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PERISTALTIC TRANSPORT OF COUPLE STRESS FLUID IN UNIFORM AND NON-UNIFORM ANNULUS THROUGH POROUS MEDIUM

V. P. Rathod[a] & *N. G. Sridhar[b]

[a]Department of studies and Research in Mathematics, Gulbarga University, Gulbarga-585 106, Karnataka, India
E-mail: rathod_vijaykumar@rediffmail.com

[b]Govt. First Grade College & P. G. Centre, Sedam-585 222, Karnataka, India
E-mail: sridhar.ng1@gmail.com

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ABSTRACT

The aim of present investigation is to study peristaltic transport through the gap between coaxial tubes, where outer tube is non uniform and inner tube is rigid. The necessary theoretical results such as viscosity, pressure gradient and friction force on inner and outer tubes have been obtained in terms of couple stress parameter through a porous medium. Out of these theoretical results the numerical solution of pressure gradient, outer friction, inert friction and flow rate are shown graphically for the better understanding.

Keywords: *Peristalsis, Couples stress fluid, Porous medium, Volume flow rate, Pressure rise and Annulus channel.*

1. INTRODUCTION:

Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a peristaltic mechanism may be involved in swallowing food through the esophagus, in urine transport from kidney to bladder through urethra, in movement of chyme in gastro–intestinal tract, in transport of spermatozoa in ductus efferentes of male reproductive tracts and in cervical canal, in movement of ovum in female fallopian tubes, in transport of lymph in lymphatic vessels, and in vasomotion of small blood vessels such as arterioles, venules and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomechanical system. Also, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces.

A number of analytical [1,8,9,10,11,12,18,24], numerical and experimental [4,21,22,25,26] studies of peristaltic flows of different fluids have been reported. A summary of most of the investigation reported up to the year 1983, has been presented by Srivastava and Srivastava [13], and some imported contribution of recent year, are reference in Srivastava and Saxsen [27]. Physiological organs are generally observed have the form of a non-uniform duct [7, 16]. In particular, the vas deferens in rhesus monkey is in the form of a diverging tube with a ration of exit to inlet dimensions of approximately four [23]. Hence, peristaltic analysis of a Newtonian fluid in a uniform geometry cannot be applied when explaining the mechanism of transport of fluid in most bio-systems. Srivastava et. al [14] and Srivastava and Srivastava [15] peristaltic transport of Newtonian and Non-Newtonian fluids in non-uniform geometries.

The fluid motion through a porous medium has been studied by many authors: Varshney [29], Raptis et al. [19], Raptis and Peridikis [20], and El-Dabe and El-Mohandis [5]. Pressure rise increases as the permeability decreases. This is because of the resistance caused by the porous medium. In the case of ureter stones this causes renal colic (ureteric colic) Ayman [2]. Habtu alemaychu and Radhakrishnamacharya [6] dispersion of a Solute in Peristaltic Motion of a couple stress fluid through a porous medium with slip condition. Studied Peristaltic Transport of a Couple Stress fluid In a Uniform and Non-Uniform Annulus by Rathod and Asha [28].

With the above discussion in mind, we propose to study peristaltic transport of a viscous incompressible fluid (creeping flow) through gap between coaxial tubes, where outer tube is non-uniform and has a sinusoidal wave traveling down its wall and inner one is a rigid, uniform tube and moving with a constant velocity. This investigation may have application in many clinical applications such as endoscopes problem. In this paper, peristaltic transport of a couple stress fluid in a uniform and non-uniform annulus through a porous medium is investigated.

*Corresponding author: *N. G. Sridhar[b], *E-mail: sridhar.ng1@gmail.com

2. FORMULATION OF THE PROBLEM:

Consider the peristaltic flow of an couple stress fluid through coaxial tubes such that the outer tubes is non-uniform and has a sinusoidal wave traveling down its wall and the inner one is rigid, uniform and moving with a constant velocity. The geometry of the wall surface are

$$r_1^* = a_1 \tag{1}$$

$$r_2^* = a_2 + bSin(\frac{2\pi}{\lambda}(x^* - ct^*)) \tag{2}$$

With $a_2(z^*) = a_{20} + kz^*$

Where a_1 is radius of inner tube, $a_2(z^*)$ is radius of outer tube at axial distance z^* form inlet, a_{20} is the radius of the outer tube at the inlet, k (<<1) is the constant whose magnitude depends on the length of the outer tube, b is the wave amplitude, λ is the wavelength, c is the propagation velocity, η is couple stress parameter, K is porous media and t is time. We choose cylindrical coordinate system (r^*, z^*) where z-axis lies along centerline of inner and outer tubes and r^* is distance measured radially.

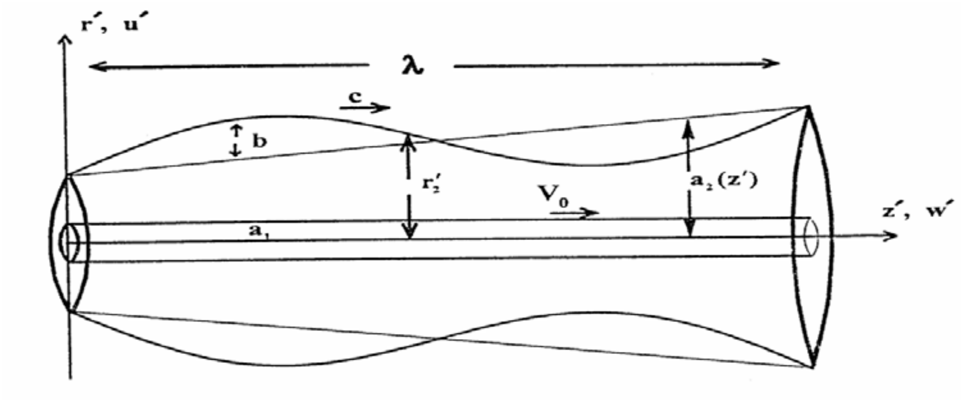

Fig.1. Geometry of the problem.

