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RELATED FIXED POINT THERREM FOR TWO PAIRS OF SETVALUED MAPPINGS ON TWO UNIFORM SPACE

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ABSTRACT

A related fixed point theorem for two pairs of set-valued mappings on two complete uniform spaces is obtained. A generalization for two compact uniform spaces is also obtained.

Keywords: *Fixed point, set-valued mappings, complete metric space, compact metric space, complete uniform space, compact uniform space.*

2000 Mathematics subject classification: *54H25, 47H10.*

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1. INTRODUCTION

The following theorems were proved by Ranjit and Rohen [12].

Theorem 1.1: *Let (X,d) and (Y,ρ) be two complete metric spaces. Let A, B be mappings of X into Y and let S, T be mappings of Y into X satisfying the inequalities*

$$d(Sy,Ty')d(SAx,TBx') \le c\max\{d(Sy,Ty')\rho(Ax,Bx'), d(x',Sy)\rho(y',Ax), d(x,x')d(Sy,Ty'), d(Sy,SAx)d(Ty',TBx')\}$$

$$\rho(Ax,Bx')\rho(BSy,ATy') \le c\max\{d(Sy,Ty')\rho(Ax,Bx'), d(x',Sy)\rho(y',Ax), \rho(y,y')\rho(Ax,Bx'), \rho(Ax,BSy)\rho(Bx',ATy')\}$$

for all x, x' in X and y, y' in Y, where $0 \le c < 1$. If one of the mappings A, B, S and T is continuous, then SA and TB have a unique common fixed point z in X and BS and AT have a unique common fixed point w in Y.

Further Az = Bz = w and Sw = Tw = z.

The following theorem was proved by Rohen [16].

Theorem 1.2: *Let (X,d_1) and (Y,d_2) be two complete metric spaces. Let A, B be mappings of X into B(Y) and let S, T be mappings of Y into B(X) satisfying the inequalities*

$$\delta_1(Sy,Ty')\delta_1(SAx,TBx') \le c\max\{\delta_1(Sy,Ty')\delta_1(Ax,Bx'), \delta_1(x',Sy)\delta_2(y',Ax), d_1(x,x')\delta_1(Sy,Ty'), \delta_1(Sy,SAx)\delta_1(Ty',TBx')\}$$

$$\delta_2(Ax,By')\delta_2(BSy,ATx') \le c\max\{\delta_1(Sy,Ty')\delta_2(Ax,Bx'), \delta_1(x',Sy)\delta_2(y',Ax), d_2(y,y')\delta_2(Ax,Bx'), \delta_2(Ax,BSy)\delta_2(Bx',ATy')\}$$

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for all x , x' in X and y , y' in Y , where $0 \le c < 1$. If one of the mappings A, B, S and T is continuous , then SA and TB have a unique common fixed point z in X and BS and AT have a unique common fixed point w in Y . Further Az = Bz = w and Sw = Tw = z.

Before coming to our main results we recall the following definitions from Fisher and Turkoglu [15].

Let (X, U_1) and (Y, U_2) be uniform spaces. Families $\{ d_1^i : i \in I,$ being indexing set}, $\{ d_2^i : i \in I \}$ of pseudometrics on X and Y respectively, are called associated families for uniformities U_1, U_2 respectively, if families

$$\beta_1 = \{ V_1(i, r) : i \in I, r > 0 \},$$
$$\beta_2 = \{ V_2(i, r) : i \in I, r > 0 \},$$

where

$$V_1(i,r) = \{(x, x') : x, x' \in X, d_1^i(x, x') < r\},$$
$$V_2(i,r) = \{(y, y') : y, y' \in Y, d_2^i(y, y') < r\}$$

are sub bases for the uniformities U_1, U_2 respectively. We may assume that β_1, β_2 themselves are bases by adjoining finite intersection of members of β_1, β_2 if necessary. The corresponding families of pseudo metrics are called an augmented associated families for U_1, U_2. An associated family for U_1, U_2 will be denoted by D_1, D_2 respectively.

