)$ be a graph and let $v\in V$ the open neighborhood and the closed neighborhood of v are denoted by $N(v)$ and $N[v]=N(v)\cup v$ respectively. If $S\subseteq V$ then $N(S)=\cup_{v\in S}N(v)$ and $N[S]=N(S)\cup S$. If $S\subseteq V$ and $u\in S$ then the private neighbor set of u with respect to S is defined by $Pn[u,S]=\{v:N[v]\cap S=\{u\}\}$.

A dominating set S of G is called a connected dominating set if the induced subgraph $\langle S\rangle$ is connected the minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by $\gamma_c(G)$. A dominating set S of a connected graph G is called a neighborhood connected dominating set (ncd-set) if the induced subgraph $\langle N(S)\rangle$ is connected. The minimum cardinality of a ncd-set of G is called the neighborhood connected domination number of G and is denoted by $\gamma_{nc}(G)$. A ncd-set S is said to be minimal if no proper subset of S is a ncd-set. A coloring of a graph G is an assignment of colors to the vertices of G such that no two adjacent vertices receive the same color. The minimum integer K for which a graph G is $k-$colorable is called the chromatic number of G and is denoted by $\chi(G)$.

A subset S of V is called an equitable dominating set if for every $v\in V-S$ there exist a vertex $u\in S$ such that $uv\in E(G)$ and $|d(u)-d(v)|\le 1$. The minimum cardinality of such an equitable dominating set is denoted by γ_e and is called the equitable domination number of G. A vertex $u\in V$ is said to be degree equitable with a vertex

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$v \in V$ if $|d(u)-d(v)| \leq 1$. If S is an equitable dominating set then any super set of S is an equitable dominating set. An equitable set S is said to be a minimal equitable dominating set if no proper subset of S is an equitable dominating set. The minimal upper equitable dominating number is Γ_e the upper equitable dominating set of G. If $u \in V$ such that $|d(u)-d(v)| \geq 2$ for every $v \in N(u)$ then u is in every equitable dominating set such points are called an equitable isolated. I_e denotes the set of all equitable isolates. An equitable dominating S of connected graph G is called an equitable connected dominating set (ecd-set) if the induced subgraph $\langle S \rangle$ is connected. The minimum cardinality of a ecd-set of G is called the equitable connected domination number of G and is denoted by $\gamma_{ec}(G)$. Let $G=(V,E)$ be a graph and let $u \in V$ the equitable neighborhood of u denoted by $N_e(u)$ is defined as $N_e(u) = \{v \in V : |v \in N(u), |d(u)-d(v)| \leq 1$. The maximum and minimum equitable degree of a point in G are denoted by $\Delta_e(G)$ and $\delta_e(G)$ that is $\Delta_e(G) = max_{u \in V(G)} |N_e(u)|$ and $\delta_e(G) = min_{u \in V(G)} |N_e(u)|$. The open equitable neighbourhood and closed equitable neighbourhood of v are denoted by $N_e(v)$ and $N_e[v] = N_e(v) \cup \{v\}$ respectively. If $S \subseteq V$ then $N_e(S) = \cup_{v \in S} N_e(v)$ and $N[S] = N_e(S) \cup S$. If $S \subseteq V$ and $u \in S$ then the private equitable neighbor set of u with respect to S is defined by $pne[u,S] = N_e[u] - N_e[S-\{u\}]$.

If G is connected graph, then a vertex cut of G is a subset R of $V(G)$ with the property that the subgraph of G induced by $V(G)-R$ is disconnected. If G is not a complete Graph, then the vertex connectivity number $k(G)$ is the minimum cardinality of a vertex cut. If G is complete graph K_p it is known that $k(G) = p-1$

2. 2-EQUITABLE DOMINATION IN A GRAPH

Definition. Let $G=(V,E)$ be a graph. An equitable dominating set S of a graph G is called 2-equitable dominating set (2-ed-set) if for any vertex v in G either $v \in S$ or v is equitable dominated by at least 2 vertices in S . The minimum cardinality of a an 2-equitable dominating set of G is called the 2-equitable domination number of G and is denoted by $\gamma_{\times 2e}(G)$.

Proposition 2.1. *The 2-equitable domination number of some standard graphs are*

(1) $\gamma_{\times 2e}(K_p) = 2$

(2) $\gamma_{\times 2e}(P_p) = \lfloor \frac{p+2}{2} \rfloor$

(3) $\gamma_{\times 2e}(C_p) = \lfloor \frac{p+1}{2} \rfloor$

(4) $\gamma_{\times 2e}(W_p) = \begin{cases} 1+\lfloor \frac{p}{2} \rfloor, & if \quad p \geq 5; \\ 2, & otherwise. \end{cases}$

(5) $\gamma_{\times 2e}(K_{r,t}) = \begin{cases} r+t, & if \ |r-t| \geq 2; \\ 4, & otherwise. \end{cases}$

Proposition 2.2. *For any graph* G ,

(1) $\gamma(G) \leq \gamma_e(G) \leq \gamma_{\times 2e}(G)$.

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(2) $\gamma_{\times 2}(G) \le \gamma_{\times 2e}(G)$.

Proof. From the definition of the 2-equitable dominating set of a graph G, it is clearly that for any graph G any 2-equitable dominating set D is also an equitable dominating set and every equitable dominating set is also dominating set. Hence $\gamma(G) \le \gamma_e(G) \le \gamma_{\times 2e}(G)$. Similarly (2) since every 2-equitable dominating set is 2-dominating set for any graph G. Hence $\gamma_{\times 2}(G) \le \gamma_{\times 2e}(G)$.

Observation 2.3. *In any graph G, if v be any vertex of equitable degree less than two, then v must be in every 2-equitable dominating set.*

A 2-equitable dominating set S is said to be minimal if no proper subset of S is 2-equitable dominating set.