The equation of motion of flow in the gap between inner and the outer tubes are

$$\frac{1}{r^*}\frac{\partial(r^*, u^*)}{\partial r^*} + \frac{\partial(w^*)}{\partial z^*} = 0 \tag{3}$$

$$\rho\{\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial r^*} + w^*\frac{\partial u^*}{\partial z^*}\} = -\frac{\partial p^*}{\partial r^*} + \mu\{\frac{\partial}{\partial r^*}(\frac{1}{r^*}\frac{\partial(r^*, u^*)}{\partial r^*}) + \frac{\partial^2 u^*}{\partial z^{*2}}\} - \eta\nabla^2(\nabla^2(u^*)) - \frac{\mu}{K}u^* \tag{4}$$

$$\rho\{\frac{\partial w^*}{\partial t^*} + u^*\frac{\partial w^*}{\partial r^*} + w^*\frac{\partial w^*}{\partial z^*}\} = -\frac{\partial p^*}{\partial z^*} + \mu\{\frac{\partial}{\partial r^*}(\frac{1}{r^*}\frac{\partial(r^*, w^*)}{\partial r^*}) + \frac{\partial^2 w^*}{\partial z^{*2}}\} - \eta\nabla^2(\nabla^2(w^*)) - \frac{\mu}{K}w^* \tag{5}$$

Where $\nabla^2 = \{\frac{\partial}{\partial r^*}(\frac{1}{r^*}\frac{\partial(r^*)}{\partial r^*})\}$

Where u^* and w^* are velocity components in the r^* and w^* direction respectively, ρ is density, p^* is pressure and μ is viscosity, η is couple stress parameter, K is porous media.

The boundary conditions are

$$u^* = 0,\ w^* = V_0^*,\ \nabla^2(w^*)\ finite\ at\ r^* = r_1^* \tag{6a}$$

$$u^* = \frac{\partial r_2^*}{\partial t^*},\ w^* = 0,\ \nabla^2(w^*) = 0\ \ at\ r^* = r_2^* \tag{6b}$$

It is convenient to non-dimensionalize variable appearing in equations (1-6) and introducing Reynolds number Re, wave number ratio δ, and velocity parameter V_0 as follows:

$$z=\frac{z^*}{\lambda^*},\ \ r=\frac{r^*}{c},\ u=\frac{\lambda u^*}{a_{20}c},\ \ p=\frac{a_{20}^2}{\lambda\mu c}p^*(z^*),\ \ t=\frac{t^* c}{\lambda},\ \mathrm{Re}=\frac{\rho c a_{20}}{\mu},\ w=\frac{w^*}{c},K=\frac{K^*}{\lambda},\ \eta=l^2\rho\gamma,$$
$$\delta=\frac{a_{20}}{\lambda},V_0=\frac{V^*_o}{c},\ \ r_1=\frac{r_1^*}{a_{20}}=\varepsilon,\ \ \ r_2=\frac{r_2^*}{a_{20}}=1+\frac{\lambda kz}{a_{20}}+\phi\sin[2\pi(z-t)] \tag{7}$$

Where, $\phi\ \ amplitude-\dfrac{b}{a_{20}}\le 1$

The equation of motion and boundary conditions in dimensionless form becomes

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z}=0 \tag{8}$$

$$\mathrm{Re}\,\delta^3\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\}=-\frac{\partial p}{\partial r}+\delta^2\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial(ru)}{\partial r})+\delta^4\frac{\partial^2 u}{\partial z^2}-\frac{\delta^2}{\gamma^2}\nabla^2(\nabla^2(u))-\delta^2h^2u \tag{9}$$

$$\mathrm{Re}\,\delta\{\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\}=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial(rw)}{\partial r})+\delta^2\frac{\partial^2 w}{\partial z^2}-\frac{1}{\gamma^2}\nabla^2(\nabla^2(w))-h^2w \tag{10}$$

Where, $\gamma=\sqrt{\dfrac{\eta}{\mu a_{20}^2}}$ couples stress parameter & $h=\sqrt{\dfrac{a_{20}}{K}}$ porous media.

The boundary conditions are:

$$u=0,\ w=V_0,\ \ \nabla^2(u,w)\ \text{finite at } r=r_1=\varepsilon, \tag{11a}$$

$$u=\frac{\partial r_2}{\partial y},\ \ w=0,\ \nabla^2(u,w)=0,\ \ \ at\ r=r_2=1+\frac{\lambda kz}{a_{20}}+\phi\sin[2\pi(z-t)] \tag{11b}$$

Using long wavelength approximation and dropping terms of order δ it follows from equations (8)-(10) that appropriate equations describing the flow in laboratory frame of reference are

$$\frac{\partial p}{\partial r}=0, \tag{12}$$

$$\frac{\partial p}{\partial z}=(\nabla^2-h^2)w-\frac{1}{\gamma^2}\nabla^2(\nabla^2(w)) \tag{13}$$

With dimensional boundary condition

$w=V_0\ \ \nabla^2(u,w)$ finite at $r=r_1=\varepsilon$,

$$w=0\ \ \ \ \nabla^2(u,w)=0\ \ \ \ at\ r=r_2=1+\frac{\lambda kz}{a_{20}}+\phi\sin[2\pi(z-t)] \tag{14}$$

Integrating equation (13) and using boundary condition (14), one finds the expression for viscosity profile as

$$w(z,t) = -\frac{1}{4(1-\frac{h^2r^2}{4})}(\frac{\partial p}{\partial z})[(r_2^2 - r_1^2)(\frac{In(\frac{r}{r_1})}{In(\frac{r_2}{r_1})}) - r^2 + r_1^2] + \frac{1}{16\gamma^2}(\frac{\partial p}{\partial z})\{(r_2^2 - r_1^2)^2(\frac{(In(\frac{r}{r_1}))^2}{(In(\frac{r_2}{r_1}))^2}) - \tag{15}$$

$$(r^2 - r_1^2)^2\} + \frac{V_0}{In(\frac{r_2}{r_1})}In(\frac{r}{r2})(\frac{1}{4\gamma^2})\{(r_2^2 - r_1^2)(\frac{In(\frac{r}{r_1})}{In(\frac{r_2}{r_1})}) - r^2 + r_1^2\} - \{1 + \frac{(1-\frac{h^2r_1^2}{4})}{(1-\frac{h^2r^2}{4})}\}\frac{V_0}{In(\frac{r_2}{r_1})}In(\frac{r}{r_2})$$