Let *A*, *B* be a non empty subset of a uniform space *X*, *Y* respectively. Define

$$P_1^*(A) = \sup \{ d_1^i(x, x') : x, x' \in A, i \in I\}$$
$$P_2^*(B) = \sup \{ d_2^i(y, y') : y, y' \in B, i \in I \}$$

where

$$\{d_1^i(x, x') : x, x' \in A, i \in I\} = P_1^*,$$
$$\{d_2^i(y, y') : y, y' \in B, i \in I\} = P_2^*.$$

Then $P_1^*(A), P_2^*(B)$ are called an augmented diameter of *A*, *B*. Further *A*, *B* are said to be $P_1^*(A) < \infty, P_2^*(B) < \infty$.

Let 2^X = {A: A is a non empty P_1^*- bounded subset of *X*}
2^Y = {B: B is a non empty P_2^*- bounded subset of *Y*}.

For each $i \in I$ and $A_1, A_2 \in 2^X$, $B_1, B_2 \in 2^Y$, define

$$\delta_1^i(A_1, A_2) = \sup\{d_1^i : x \in A_1, x' \in A_2\}$$
$$\delta_2^i(B_1, B_2) = \sup\{d_2^i : y \in B_1, y' \in B_2\}$$

Let (X, U_1) and (X, U_2) be uniform spaces and let $\mathrm{U}_1 \in U_1$ and $\mathrm{U}_2 \in U_2$ be arbitrary entourages.

For each $A \in 2^X, B \in 2^Y,$ define

$$\mathrm{U}_1[\mathrm{A}] = \{ x' \in X : (x, x') \in \mathrm{U}_1 \text{ for some } x \in A \}$$
$$\mathrm{U}_2[\mathrm{B}] = \{ y' \in Y : (y, y') \in \mathrm{U}_2 \text{ for some } y \in B \}$$

The uniformities 2^{U_1} on 2^X and 2^{U_2} on 2^Y are defined by bases

$$2^{\beta_1} = \{\tilde{\mathrm{U}}_1 : \mathrm{U}_1 \in U_1\},\ 2^{\beta_2} = \{\tilde{\mathrm{U}}_2 : \mathrm{U}_2 \in U_2\}$$

where

$$\tilde{\mathrm{U}}_1 = \{(A_1, A_2) \in 2^X \times 2^Y : A_1 \times A_2 \in \mathrm{U}_1\} \cup \Delta,$$
$$\tilde{\mathrm{U}}_2 = \{ (B_1, B_2) \in 2^Y \times 2^Y : B_1 \times B_2 \in \mathrm{U}_2\} \cup \Delta.$$

where Δ denotes the diagonal on $X \times X$ and $Y \times Y$. The augmented families P_1^*, P_2^* also induce uniformities U_1^* on $2^X, U_2^*$ on 2^Y defined by bases

$$\beta_1^* = \{V_1^*(i,r) : i \in I, r > 0\},$$
$$\beta_2^* = \{V_2^*(i,r) : i \in I, r > 0\}.$$

where

$$V_1^*(i,r) = \{(A_1, A_2) : A_1, A_2 \in 2^X : \delta_1^i(A_1, A_2) < r\} \bigcup \Delta$$
$$V_2^*(i,r) = \{(B_1, B_2) : B_1, B_2 \in 2^Y : \delta_2^i(B_1, B_2) < r\} \bigcup \Delta$$

Uniformities 2^{U_1} and U_1^* on 2^X are uniformly isomorphic and uniformities 2^{U_2} and U_2^* on 2^Y are uniformly isomorphic. The spaces $(2^X, U_1^*)$ is thus a uniform space called the hyperspace of (X, U_1). The $(2^Y, U_2^*)$ is also a uniform space called the hyperspace of (Y, U_2).

Now let $\{A_n : n = 1,2,........\}$ be a sequence of non empty subsets of uniform space (X, U). We say that sequence $\{A_n\}$ converge to subset A of X if

(i) each point a in A is the limit of a convergent sequence $\{a_n\}$, where a_n is in A_n for n =1,2,…….

(ii) for arbitrary $\varepsilon > 0$, there exist an integer N such that $A_n \subseteq A_\varepsilon$ for $n > N$,

where $A_\varepsilon = \bigcup_{x \in A} U(x) = \{ y \in X : d_i(x,y) < \varepsilon$ for some x in A, $i \in I \}$

A is then said to be a limit of the sequence $\{A_n\}$. It follows easily from the definition that if A is the limit of a sequence $\{A_n\}$, then A is closed.

