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AN EPQ MODEL PERMITTING TWO LEVELS OF TRADE CREDIT OFFERING PRICE DISCOUNT WITH TIME

***C. Sugapriya**
Dept of Mathematics, Roever Engg. College, Perambalur, Tamilnadu, India
E-mail: sugapriya_mmca@yahoo.co.in

****K. Jeyaraman**
Dean of science and humanities, PSNA college of Engg. &Tech., Tamilnadu, India

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ABSTRACT

Sugapriya and Jeyaraman (2011) discussed an EPQ model permitting two levels trade credit period without price discount. In this paper, price discount is allowed to maintain the demand. Since the price discount can be facilitated one if the production cost is known, production and demand rate is constant over an infinite planning horizon under two levels of trade credit period to determine the retailers optimal replenishment cycle time

This work is based on the investigation by Chung K.J and Huang Y.F (2003) allowing permissible delay in payments and the model developed by Huang Y.F (2004) offering the price discount.

Keywords: *EPQ, Price discount, Deterioration, Trade credit, production rate, demand rate.*

1.1 INTRODUCTION

The Economic Production Quantity model (also known as the EPQ model) is an extension of the Economic Order Quantity model. The EPQ model was developed by E.W. Taft in 1918. The difference being that the EPQ model assumes orders are received incrementally during the production process. The function of this model is to balance the inventory holding cost and the average fixed ordering cost. Economic Order Quantity is also said to be the amount of orders that minimizes total variable costs required to order and hold inventory or size of an order at which the total of procurement cost and inventory carrying cost is at minimum.

Kuo-Lung Hou (2007) discussed the EPQ model with the setup cost and process quality is functions of capital expenditure. An efficient procedure is developed to find the optimal production run length, setup cost and process quality.

Yang P. C and Wee H. M (2002) developed a single-vendor, multi-buyers production–inventory policy for a deteriorating item with a constant production and demand rate.

Suresh Kumar Goyal and Chun-Tao Chang (2008) established an appropriate model for a customer to determine its optimal special order quantity when the supplier offers a special extended permissible delay for one time only during a specified period. Liang-Yuh Ouyang et al (2006) determined the optimal replenishment policies under conditions of non- instantaneous receipt and permissible delay in payments. The optimal order quantity, order cycle and order receipt period are calculated so that the total relevant cost per unit time is minimized.

Balkhi Z.T and Benkherouf L developed (1996) a fixed production schedule for deteriorating items where demand and production are allowed to vary with time in an arbitrary way and the rate of deterioration is constant. Jamal A. M. M et al (1997) developed a model to determine an optimal ordering policy for deteriorating items under permissible delay of payment and allowable shortage.

Dave U (1987) studied an EOQ inventory model developed for deteriorating items for which the supplier allows delayed payments for setling the replenishment account. Jinn-Tsair Teng et al (2005) discussed with economic production quantity model to allow for time-varying cost and the total cost is a convex function of the number of replenishments which reduces the search for the optimal solution to find a local minimum.

**Corresponding author: *C. Sugapriya, *E-mail: sugapriya_mmca@yahoo.co.in*

The expected overall costs for an imperfect EPQ model with backlogging permitted is less than or equal to that of the one without backlogging. Yuan-Shyi Peter Chiu et al (2006) included intangible backorder cost in mathematical analysis, an optimal lot-size policy that minimizes expected total costs as well as satisfied the maximal shortage level constraint for the EPQ model.

Tai-Yue Wang and Long-Hui Chen (2007) presented a production lot size model for deteriorating items with time-varying demand. The replenishment cycle and deterioration rates are allowed to vary over a finite planning horizon.

The Retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory cycle time. The retailer's will do order less quantity to take the permissible delays in payments more frequently and difference between unit selling price per item and unit purchasing price per item also larger dealt by Yung-Fu Huang and Kuang-Hua Hsu (2007).