The instantaneous volume flow rate Q (z, t) is given by

$$Q(z,t) = \int_{r_1}^{r_2} 2\pi r w dr = -\frac{\pi}{8}[-\frac{V_o}{4\gamma^2}(r_2^2 - r_1^2)\{(r_2^2 - 3r_1^2)In\frac{r_2}{r_1} - 4(r_2^2 - r_1^2)In(1-\frac{r_1}{r_2})(1 + In\frac{r_2}{r_1} -$$

$$2In(-1+\frac{r_2}{r_1})) + 4(r_2^2 - r_1^2)(-1 + In(-1+\frac{r_2}{r_1}))\} - \{((\frac{\partial p}{\partial z})(r_1 - r_2)^3(r_1 + r_2)^3(-3 + 2(In\frac{r_2}{r_1})^2 -$$

$$6(-1 + In(-1+\frac{r_2}{r_1}))In(-1+\frac{r_2}{r_1}))/(12\gamma^2(In\frac{r_2}{r_1})^2))\} + \frac{1}{In\frac{r_2}{r_1}}\{4V_o(r_1^2 - r_2^2 + 2(r_2^2 - r_1^2)In(1-\frac{r_1}{r_2})$$

$$+\frac{1}{h}((-4 + h^2r_1^2)(In(1-\frac{r_1}{r_2})(In(2 + hr_1 - hr_2) - In(2 - hr_1 + hr_2)) - Poly\log(2, \frac{h(r_1 - r_2)}{2}) +$$

$$Poly\log(2, \frac{h(-r_1 + r_2)}{2}))))\} + \frac{1}{h^4}\{4(\frac{\partial p}{\partial z})(2h^2(r_2^2 - r_1^2) + 2h^2r_1^2 In(4 + h^2(r_2^2 - r_1^2)) +$$

$$8In(-4 + h^2(r_2^2 - r_1^2)) + (\frac{1}{In\frac{r_2}{r_1}})(-2h^2(r_2^2 - r_1^2)((-In4 + In(2 + hr_1 - hr_2) + In(2 - hr_1 + hr_2))$$

$$In(-1+\frac{r_2}{r_1}) + Poly\log(2, \frac{h(r_1 - r_2)}{2}) + Poly\log(2, \frac{h(-r_1 + r_2)}{2}))))\}] \tag{16}$$

$$where, h = \sqrt{\frac{a_{20}}{K}}$$

From equation (16), we have

$$\frac{\partial p}{\partial z} = [(8Q/\pi) - \{\frac{V_o}{4\gamma^2}(r_2^2 - r_1^2)[(r_2^2 - 3r_1^2)In\frac{r_2}{r_1} - 4(r_2^2 - r_1^2)In(1-\frac{r_1}{r_2})(1 + In\frac{r_2}{r_1} - 2In(-1+\frac{r_2}{r_1})) - 4(r_2^2 - r_1^2)$$

$$(-1 + In(-1+\frac{r_2}{r_1}))) - \frac{1}{In\frac{r_2}{r_1}}(4V_o(r_1^2 - r_2^2 + 2(r_2^2 - r_1^2)In(1-\frac{r_1}{r_2}) + \frac{1}{h}((-4 + h^2r_1^2)(In(1-\frac{r_1}{r_2})(In(2 + hr_1 - hr_2)$$

$$-In(2 - hr_1 + hr_2)) - Poly\log(2, \frac{h(r_1 - r_2)}{2}) + Poly\log(2, \frac{h(-r_1 + r_2)}{2})))))\}]/[((((r_1 - r_2)^3(r_1 + r_2)^3$$

$$(-3 + 2(In\frac{r_2}{r_1})^2 - 6(-1 + In(-1+\frac{r_2}{r_1}))In(-1+\frac{r_2}{r_1}))) / (12\gamma^2(In\frac{r_2}{r_1})^2)) - \frac{4}{h^4}(2h^2(r_2^2 - r_1^2) - 2h^2r_1^2$$

$$In(4-h^2(r_2^2-r_1^2))+8In(-4+h^2(r_2^2-r_1^2))+(\frac{1}{In\frac{r_2}{r_1}})(-2h^2(r_2^2-r_1^2)((-In(4)+In(2+hr_1-hr_2)$$

$$+In(2-hr_1+hr_2))In(-1+\frac{r_2}{r_1})+Poly\log(2,\frac{h(r_1-r_2)}{2})+Poly\log(2,\frac{h(-r_1+r_2)}{2}))))] \quad (17)$$

The pressure rise $\Delta p_L(t)$ and friction force (at the wall) on outer and inner tubes $F_L^{(o)}(t)$ *and* $F_L^{(i)}(t)$ respectively, in a tube of length L, in their non-dimensional forms, are given by

$$\Delta p_L(t)=\int_0^A \frac{\partial p}{\partial z}dz \quad (18)$$

$$\Delta F_L^{(o)}(t)=\int_0^A r_2^2(-\frac{\partial p}{\partial z})dz, \quad (19)$$

$$\Delta F_L^{(i)}(t)=\int_0^A r_1^2(-\frac{\partial p}{\partial z})dz, \quad (20)$$

Where A = L / λ ,

Substituting from equation (17) in equations (18)-(20) and with $r_1=\varepsilon$ and

$r_2(z,t)=1+\frac{\lambda kz}{a_{20}}+\phi\sin[2\pi(z-t)]$, we get

$$\Delta p_L(t)=\int_0^A [(8Q/\pi)-\{\frac{V_o}{4\gamma^2}(r_2^2-r_1^2)[(r_2^2-3r_1^2)In\frac{r_2}{r_1}-4(r_2^2-r_1^2)In(1-\frac{r_1}{r_2})(1+In\frac{r_2}{r_1}-2In(-1+\frac{r_2}{r_1}))$$

$$-4(r_2^2-r_1^2)(-1+In(-1+\frac{r_2}{r_1})))-\frac{1}{In\frac{r_2}{r_1}}(4V_o(r_1^2-r_2^2+2(r_2^2-r_1^2)In(1-\frac{r_1}{r_2})+\frac{1}{h}((-4+h^2r_1^2)(In(1-\frac{r_1}{r_2})$$

$$(In(2+hr_1-hr_2)-In(2-hr_1+hr_2))-Poly\log(2,\frac{h(r_1-r_2)}{2})+Poly\log(2,\frac{h(-r_1+r_2)}{2})))))\}]/$$