The following lemma was proved by Fisher and Turkoglu [15].

Lemma 1.3: *If $\{A_n\}$ and $\{B_n\}$ are sequences of bounded non empty subsets of a complete uniform space (X, U) which converge to the bounded subsets A and B respectively , then sequence $\{\delta_i(A_n, B_n)\}$ converges to $\delta_i(A, B)$.*

2. MAIN RESULTS

We prove the following theorems.

Theorem 2.1: *Let (X, U_1) and (X, U_2) be two complete Hausdorff uniform spaces defined by $\{d_1^i, i \in I\} = P_1^*, \{d_2^i, i \in I\} = P_2^*$, and $(2^X, U_1^*), (2^Y, U_2^*)$ hyperspaces, let $Q, R : X \to 2^Y$ and $S, T : Y \to 2^X$ satisfying the inequalities*

$$\delta_1^i(Sy, Ty')\delta_1^i(SQx, TRx') \le c_i \max\{\delta_1^i(Sy, Ty')\delta_2^i(Qx, Rx'), \delta_1^i(x', Sy)\delta_2^i(y', Qx), d_1^i(x, x')\delta_1^i(Sy, Ty'), \delta_1^i(Sy, SQx)\delta_1^i(Ty', TRx')\} \quad (1)$$

$$\delta_2^i(Qx, Rx')\delta_2^i(RSy, QTy') \le c_i \max\{\delta_1^i(Sy, Ty')\delta_2^i(Qx, Rx'), \delta_1^i(x', Sy)\delta_2^i(y', Qx), d_2^i(y, y')\delta_2^i(Qx, Rx'), \delta_2^i(Qx, RSy)\delta_2^i(Rx', QTy')\} \quad (2)$$

for all $i \in I$ and x, x' in X and y, y' in Y , where $0 \le c_i < 1$. If one of the mappings Q, R, S, T is continuous, then SQ and TR have a unique common fixed point z in X and RS and QT have a unique fixed point w in Y. Further, Qz = Rz = w and Sw = Tw = z.

Proof: Let x_0 be an arbitrary point in X and define sequences $\{x_n\}$ and $\{y_n\}$ in X and Y respectively as follows. Choose a point y_1 in Qx_0, a point x_1 in Sy_1, a point y_2 in Rx_1 and a point x_2 in Ty_2. In general, having chosen x_{2n-2} in X, choose a point y_{2n+1} in Qx_{2n}, a point x_{2n-1} in Sy_{2n-1}, a point y_{2n} in Rx_{2n-1} and a point x_{2n} in Ty_{2n} for n = 1, 2 …

Let $\mathrm{U}_1 \in U_1$ be an arbitrary entourage. Since β_1 is a base for U_1, there exists $V_1(i,r) \in \beta_1$ such that $V_1(i,r) \subseteq \mathrm{U}_1$.

Using inequality (1), we get

$$d_1^i(x_{2n-1},x_{2n})d_1^i(x_{2n+1},x_{2n})=\delta_1^i(Sy_{2n-1},Ty_{2n})\delta_1^i(SQx_{2n},TRx_{2n-1})$$

$$\le c_i \max\{\delta_1^i(Sy_{2n-1}Ty_{2n})\delta_2^i(Qx_{2n},Rx_{2n-1}),\delta_1^i(x_{2n-1},Sy_{2n-1})\delta_2^i(y_{2n},Qx_{2n}),$$
$$d_1^i(x_{2n},x_{2n-1})\delta_1^i(Sy_{2n-1},Ty_{2n}),\delta_1^i(Sy_{2n-1},SQx_{2n})\delta_1^i(Ty_{2n},TRx_{2n+1})\}$$

$$=c_i \max\{d_1^i(x_{2n-1},x_{2n})d_2^i(y_{2n+1},y_{2n}),[d_1^i(x_{2n-1},x_{2n})]^2\}$$

from which it follows that

$$d_1^i(x_{2n+1},x_{2n})\le c_i \max\{d_2^i(y_{2n+1},y_{2n}),d_1^i(x_{2n-1},x_{2n})\} \tag{3}$$

Let $\mathrm{U}_2 \in U_2$ be an arbitrary entourage. Since β_2 is a base for U_2, there exists $V_2(i,r)\in\beta_2$ such that $V_2(i,r)\subseteq \mathrm{U}_2$.