1.2 BASIC ASSUMPTIONS AND NOTATIONS

The notations used for the development of Economic Production Quantity (EPQ) models are listed below:

p : production rate per unit time.
d : actual demand for the product per unit time.
A : set up cost.
θ : a constant deterioration rate (unit/unit time).
h : inventory carrying cost per unit time.
r : price discount per unit time.
K : production cost per unit time.
I_e : interest earned per $ unit time.
I_p : interest charged per $ in stocks per unit time by the supplier.
M : the retailer's trade credit period offered by supplier in time units.
N : the customer's trade credit period offered by retailer in time units.
T : optimal cycle time.
T_1 : production period.
T_2 : time during which there is no production of the product i.e., $T_2 = T - T_1$.
$I_1(t)$: inventory level of the product during the production period, i.e, $0 \le t \le T_1$.
$I_2(t)$: inventory level of the product during the period when there is no production i.e., $T_1 \le t \le T_2$.
I(M) : maximum inventory level of the product.
TVC (T) : total cost per unit time.

The following assumptions are used for the development of Economic Production Quantity models.

1. The demand rate for the product is known and is finite.
2. Shortage is not allowed.
3. An infinite planning horizon is assumed.
4. Once a unit of the product is produced, it is available to meet the demand.
5. Once the product starts deterioration, the production is terminated and the Price discount is provided.
6. The deterioration follows an exponential distribution.
7. There is no replacement or repair for a deteriorated item.
8. $I_p \ge I_e$, $M \ge N$.
9. When $T \ge M$, the account is settled at T = M and the retailer starts paying for the interest charges on the items in stock with rate I_p. When $T \le M$, the account is settled at T=M and the retailer does not need to pay any interest charge.
10. The retailer can accumulate revenue and earn interest after its customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

1.3 MODEL DEVELOPMENT

The price discount and trade credit play an important role in this model. At time t =0, the inventory level is zero. The production and supply start simultaneously the inventory piles up at a rate of p-d in the interval [0, T_1]. There is no deterioration and the inventory reaches the maximum level I(M) at t= T_1. After the time T_1, the inventory starts deterioration and supply is continued at a discount rate. There is no fall in demand. When the inventory reduces to zero the production run begins. The inventory level of the product at time t over period [0 , T] has been represented by the differential equations:

$$\frac{dI_1(t)}{dt} = p - d \text{ for } 0 \le t \le T_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d \text{ for } 0 \le t \le T_2 \tag{2}$$

The boundary conditions associated with these equations are $I_1(0) = 0$ and $I_2(T_2) = 0$.

$$I_1(t) = (p-d)t \text{ for } 0 \le t \le T_1 \tag{3}$$

$$I_2(t) = \frac{d}{\theta}\left[e^{\theta(T_2 - t)} - 1\right] \text{ for } 0 \le t \le T_2 \tag{4}$$

Price discount: Price discount is offered as a fraction of production cost for the units in the period $[0, T_2]$. The equation associated with price discount represented as

$$\text{PD} = \frac{kr}{T}\int_0^{T_2} d.dt$$

$$= \frac{krdT_2}{T} \tag{5}$$

Using Sugapriya and Jeyaraman (2011) paper,the total cost per unit time is represented as TVC(T) = Production cost+ Setup cost +Holding cost +Deterioration cost +Price discount

= PC+SC+HC+DC+PD

$$\text{TVC(T)} = pk\frac{T_1}{T} + \frac{A}{T} + \frac{h}{T}\left[(p-d)\frac{T_1^2}{2} + \frac{d}{\theta^2}\left(e^{\theta T_2} - \theta T_2 - 1\right)\right] + \frac{kd\theta T_2^2}{2T} + \frac{krdT_2}{T} \tag{6}$$

From assumptions (8) and (9) there are three cases to discuss capital opportunity cost per unit time.