$$[((((r_1-r_2)^3(r_1+r_2)^3(-3+2(In\frac{r_2}{r_1})^2-6(-1+In(-1+\frac{r_2}{r_1}))In(-1+\frac{r_2}{r_1})))/(12\gamma^2(In\frac{r_2}{r_1})^2))$$

$$-\frac{4}{h^4}(2h^2(r_2^2-r_1^2)-2h^2r_1^2In(4-h^2(r_2^2-r_1^2))+8In(-4+h^2(r_2^2-r_1^2))$$

$$+(\frac{1}{In\frac{r_2}{r_1}})(-2h^2(r_2^2-r_1^2)((-In(4)+In(2+hr_1-hr_2)+In(2-hr_1+hr_2))In(-1+\frac{r_2}{r_1})$$

$$+Poly\log(2,\frac{h(r_1-r_2)}{2})+Poly\log(2,\frac{h(-r_1+r_2)}{2}))))]dz, \quad (21)$$

$$\Delta F_L^{(o)}(t)=\int_0^A [\{-(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2\}\{(8Q/\pi)-\{\frac{V_o}{4\gamma^2}(r_2^2-r_1^2)[(r_2^2-3r_1^2)In\frac{r_2}{r_1}-4(r_2^2-r_1^2)$$
$$In(1-\frac{r_1}{r_2})(1+In\frac{r_2}{r_1}-2In(-1+\frac{r_2}{r_1}))-4(r_2^2-r_1^2)(-1+In(-1+\frac{r_2}{r_1})))-\frac{1}{In\frac{r_2}{r_1}}(4V_o(r_1^2-r_2^2+2(r_2^2-r_1^2)$$
$$In(1-\frac{r_1}{r_2})+\frac{1}{h}((-4+h^2r_1^2)(In(1-\frac{r_1}{r_2})(In(2+hr_1-hr_2)-In(2-hr_1+hr_2))-Poly\log(2,\frac{h(r_1-r_2)}{2})$$
$$+Poly\log(2,\frac{h(-r_1+r_2)}{2}))))\}\}]/[((((r_1-r_2)^3(r_1+r_2)^3(-3+2In(\frac{r_2}{r_1})^2-6(1+In(-1+\frac{r_2}{r_1}))$$
$$In(-1+\frac{r_2}{r_1})))/(12\gamma^2(In\frac{r_2}{r_1})^2))-\frac{4}{h^4}(2h^2(r_2^2-r_1^2)-2h^2r_1^2In(4-h^2(r_2^2-r_1^2))$$
$$+8In(-4+h^2(r_2^2-r_1^2))+(\frac{1}{In\frac{r_2}{r_1}})(-2h^2(r_2^2-r_1^2)((-In(4)+In(2+hr_1-hr_2)+In(2-hr_1+hr_2))$$
$$In(-1+\frac{r_2}{r_1})+Poly\log(2,\frac{h(r_1-r_2)}{2})+Poly\log(2,\frac{h(-r_1+r_2)}{2}))))]dz, \tag{22}$$

$$\Delta F_L^{(i)}(t)=\int_0^A [\{-\varepsilon^2\}\{(8Q/\pi)-\{\frac{V_o}{4\gamma^2}(r_2^2-r_1^2)[(r_2^2-3r_1^2)In\frac{r_2}{r_1}-4(r_2^2-r_1^2)In(1-\frac{r_1}{r_2})(1+In\frac{r_2}{r_1}-$$
$$2In(-1+\frac{r_2}{r_1}))-4(r_2^2-r_1^2)(-1+In(-1+\frac{r_2}{r_1})))-\frac{1}{In\frac{r_2}{r_1}}(4V_o(r_1^2-r_2^2+2(r_2^2-r_1^2)In(1-\frac{r_1}{r_2})+$$
$$\frac{1}{h}((-4+h^2r_1^2)(In(1-\frac{r_1}{r_2})(In(2+hr_1-hr_2)-In(2-hr_1+hr_2))-Poly\log(2,\frac{h(r_1-r_2)}{2})+$$
$$Poly\log(2,\frac{h(-r_1+r_2)}{2}))))\}\}]/[((((r_1-r_2)^3(r_1+r_2)^3(-3+2In(\frac{r_2}{r_1})^2-6(1+In(-1+\frac{r_2}{r_1}))$$
$$In(-1+\frac{r_2}{r_1})))/(12\gamma^2(In\frac{r_2}{r_1})^2))-\frac{4}{h^4}(2h^2(r_2^2-r_1^2)-2h^2r_1^2In(4-h^2(r_2^2-r_1^2))+$$
$$8In(-4+h^2(r_2^2-r_1^2))+(\frac{1}{In\frac{r_2}{r_1}})(-2h^2(r_2^2-r_1^2)((-In(4)+In(2+hr_1-hr_2)+In(2-hr_1+hr_2))$$
$$In(-1+\frac{r_2}{r_1})+Poly\log(2,\frac{h(r_1-r_2)}{2})+Poly\log(2,\frac{h(-r_1+r_2)}{2}))))]dz. \tag{23}$$

The limiting of equation (15-17), as r_1 tends to zero gives forms of axial velocity and pressure gradient for peristaltic flow in non-uniform tube(without endoscope, $\varepsilon=0$), are

$$w(r,z,t)=-\frac{1}{4(1-\frac{h^2r^2}{4})}(\frac{\partial p}{\partial z})[(r_2^2-r^2)]+\frac{1}{16\gamma^2}(\frac{\partial p}{\partial z})(r_2^4-r^4), \tag{24}$$

$$\frac{\partial p}{\partial z}=-\frac{h^4Q}{\pi[h^2r_2^2+4In(h^2r_2^2-4)]} \tag{25}$$

Hence, pressure rise and outer friction force, in this case respectively, take the form

$$\Delta p_L(t) = -\int_0^A \frac{h^4 Q(z,t)/\pi}{h^2(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2 + 4In(h^2(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2-4)}dz \tag{26}$$

$$\Delta F_L^{(o)}(t) = \int_0^A \frac{h^4 (Q(z,t)/\pi)(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2}{h^2(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2 + 4In(h^2(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2-4)}dz \tag{27}$$

Putting k=0 in equations (26) and (27), reduces to the expressions for pressure rise and friction force in a uniform tube. The analytical interpretation of our analysis with other theories are difficult to make at this stage, as the integral equations (26) and (27) are not solvable in closed form , neither for non-uniform nor uniform geometry (k=0).Thus, further studies of our analysis are only possible after numerical evaluation of these integrals.