Similarly, applying inequality (2), we get

$$[d_2^i(y_{2n},y_{2n+1})]^2=\delta_2^i(Qx_{2n-1},x_{2n})\delta_2^i(RSy_{2n-1},QTy_{2n})$$

$$\le c_i \max\{d_1^i(x_{2n-1},x_{2n})d_2^i(y_{2n},y_{2n+1}),d_2^i(y_{2n-1},y_{2n})d_2^i(y_{2n},y_{2n+1}),d_2^i(y_{2n},y_{2n})d_2^i(y_{2n+1},y_{2n+1})\}$$

It follows that

$$d_2^i(y_{2n},y_{2n+1})\le c_i \max\{d_1^i(x_{2n-1},x_{2n}),d_2^i(y_{2n-1},y_{2n})\} \tag{4}$$

We can write as

$$d_1^i(x_{n+1},x_n)\le c_i \max\{d_2^i(y_n,y_{n+1}),d_1^i(x_{n-1},x_n)\}$$
and
$$d_2^i(y_n,y_{n+1})\le c_i \max\{d_1^i(x_{n-1},x_n),d_2^i(y_{n-1},y_n)\}$$

It now follows easily by induction that

$$d_1^i(x_n,x_{n+1})\le c_i^n \max\{d_1^i(x,x_1),d_2^i(y_1,y_2)\}$$
$$d_2^i(y_n,y_{n+1})\le c_i^{n-1} \max\{d_1^i(x,x_1),d_2^i(y_1,y_2)\}$$

for n = 1,2,3,……... Since $c_i < 1$, it follows that there exists p such that $d_1^i(x_n,x_m)<r$ and hence $(x_n,x_m)\in \mathrm{U}_1$ for all n, $m \ge p$. Therefore, sequence $\{x_n\}$ is a Cauchy sequence in d_1^i- uniformity on X. Similarly, it follows that the sequence $\{y_n\}$ is a Cauchy sequence in d_2^i- uniformity on Y.

Let $F_p=\{x_n : n\ge p\}$ for all positive integers p and let B_1 be the filter basis $\{F_p : p = 1, 2\ldots\}$. Then, since $\{x_n\}$ is a d_1^i- Cauchy sequence for each $i\in I$, it is easy to see that the filter basis B_1 is a Cauchy filter in the uniform space (X,U_1). Since (X,U_1) is a complete Hausdorff space, the Cauchy filter $\mathrm{B}_1=\{F_p\}$ converges to a unique point z in X.

Similarly, the Cauchy filter $\mathrm{B}_2=\{F_k\}$ converges to a unique point w in Y.

Applying inequality (1), we have

$$\delta_1^i(Sw,x_{2n})\delta_1^i(SQz,x_{2n+1})=\delta_1^i(Sw,Ty_{2n})\delta_1^i(SQz,TRx_{2n})$$

$$\le c_i \max\{\delta_1^i(Sw,x_{2n})\delta_2^i(Qz,y_{2n+1}),\delta_1^i(x_{2n},Sw)\delta_2^i(y_{2n},Qz),d_1^i(z,x_{2n})\delta_1^i(Sw,x_{2n}),$$

$$\delta_1^i(Sw, SQz)d_1^i(x_{2n}, x_{2n+1})\}$$

Letting n tends to infinity, we have

$$\delta_1^i(Sw,z)\delta_1^i(SQz,z) \le c_i \max\{\delta_1^i(Sw,z)\delta_2^i(Qz,w), \delta_1^i(z,Sw)\delta_2^i(w,Qz), d_1^i(z,z)\delta_1^i(Sw,z), \delta_2^i(Sw,SQz)d_1^i(z,z)\}$$