1.3.1 Case: 1 $(T_2 \ge M)$

Capital opportunity cost per unit time= $\frac{kI_p}{T}\int_M^{T_2} I_2(t)dt - \frac{kI_e}{T}\int_N^M dt.dt$

$$= \frac{kI_p d}{T\theta^2}\left(e^{\theta\left[\frac{p-d}{p}T - M\right]} - \theta\left[\frac{p-d}{p}T - M\right] - 1\right) - \frac{kI_e d}{2T}\left(M^2 - N^2\right)$$

Total cost per unit time, $TVC_1(T) = pk\frac{T_1}{T} + \frac{A}{T} + krd\left(\frac{T_2}{T}\right) + \frac{kd\theta T_2^2}{2T}$

$$+ \frac{h}{T}\left[(p-d)\frac{T_1^2}{2} + \frac{d}{\theta^2}\left(e^{\theta T_2} - \theta T_2 - 1\right)\right] + krd\left(\frac{T_2}{T}\right)$$

$$+ \frac{kI_p d}{T\theta^2}\left(e^{\theta\left[\frac{p-d}{p}T - M\right]} - \theta\left[\frac{p-d}{p}T - M\right] - 1\right) - \frac{kI_e d}{2T}\left(M^2 - N^2\right) \text{ if T>0} \tag{7}$$

1.3.2 Case: 2 $\left(N \leq T_2 \leq M\right)$

Capital opportunity cost per unit time= $-\frac{kI_e}{T}\left[\int_N^{T_2} dt.dt + dT_2(M - T_2)\right]$

$\therefore$ Total cost per unit time, $TVC_2(T) = pk\frac{T_1}{T} + \frac{A}{T} +$

$$\frac{h}{T}\left[(p-d)\frac{T_1^2}{2} + \frac{d}{\theta^2}\left(e^{\theta T_2} - \theta T_2 - 1\right)\right] + \frac{kd\theta T_2^2}{2T} + krd\left(\frac{T_2}{T}\right) - \frac{kI_e d}{2T}\left[2T_2 M - N^2 - T_2^2)\right] \quad (8)$$

1.3.3 Case: 3 $\left(T_2 < N\right)$

Capital opportunity cost per unit time= $-\frac{kI_e}{T}\left[\int_N^{M} dT_2.dt\right]$

$\therefore$ Total cost per unit time, $TVC_3(T) = pk\frac{T_1}{T} + \frac{A}{T} +$

$$\frac{h}{T}\left[(p-d)\frac{T_1^2}{2} + \frac{d}{\theta^2}\left(e^{\theta T_2} - \theta T_2 - 1\right)\right] + \frac{kd\theta T_2^2}{2T} + krd\left(\frac{T_2}{T}\right) - \frac{kI_e d(p-d)}{p}[M - N] \quad (9)$$

According to the above arguments,

$$TVC(T) = \begin{cases} TVC_1(T), & if \quad M \leq N \\ TVC_2(T), & if \quad N \leq T < M \\ TVC_3(T), & if \quad 0 < T < N \end{cases}$$

$TVC_1(M) = TVC_2(M)$ and $TVC_2(N) = TVC_3(N)$, $TVC(T)$ is continuous and well defined.

1.4 CONVEXITY

Here it has been shown that the three inventory functions derived in the above section are convex on their appropriate domains.

1.4.1 Theorem:

- $TVC_1(T)$ is convex on $[M, \infty)$.
- $TVC_2(T)$ is convex on $(0, \infty)$.
- $TVC_3(T)$ is convex on $(0, \infty)$.
- $TVC(T)$ is convex on $(0, \infty)$.

Before proving Theorem, the following required lemma is stated and proved.