3. NUMERICAL RESULT AND DISCUSSION:

To discuss the results obtained above quantitatively we shall assume form of instantaneous volume flow rate Q (z, t), periodic in (z-t) as [3, 14]

$$\frac{Q(z,t)}{\pi} = \frac{\bar{Q}}{\pi} - \frac{\phi^2}{2} + 2\phi\sin(2\pi(z-t)) + \frac{2\lambda kz}{a_{20}}\phi\sin(2\pi(z-t)) + \phi^2\sin^2(2\pi(z-t))$$

Where $\bar{Q}$ is time average of flow over one period of wave .This form of Q(z, t) has been assumed in view of fact that the constant value of Q(z,t) gives $\Delta P_L(t)$ always negative, and hence will be no pumping action. Using this form of Q (z, t), we shall now compute dimensionless pressure rise $\Delta P_L(t)$, inner friction force $F_L^{(i)}(t)$ (on inner surface) and outer friction force $F_L^{(o)}(t)$ (on outer tube) over tube length for various value of dimensionless time t, dimensionless flow average $\bar{Q}$, amplitude ratio ϕ , radius ratio ε, couple stress parameter η , K is porous media and velocity of inner tube V_0. Average rise in pressure $\Delta \bar{P}_L$, outer friction force $F_{(L)}^{-(o)}$ and inner friction force $F_L^{-(i)}(t)$ are then evaluated by averaging $\Delta P_L(t), F_L^{(o)}(t)$ and $F_L^{(i)}(t)$ over one period of wave. As integrals in equations (21)-(23) are not integrable in closed form, they are evaluated numerically using digital computer. Following Srivastava [15], we use the value of the various parameters in as:

a_{20}=1.25cm, L= λ =8.01cm, $k = \frac{3a_{20}}{\lambda}$.

Furthermore, since most routine upper gastrointestinal endoscopes are between 8-11 mm in diameter as reported Cotton and Williams [17] and radius of small intestine is 1.25 cm as reported in Srivastava [15] then radius ratio ε, take values 0.32, 0.38, and 0.44.

Fig.2. Shown the effect of porous medium of the pressure rise ($\Delta \bar{P}_L$) at $\gamma = 0.2, \phi = 0.4$, v_0 =0, $\varepsilon = 0.32, \bar{Q}$ = 0 and K = 2,2.5 & 3.It is noted that the pressure rise increases as the porous medium increases.

Figs.3 and 14 represent variation of dimensionless pressure with dimensionless time t for ϕ=0.4, V_0=0, K=2 and radius ratio ε = 0.44, 0.55, and 0.66 in the case of uniform and non-uniform tube respectively. The difference of pressure for different values of ε becomes smaller as the radius ratio increases, i.e., as inner radius of tube increases. It can also be seen that effect of increasing flow rate is to reduce pressure rise for various values of ε .

Fig.4. Represent variation of dimensionless pressure rise with dimensionless time t for ϕ =0.4, V_0= 0, ε = 0.44,K=2 and dimensionless flow average $\bar{Q}$ = 0, 0.22, 0.66. The result shows that pressure rise increases as flow average decreases.

Figs.5 and 15 represent variation of dimensionless pressure rise with dimensionless time t for ϕ =0.4, ε = 0.32,K=2 and velocity V_0= -1, 0, 1 for non-uniform and uniform tube respectively. The result shows that pressure rise increases as inner tube velocity increases, i.e., pressure rise for endoscope increases as inner tube moves in the direction of peristaltic waves.

Figs.6-13 shows inner friction force (on inner surface) and outer friction forces (on outer surface) are plotted versus dimensionless time t and flow average $\bar{Q}$ with K is porous media and velocity of inner tube V_0. The average rise in pressure. Here it is observed that as radius ration increases there is decrease in inner friction force. It is noticed that inner friction force behaves similar to outer friction force for same values of parameter. Moreover, outer friction force is greater than inner friction force at same values of parameter.

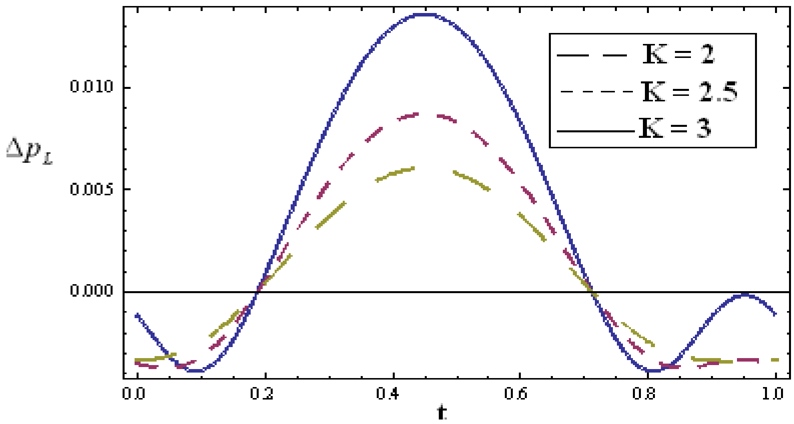

Fig.2. Variation of Pressure rise over the length of a non-uniform annulus at γ = 0.2, ϕ = 0.4, v_0 =0, ε = 0.32, $\bar{Q}$ = 0 & different values of K.

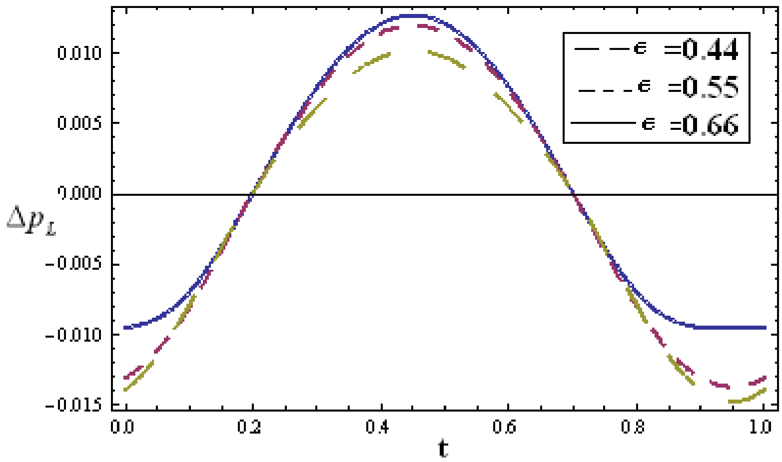

Fig.3. Variation of Pressure rise over the length of a non-uniform annulus at γ = 0.2, ϕ = 0.4, v_0 = 0, $\bar{Q}$ = 0.22, K = 2 & different values of ε .