$$\le c_i\delta_1^i(Sw,z)\delta_2^i(Qz,w)$$

and so either

$$Sw = z \tag{5}$$

or

$$\delta_1^i(SQz,z) \le c_i\delta_2^i(Qz,w) \tag{6}$$

Again, applying inequality (1), we have

$$\delta_1^i(x_{2n},Tw)\delta_1^i(x_{2n+1},TRz) \le c_i \max\{\delta_1^i(x_{2n},Tw)\delta_2^i(y_{2n+1},Rz), \delta_1^i(z,x_{2n+1})\delta_2^i(w,Qx_{2n}), d_1^i(x_{2n},z)\delta_1^i(x_{2n},Tw), d_1^i(x_{2n},x_{2n+1})\delta_1^i(Tw,TRz)\}$$

Letting n tends to infinity, we have

$$\delta_1^i(z,Tw)\delta_1^i(z,TRz) \le c_i \max\{\delta_1^i(z,Tw)\delta_2^i(w,Rz), d_1^i(z,z)\delta_2^i(w,Qz), d_1^i(z,z)\delta_1^i(z,Tw),\ d_1^i(z,z)\delta_1^i(Tw,TRz)\}$$

and so either

$$Tz = w \tag{7}$$

or

$$\delta_1^i(z,TRz) \le c_i\delta_2^i(w,Rz) \tag{8}$$

Applying inequality (2), we have

$$\delta_2^i(Qz,y_{2n+1})\delta_2^i(RSw,y_{2n+1}) = \delta_2^i(Qz,Bx_{2n})\delta_2^i(RSw,Ax_{2n})$$

$$\le c_i \max\{\delta_1^i(Sw,x_{2n})\delta_2^i(Qz,y_{2n+1}), \delta_1^i(x_{2n},Sw)\delta_2^i(w,Qz), d_2^i(w,y_{2n})\delta_2^i(Qz,y_{2n+1}), \delta_2^i(Qz,RSw)d_2^i(y_{2n+1},y_{2n+1})\}$$

Letting n tends to infinity, we have

$$\delta_2^i(Qz,w)\delta_2^i(RSw,w) \le c_i\{\delta_1^i(Sw,z)\delta_2^i(Qz,w)\}$$

and so either

$$Qz = w \tag{9}$$

or

$$\delta_2^i(RSw,w) \le c_i\delta_1^i(Sw,z) \tag{10}$$

Again applying inequality (2) and letting n tend to infinity, we have

$$\delta_2^i(w,Rz)\delta_2^i(w,QTw) \le c_i\{\delta_1^i(z,Tw)\delta_2^i(w,Rz)\}$$

and so either

$$Rz = w \tag{11}$$

Or

$$\delta_2^i(w,QTw) \le c_i\delta_1^i(z,Tw) \tag{12}$$

If Q is continuous, then

$$w = \lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} Qx_{2n} = Qz$$

$$\therefore Sw = SQz$$

If inequality (6) holds, then

$$z = Sw = SQz$$

and so equation (5) will necessarily hold.

If R is continuous, then

$$w = \lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} Rx_{2n} = Rz$$

$$\therefore Tw = TRz$$

If inequality (8) holds, then

$$z = Tw = TRz$$

and so equation (7) will necessarily hold.

If S is continuous, then

$$z = \lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} Sy_{2n} = Sw$$

$$\therefore Rz = RSw$$

If inequality (10) holds, then

$$w = Rz = RSw$$

and so equation (9) will necessarily hold .

If T is continuous, then

$$z = \lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} Ty_{2n} = Tw$$

$$\therefore Qz = QTw$$

If inequality (12) holds, then

$$w = Qz = QTw$$

and so equation (11) will necessarily hold.

To prove uniqueness, suppose that SQ and TR have a second common fixed point z' in X and RS and QT have a second fixed point w' in Y.