1.4.2 Lemma:

$$e^{\theta\left(\frac{p-d}{p}T - m\right)} - 1 - \theta\left(\frac{p-d}{p}\right)Te^{\theta\left(\frac{p-d}{p}T - M\right)} + \left(\frac{p-d}{p}\right)^2\frac{T^2\theta^2}{2}e^{\theta\left(\frac{p-d}{p}T - M\right)} + \theta M - \frac{\theta^2\left(M^2 - N^2\right)}{2} > 0$$

if $T_2 \geq M$

Proof:

Let $g(T) = e^{\theta\left(\frac{p-d}{p}T-M\right)} - 1 - \theta\left(\frac{p-d}{p}\right)Te^{\theta\left(\frac{p-d}{p}T-M\right)} + \left(\frac{p-d}{p}\right)^2\frac{T^2\theta^2}{2}e^{\theta\left(\frac{p-d}{p}T-M\right)} + \theta M - \frac{\theta^2\left(M^2-N^2\right)}{2} > 0,$

Then we have $g'(T) = \left(\frac{p-d}{p}\right)^2\frac{T^2\theta^3}{2}e^{\theta\left(\frac{p-d}{p}T-M\right)}$, so g(T) is increasing on $[M,\infty)$ and g(T)>g(M) = $\frac{\theta^2\left(\frac{p-d}{p}\right)^2 N^2}{2} > 0$ if $T \geq M$. This completes the proof.

Proof of Theorem:

From equation (5.11), $TVC_1^{'}(T) = -\frac{A}{T^2} + h(p-d)\frac{d^2}{2p^2}$

$$+\frac{d(k\theta+h)}{T^2\theta^2}\left(\theta T\left[\frac{p-d}{p}\right]e^{\theta\left[\frac{p-d}{p}T\right]} - e^{\theta\left[\frac{p-d}{p}T\right]} + 1\right)$$

$$+\frac{kI_pd}{T^2\theta^2}\left(\theta T\left[\frac{p-d}{p}\right]e^{\theta\left[\frac{p-d}{p}T-M\right]} - e^{\theta\left[\frac{p-d}{p}T-M\right]} + 1 - \theta M\right) + \frac{kI_ed}{2T^2}\left(M^2-N^2\right)$$

$$TVC_1^{''}(T) = \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2}\left(e^{\theta\left[\frac{p-d}{p}T\right]}\left(1-\theta\left(\frac{p-d}{p}\right)T + \left(\frac{p-d}{p}\right)^2\frac{\theta^2T^2}{2}\right) - 1\right)$$

$$+\frac{2kI_pd}{T^3\theta^2}\begin{pmatrix} e^{\theta\left[\frac{p-d}{p}T-M\right]} - 1 - \theta T\left[\frac{p-d}{p}\right]e^{\theta\left[\frac{p-d}{p}T-M\right]} + \\ \frac{\theta^2}{2}\left[\frac{p-d}{p}\right]^2 T^2 e^{\theta\left[\frac{p-d}{p}T-M\right]} + \theta M \end{pmatrix} - \frac{kI_ed}{T^3}\left(M^2-N^2\right)$$

$$\geq \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2}\left(e^{\theta\left[\frac{p-d}{p}T\right]}e^{-\theta\left[\frac{p-d}{p}T\right]} - 1\right)$$

$$+\frac{2kI_pd}{T^3\theta^2}\begin{pmatrix} e^{\theta\left[\frac{p-d}{p}T-M\right]} - 1 - \theta T\left[\frac{p-d}{p}\right]e^{\theta\left[\frac{p-d}{p}T-M\right]} + \\ \frac{\theta^2}{2}\left[\frac{p-d}{p}\right]^2 T^2 e^{\theta\left[\frac{p-d}{p}T-M\right]} + \theta M - \frac{\theta^2}{2}\left(M^2-N^2\right) \end{pmatrix}$$

$$= \frac{2A}{T^3} + \frac{2kI_pd}{T^3\theta^2}\begin{pmatrix} e^{\theta\left[\frac{p-d}{p}T-M\right]} - 1 - \theta T\left[\frac{p-d}{p}\right]e^{\theta\left[\frac{p-d}{p}T-M\right]} + \\ \frac{\theta^2}{2}\left[\frac{p-d}{p}\right]^2 T^2 e^{\theta\left[\frac{p-d}{p}T-M\right]} + \theta M - \frac{\theta^2}{2}\left(M^2-N^2\right) \end{pmatrix}$$

Lemma imply that $\frac{d^2TVC_1(T)}{dT^2} > 0$ if T $\geq$ M. i.e., the second derivative is found to be positive. It is the basic requirement for T to be the minimum total cost in the EPQ model.