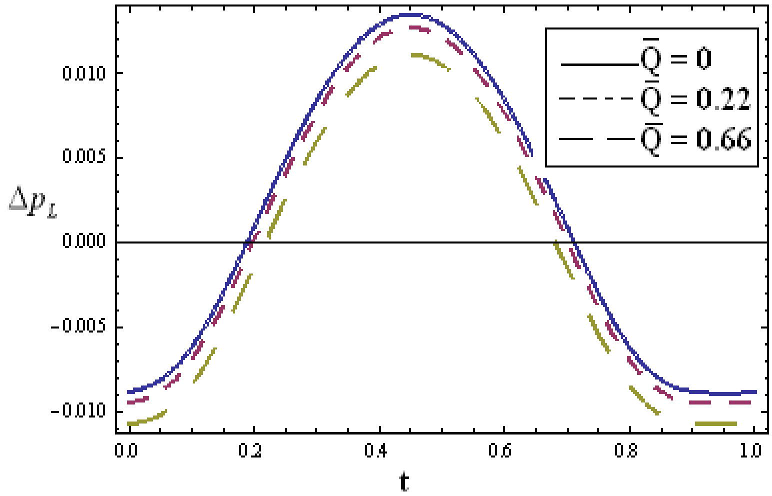

Fig.4. Variation of Pressure rise over the length of a non-uniform annulus at $\gamma = 0.2$, $\phi = 0.4$, $v_0 = 0$, $\varepsilon = 0.44$, K = 2 & different values of $\bar{Q}$.

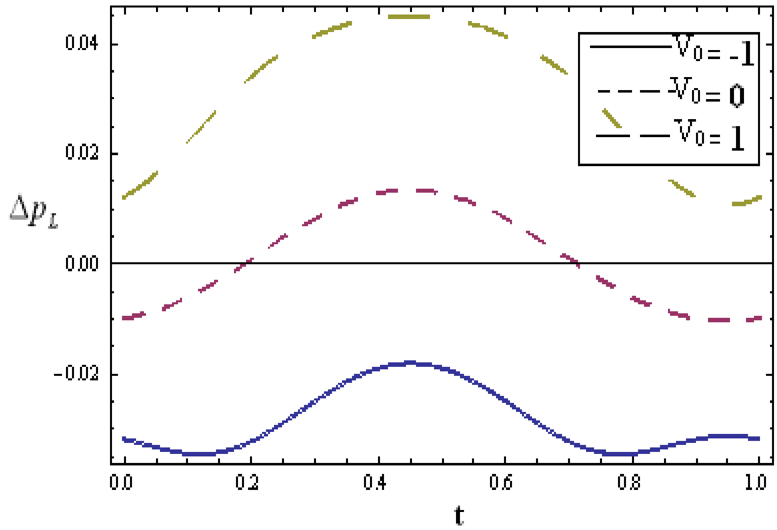

Fig.5. Variation of Pressure rise over the length of a non-uniform annulus at $\gamma = 0.2$, $\phi = 0.4$, $\gamma = 0.6$, $\varepsilon = 0.32$, K = 2, $\bar{Q} = 0$ & different values of v_0

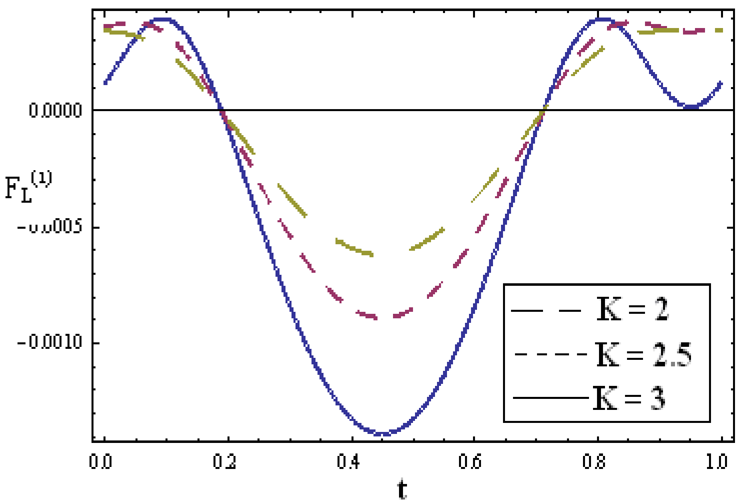

Fig.6. Variation of inner friction force of non-uniform annulus at $\gamma = 0.2, \phi = 0.4$, $v_0 = 0$, $\varepsilon = 0.32$, $\bar{Q} = 0$ & different values of K.

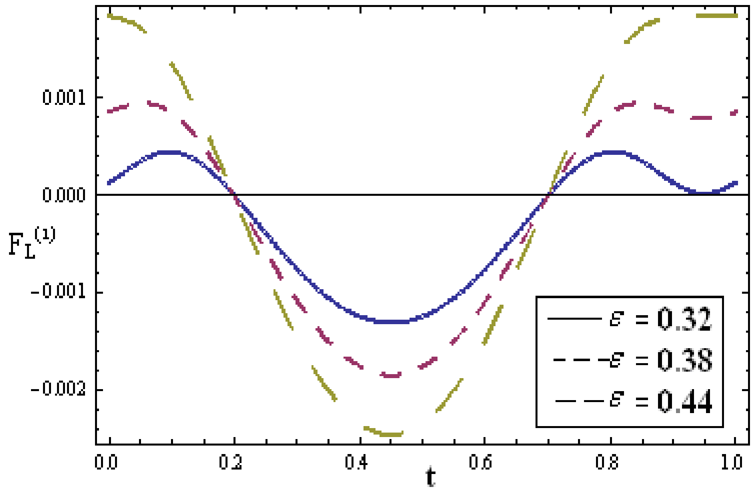

Fig.7. Variation of inner friction force of non-uniform annulus at $\gamma = 0.2, \phi = 0.4$, $v_0 = 0$, $\bar{Q} = 0.22$, K = 2 & different values of ε.