Applying inequality (1), we have

$$[d_1^i(z,z')]^2 = [\delta_1^i(SQz, TRz')]^2$$

$$\leq c_i \max\{d_1^i(z,z')\delta_2^i(Qz,Rz'), d_1^i(z',z)\delta_2^i(Rz',Qz), [d_1^i(z,z')]^2\}$$

$$= c_i \max\{d_1^i(z,z')\delta_2^i(Qz,Rz'),[d_1^i(z,z')]^2\}$$

$$\therefore d_1^i(z,z') \le c_i\delta_2^i(Qz,Rz') \tag{13}$$

Again applying inequality (2), we have

$$\begin{aligned}[\delta_2^i(Qz,Rz')]^2 &= \delta_2^i(Qz,Rz')\delta_2^i(RSQz',QTRz)\\ &\le c_i \max\{d_1^i(z,z')\delta_2^i(Qz,Rz'),\delta_2^i(Qz',Rz)\delta_2^i(Qz,Rz'),\delta_2^i(Qz,Rz')\delta_2^i(Rz',Qz)\}\end{aligned}$$

$$\therefore \delta_2^i(Qz,Rz') \le c_i d_1^i(z,z') \tag{14}$$

From (13) and (14), it follows that

$$d_1^i(z,z') \le c_i\delta_2^i(Qz,Rz') \le c_i d_1^i(z,z') \text{ and so } z = z'$$

since $c_i < 1$, proving the uniqueness of the fixed point *z* of *SQ* and *TR*. It follows similarly that *w* is the unique common fixed point of *RS* and *QT*.

If we let *Q* and *R* be single valued mappings of *X* into *Y* and let *S* and *T* be single valued mappings of *Y* into *X*, we obtain the following result.

Corollary 2.1: *Let (X,U_1) and (X,U_2) be two complete Hausdorff uniform spaces. If Q, R be mappings of X into Y and S, T be mappings of Y into X satisfying the inequalities*

$$\begin{aligned}d_1^i(Sy,Ty')d_1^i(SQx,TRx') \le c_i \max\{&d_1^i(Sy,Ty')d_2^i(Qx,Rx'),d_1^i(x',Sy)d_2^i(y',Qx),\\ &d_1^i(x,x')d_1^i(Sy,Ty'),d_1^i(Sy,SQx)d_1^i(Ty',TRx')\}\end{aligned}$$

$$\begin{aligned}d_2^i(Qx,Rx')d_2^i(RSy,QTy') \le c_i \max\{&d_1^i(Sy,Ty')d_2^i(Qx,Rx'),d_1^i(x',Sy)d_2^i(y',Qx),\\ &d_2^i(y,y')d_2^i(Qx,Rx'),d_2^i(Qx,RSy)d_2^i(Rx',QTy')\}\end{aligned}$$

for all $i \in I$ and x, x' in X and y, y' in Y, where $0 \le c_i < 1$. If one of the mappings Q, R, S, T is continuous, then SQ and TR have a unique common fixed point z in X and RS and QT have a unique fixed point w in Y. Further, Qz = Rz = w and Sw = Tw = z.

Theorem2 .2: *Let (X,U_1) and (X,U_2) be two compact Hausdorff uniform spaces defined by $\{d_1^i, i \in I\} = P_1^*, \{d_2^i, i \in I\} = P_2^*$, and $(2^X,U_1^*),(2^Y,U_2^*)$ hyperspaces, let Q, R : $X \to 2^Y$ be continuous mappings and S, T : $Y \to 2^X$ be continuous mappings satisfying the inequalities*

$$\begin{aligned}\delta_1^i(Sy,Ty')\delta_1^i(SQx,TRx') < \max\{&\delta_1^i(Sy,Ty')\delta_2^i(Qx,Rx'),\delta_1^i(x',Sy)\delta_2^i(y',Qx),\\ &d_1^i(x,x')\delta_1^i(Sy,Ty'),\delta_1^i(Sy,SQx)\delta_1^i(Ty',TRx')\}\end{aligned} \tag{15}$$

$$\begin{aligned}\delta_2^i(Qx,Rx')\delta_2^i(RSy,QTy') < \max\{&\delta_1^i(Sy,Ty')\delta_2^i(Qx,Rx'),\delta_1^i(x',Sy)\delta_2^i(y',Qx),\\ &d_2^i(y,y')\delta_2^i(Qx,Rx'),\delta_2^i(Qx,RSy)\delta_2^i(Rx',QTy')\}\end{aligned} \tag{16}$$

for all $i \in I$ and x, x' in X and y, y' in Y. Then SQ and TR have a unique common fixed point z in X and RS and QT have a unique fixed point w in Y. Further, Qz = Rz = w and Sw = Tw = z.