$\therefore TVC_1(T)$ is convex on [M, ∞).

$$TVC_2^{'}(T) = -\frac{A}{T^2} + h(p-d)\frac{d^2}{2p^2} + \frac{d(k\theta+h)}{T^2\theta^2}\left(\theta T\left[\frac{p-d}{p}\right]e^{\theta\left[\frac{p-d}{p}T\right]} - \theta\left[\frac{p-d}{p}T\right] + 1\right)$$
$$-\frac{kI_e d}{2T^2}\left(N^2 - T^2\right)$$

$$TVC_2^{''}(T) = \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2}\left(e^{\theta\left[\frac{p-d}{p}T\right]}\left(1-\theta\left(\frac{p-d}{p}\right)T + \left(\frac{p-d}{p}\right)^2\frac{\theta^2T^2}{2}\right) - 1\right) + \frac{kI_e dN^2}{T^3}$$
$$\geq \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2}\left(e^{\theta\left[\frac{p-d}{p}T\right]}e^{-\theta\left[\frac{p-d}{p}T\right]} - 1\right) + \frac{kI_e dN^2}{T^3}$$
$$= \frac{2A}{T^3} + \frac{kI_e dN^2}{T^3} > 0.$$

and

$$TVC_3^{'}(T) = -\frac{A}{T^2} + h(p-d)\frac{d^2}{2p^2} + \frac{d(c\theta+h)}{T^2\theta^2}\left(\theta T\left[\frac{p-d}{p}\right]e^{\theta\left[\frac{p-d}{p}T\right]} - \theta\left[\frac{p-d}{p}T\right] + 1\right).$$

$$TVC_3^{''}(T) = \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2}\left(e^{\theta\left[\frac{p-d}{p}T\right]}\left(1-\theta\left(\frac{p-d}{p}\right)T + \left(\frac{p-d}{p}\right)^2\frac{\theta^2T^2}{2}\right) - 1\right).$$

$$\geq \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2}\left(e^{\theta\left[\frac{p-d}{p}T\right]}e^{-\theta\left[\frac{p-d}{p}T\right]} - 1\right).$$
$$= \frac{2A}{T^3} > 0.$$

i.e., the second derivative is found to be positive. It is the basic requirement for T to be the minimum total cost in the EPQ model. Therefore, $TVC_2^{'}(T)$ and $TVC_3^{'}(T)$ is convex on (0, ∞), respectively.

The case 1 implies that $TVC_1^{'}(T)$ is increasing on [M, ∞). case2 and case3 implies that $TVC_2^{'}(T)$ and $TVC_3^{'}(T)$ is increasing on (0, M]. Since $TVC_1^{'}(M) = TVC_2'(M)$ and $TVC_2'(N) = TVC_3'(N)$, then $TVC'(T) =$ is increasing on T>0. Consequently TVC (T) is convex on T>0. Combining the above arguments completed the proof.

1.5 DETERMINATION OF THE OPTIMAL REPLENISHMENT CYCLE TIME T

Consider the following equations:

$$TVC_1^{'}(T) = 0 \tag{10}$$
$$TVC_2^{'}(T) = 0 \tag{11}$$
$$TVC_3^{'}(T) = 0 \tag{12}$$

If the solution of Equation (10), (11) and (12) exists, then it is unique.

1.6 CONCLUSION

This study investigates EPQ model for retailer's inventory system to minimize the cost under two level credit and price discount by determining the retailers optimal replenishment cycle time. Using teorem1 the convexity can also be proved. Theorem 1.4.1 establishes the convexity of the cost functions. The supplier permits trade credit period enhancing the demand of the retailer. The supplier can also offer price discount on the products to promote the sales.

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