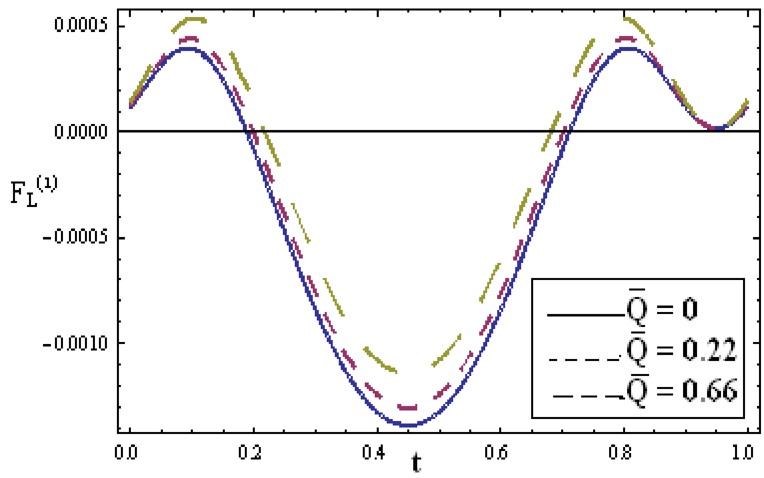

Fig.8. Variation of inner friction force of non-uniform annulus at $\gamma = 0.2$, $\phi = 0.4$, $v_0 = 0$, $\varepsilon = 0.32$, K = 2 & different values of $\bar{Q}$.

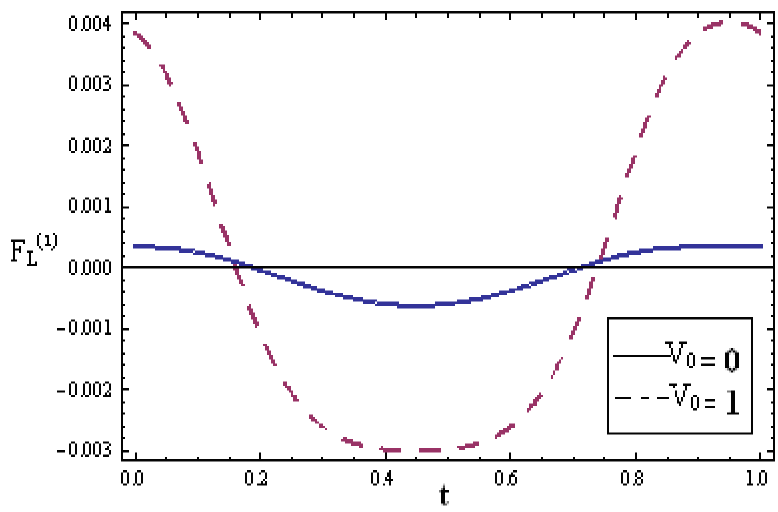

Fig.9. Variation of inner friction force of non-uniform annulus at $\gamma = 0.2$, $\phi = 0.4$, $\varepsilon = 0.32$, K = 3, $\bar{Q} = 0$ & different values of v_0.

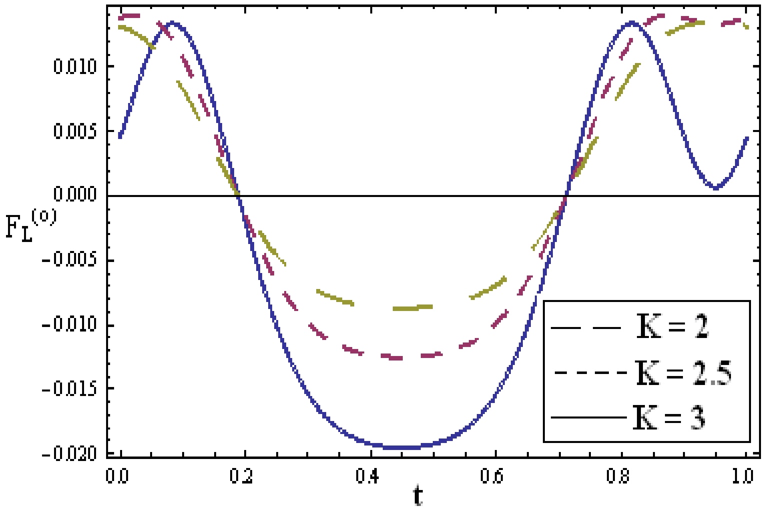

Fig.10. Variation of the outer force over the length of a non-uniform at $\gamma = 0.2$, $\phi = 0.4$, $v_0 = 0$, $\varepsilon = 0.32$, $\bar{Q} = 0$ & different values of K.

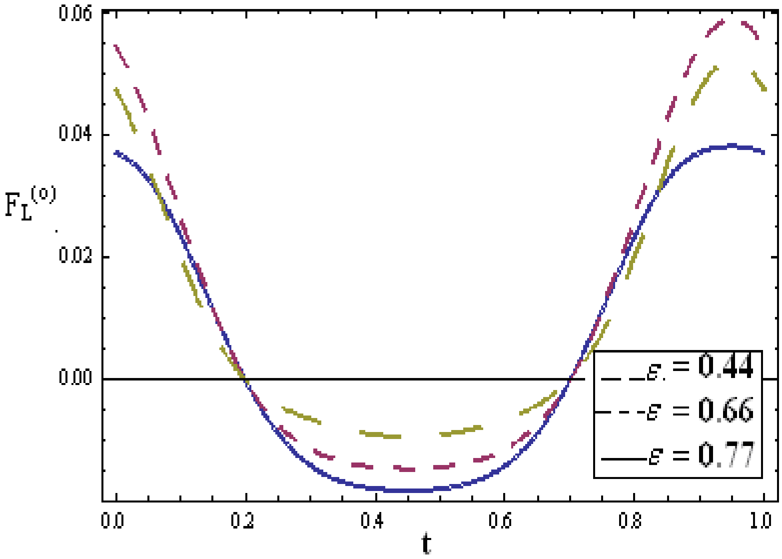

Fig.11. Variation of the outer force over the length of a non-uniform at $\gamma = 0.2$, $\phi = 0.4$. $v_0 = 0$, $\bar{Q} = 0.22$, K = 2 & different values of ε.