Proof: Suppose first of all that the right hand side of inequalities (15) and (16) are never zero. Then the functions

$$f(x,x') = \frac{\delta_1^i(Sy,Ty')\delta_1^i(SQx,TRx')}{\max\{\delta_1^i(Sy,Ty')\delta_2^i(Qx,Rx'),\delta_1^i(x',Sy)\delta_2^i(y',Qx),d_1^i(x,x')\delta_1^i(Sy,Ty'),\delta_1^i(Sy,SQx)\delta_1^i(Ty',TRx')\}}$$

and

$$g(x,x') = \frac{\delta_2^i(Qx,Rx')\delta_2^i(RSy,QTy')}{\max\{\delta_1^i(Sy,Ty')\delta_2^i(Qx,Rx'),\delta_1^i(x',Sy)\delta_2^i(y',Qx),d_2^i(y,y')\delta_2^i(Qx,Rx'),\delta_2^i(Qx,RSy)\delta_2^i(Rx',QTy')\}}$$

are continuous and so attain their maximum values a, b respectively . It follows from inequalities (15) and (16) that $a, b < 1$. Then, with $c = \max\{a, b\}$ we see that the condition of theorem 2.1 are satisfied and so that theorem is proved in this case.

Now suppose that the right hand side of inequality (15) takes the value zero for points $x = z$ and then

$$SQz = TRz = z$$

Putting $Qz = Rz = w$, we have

$$Sw = Tw = z$$

To prove the uniqueness, suppose that z' is a second distinct common fixed point of SQ and TR and RS and QT have a second distinct common fixed point in Y.

Then using inequalities (15) and (16), we have

$$[d_1^i(z,z')]^2 = [\delta_1^i(SQz,TRz')]^2$$

$$\therefore d_1^i(z,z') \le c\delta_2^i(Qz,Rz') \tag{17}$$

and $[\delta_1^i(Qz,Rz')]^2 = \delta_2^i(Qz,Rz')\delta_2^i(RSAz',QTRz)$

$$\therefore \delta_2^i(Qz,Rz') \le cd_1^i(z,z') \tag{18}$$

From (17) and (18), we have

$d_1^i(z,z') \le c\delta_1^i(Qz,Rz') \le c^2 d_1^i(z,z')$ and so $z = z'$ since $c < 1$, proving the uniqueness of the fixed point of SQ and TR. The uniqueness of w can be proved

Similarly, This completes the proof of the theorem.

If we let Q and R be single valued mappings of X into Y and let S and T be single valued mappings of Y into X, we obtain the following result.

Corollary 2.2: *Let (X,U_1) and (X,U_2) be two compact Hausdorff uniform spaces. Let Q, R be continuous mappings of X into Y and S, T be continuous mappings of Y into X satisfying the inequalities*

$$\begin{aligned} d_1^i(Sy,Ty')d_1^i(SQx,TRx') < \max\{&d_1^i(Sy,Ty')d_2^i(Qx,Rx'), d_1^i(x',Sy)d_2^i(y',Qx),\\ &d_1^i(x,x')d_1^i(Sy,Ty'), d_1^i(Sy,SQx)d_1^i(Ty',TRx')\}\end{aligned}$$

$$\begin{aligned} d_2^i(Qx,Rx')d_2^i(RSy,QTy') < \max\{&d_1^i(Sy,Ty')d_2^i(Qx,Rx'), d_1^i(x',Sy)d_2^i(y',Qx),\\ &d_2^i(y,y')d_2^i(Qx,Rx'), d_2^i(Qx,RSy)d_2^i(Rx',QTy')\}\end{aligned}$$

for all $i \in I$ and x, x' in X and y, y' in Y for which the right-hand sides of the inequalities are positive, then SQ and TR have a unique common fixed point z in X and RS and QT have a unique fixed point w in Y. Further, Qz = Rz = w and Sw = Tw = z.

Remark: Results of [12] and [16] can be obtained by replacing metric spaces in place of uniform spaces of our results.

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