Theorem 2.4. *Let $G = (V, E)$ be a graph. An 2-equitable dominating set S of G is minimal if and only if for every vertex $v \in S$, either*

(i) $|N_e(v) \cap S| < 2$, or

(ii) There exists a vertex $u \in V - S$ such that $|N_e(v) \cap S| = 2$ and $u \in N_e(v)$.

Proof. Let S be a minimal 2-equitable dominating set of G. Suppose that the two condition (i) and (ii) are not hold. That is there exist a vertex v in S such that $|N_e(v) \cap S| \ge 2$ and for every vertex $u \in V - S$ either $|N_e(v) \cap S| > 2$ or $u \notin N_e(v)$ and consider $S' = S - \{v\}$ and since v equitable adjacent to at least 2 vertices of S'. Therefore S' is an 2-equitable dominating set, a contradiction with minimality of S.

Conversely, let S be an 2-equitable dominating set of G satisfying the conditions (i)and (ii). Consider $S' = S - \{v\}$ for any vertex $v \in S$. Now if (i) holds then S' is not 2-equitable dominating set, and if (ii) holds then there exist a vertex $u \in V - S$ such that $|N_e(v) \cap S| = 2$ and $u \in N_e(v)$ and in this case S' is not an 2-equitable dominating set(because S' not 2-equitable dominate u). Therefore in the two cases S' is not an 2-equitable dominating set. Hence S is S' a minimal 2-equitable dominating set of G.

Proposition 2.5. *For any graph G with p vertices,*

(1) $2 \le \gamma_{\times e}(G) \le p$

(2) $\gamma_{\times e}(G) = p$ if and only if $\Delta_e(G) < 2$.

$\gamma_{\times e}(G) = 2$ if there exist at least two vertices $v, u \in G$, such that $deg_e(v) = deg_e(u) = p-1$ or $deg_e(v) = deg_e(u) = p-2$

Theorem 2.6. *Let G be a graph with $\delta_e(G) \ge 2$. If S is a minimal 2-equitable dominating set, then $V - S$ contains a minimal equitable dominating set.*

Proof. Let S be a minimal 2-equitable dominating set of G. Suppose that $V - S$ is not an equitable dominating set, then there exist at least one vertex $v \in S$ which is not equitable adjacent to any vertex in $V - S$. Therefore is equitable adjacent to at least two vertices in S. Then $S - \{v\}$ is an 2-equitable dominating set a contradiction. Hence every vertex in S must be equitable adjacent to at least one vertex in $V - S$. Hence $V - S$ is an equitable dominating set which contains minimal equitable dominating set.

Observation 2.7. *Every 2-equitable dominating set of a graph G contains the leaves and support vertices of G.*

Proposition 2.8. *Let G be a connected graph has no non-equitable edge and H is spanning subgraph of G. Then $\gamma_{\times 2e}(G) \le \gamma_{\times 2e}(H)$.*

Proof. Let G be a connected Graph and let H is the spanning subgraph of H. Suppose That D is the minimum 2-equitable dominating set of G. Then D also an 2-equitable dominate all the vertices in $V(H)-D$ that is D is an 2-equitable dominating set in H. Hence $\gamma_{\times 2e}(G) \leq \gamma_{\times 2e}(H)$.

Theorem 2.9. *For any graph* G, $\gamma_e(G)+1 \leq \gamma_{\times 2e}(G)$.

Proof. Let S be γ_e -set. Then for any vertex $v \in S$, $S-\{v\}$ is equitable dominating set. Hence $\gamma_e(G)+1 \leq \gamma_{\times 2e}(G)$. Further if $G = C_4$, the equality hold.

3. CONNECTED 2-EQUITABLE DOMINATION IN A GRAPH

Definition. Let $G=(V,E)$ be a graph. An 2-equitable dominating set $D \subseteq V(G)$ if the subgraph of G induced by D is connected. The connected 2-equitable domination of G is the size of its smallest connected 2-equitable dominating set, and is denoted by $\gamma_{\times 2ce}$.

Proposition 3.1. *The connected 2-equitable domination number of some standard graphs are*

(1) $\gamma_{\times 2ce}(K_p) = 2$

(2) $\gamma_{\times 2ce}(P_p) =$

(3) $\gamma_{\times 2ce}(C_p) = p-1$

(4) $\gamma_{\times 2ce}(W_p) = \begin{cases} p-1, & if \quad p \geq 5; \\ 2, & otherwise. \end{cases}$

(5) $\gamma_{\times 2ce}(K_{r,t}) = \begin{cases} r+t, & if \ |r-t| \geq 2; \\ 4, & otherwise. \end{cases}$

Observation 3.2. *For any tree* T *with* p *vertices,*

$$\gamma_{\times 2ce}(T) = p.$$

It is clear for any graph G, any connected 2-equitable dominating set is 2-equitable dominating set and any 2-equitable dominating set is equitable dominating set, and also any equitable dominating set is dominating set, then the following proposition is straightforward.

Proposition 3.3. *For any Graph,* G

$$\gamma(G) \leq \gamma_e(G) \leq \gamma_{\times 2e}(G) \leq \gamma_{\times 2ce}(G).$$

Theorem 3.4. *For any connected graph* G *with diameter equal to* k.

$$k-1 \leq \gamma_{\times 2ce}(G).$$

Proof. Let u and v be any two vertices such that the distance between them $d(u,v) = k$, if $u,v \in D$, then D has at least $k+1$ vertices, if $u \in D$ but $v \notin D$, then since v must be adjacent to at least two vertices in D. hence $|D| \geq k$, and if u and v both not in D clearly $|D| \geq k-1$.

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