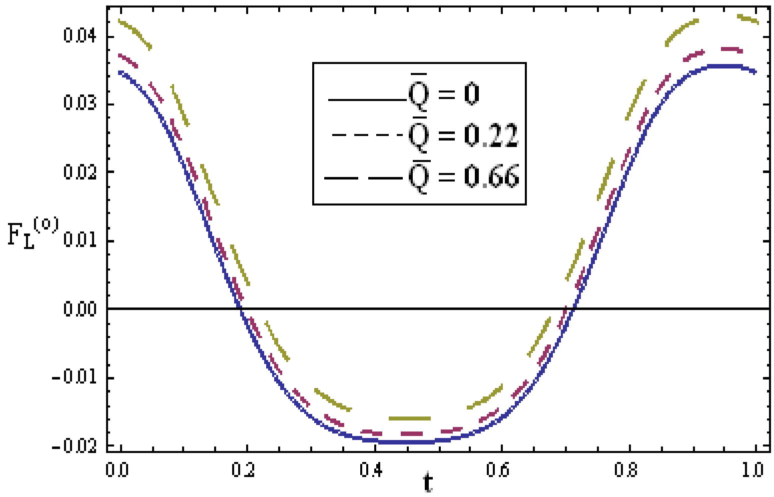

Fig.12. Variation of the outer force over the length of a non-uniform at $\gamma = 0.2$, $\phi = 0.4$, $v_0 = 0$, $\varepsilon = 0.44$, K = 2 & different values of $\bar{Q}$.

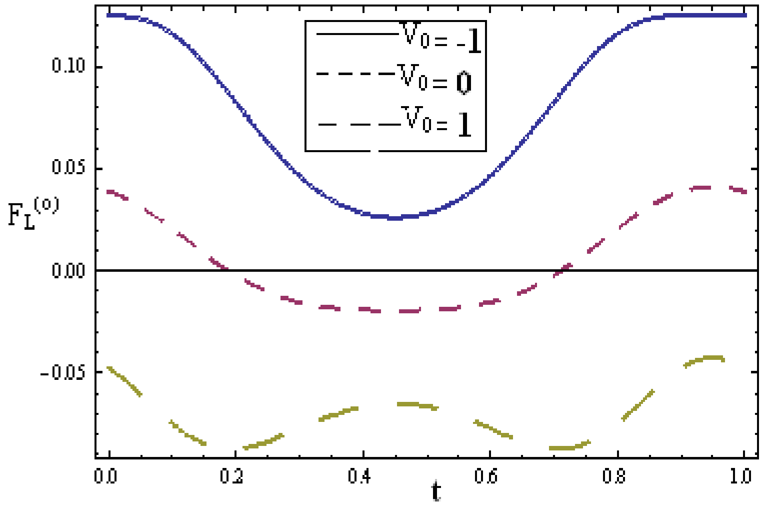

Fig.13. Variation of the outer force over the length of a non-uniform at $\gamma = 0.2$, $\phi = 0.4$, $\varepsilon = 0.32$, K = 3, $\bar{Q} = 0$ & different values of v_0

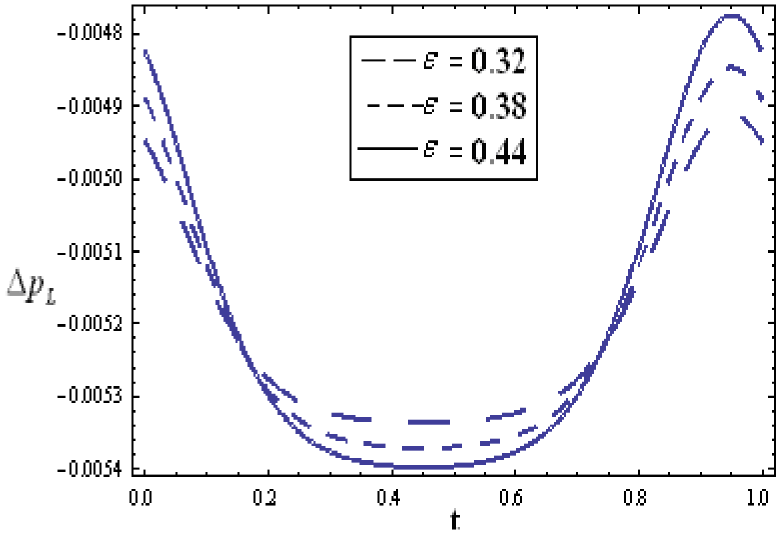

Fig.14. Variation of Pressure rise over the length of a uniform annulus at $\gamma = 0.2$, $\phi = 0.4$, $v_0 = 0$, $\bar{Q} = 0.22$, K = 2 & different values of ε .

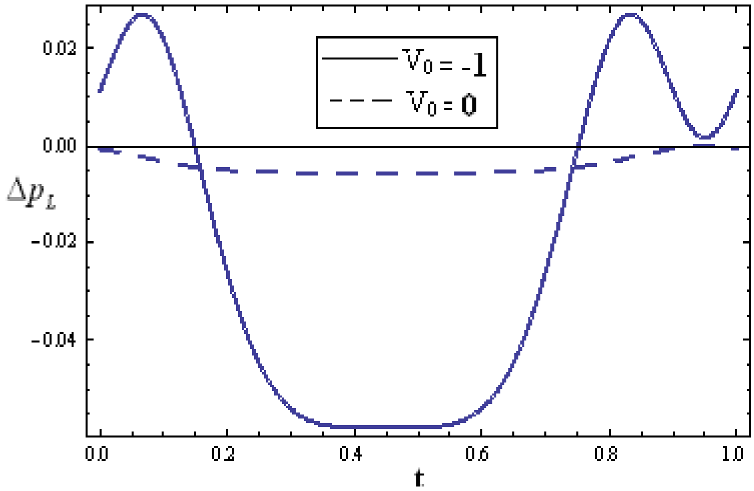

Fig.15. Variation of Pressure rise over the length of a uniform annulus at $\gamma = 0.2$, $\phi = 0.4$, $\gamma = 0.6$, $\varepsilon = 0.32$, K = 2, $\bar{Q} = 0$ & different values of v_0.

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