



## BOOLEAN LIKE SEMI RING OF FRACTIONS

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### ABSTRACT

*In this paper we construct the fractions of a Boolean like semi ring and establish that Boolean like semi ring of fraction is Boolean like ring of Foster [1].*

*Key words: Boolean like semi ring, Boolean like ring, ring of fractions.*

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### INTRODUCTION

The concept of Boolean like rings is originally due to Foster A.L. [1]. Later Swarminathan [6, 7] has extensively studied the geometry of Boolean like rings. Recently Venkateswarlu et al [8] introduced the notion of Boolean like semi rings by generalizing the concept of Boolean like rings of Foster. Venkateswarlu and Murthy [8, 9,10 & 11] have made an extensive study of Boolean like semi rings to name some:ideals, Prime ideals, Maximal ideals, nil radical and Jacobson radical of Boolean like semi rings. Further they have generalized most of the concepts of commutative theory of rings to the class of Boolean like semi rings. In fact it is observed in [8] that Boolean like semi rings are special classes of left near rings. In this paper we further investigate the theory of Boolean like semi rings by introducing the notion of fractions of Boolean like semi rings and prove that fractions of Boolean like semi rings are precisely the Boolean like rings of Foster [1]. This paper is divided into two sections. The first section is devoted to collect certain definition and results concerning Boolean like semi rings from [8]. In section 2 we introduce the notion of fractions of Boolean like rings and prove that every Boolean like semi ring of fractions is a Boolean like ring of Foster (see Theorem 2.9)

### 1. PRELIMINARIES

Here, we recall certain definitions and results on Boolean like semi rings from [8].

**Definition 1.1:** A non empty set  $R$  together with two binary operations  $+$  and  $\cdot$  satisfying the following conditions is called Boolean like semi ring;

1.  $(R, +)$  is an abelian group;
2.  $(R, \cdot)$  is a semi group;
3.  $a \cdot (b + c) = a \cdot b + a \cdot c$ ;
4.  $a + a = 0$  for all  $a$  in  $R$ ;
5.  $ab(a + b + ab) = ab$  for all  $a, b, c \in R$ .

**Lemma 1.2:** Let  $R$  be a Boolean like semi ring. Then  $a \cdot 0 = 0$  for all  $a$  in  $R$ .

**Definition 1.3:** A Boolean like semi ring  $R$  is said to be weak commutative if  $abc = acb$  for all  $a, b$  and  $c \in R$ .

**Lemma 1.4:** Let  $R$  be weak commutative Boolean like semi ring. Then  $0 \cdot a = 0$  for all  $a \in R$ .

**Lemma 1.5:** Let  $R$  be weak commutative Boolean like semi ring and let  $m$  and  $n$  be integers. Then, (i)  $a^m a^n = a^{m+n}$   
(ii)  $(a^m)^n = a^{mn}$  (iii)  $(ab)^n = a^n b^n$

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## 2. CONSTRUCTION OF BOOLEAN LIKE SEMI RING OF FRACTIONS

Here we introduce the construction of  $S^{-1}R$  from a Boolean like semi ring  $R$ .

**Definition 2.1:** A non empty subset  $S$  of  $R$  is called multiplicatively closed whenever  $a, b \in S$  implies  $ab \in S$

Now we prove the following two lemmas which we use in the sequel.

**Lemma 2.2:** Let  $R$  be a weak commutative Boolean like semi ring. Then  $ba^2 + b^2a = ba + (ba)^2 \quad \forall a, b \in R$ .

$$\begin{aligned}
 \text{Proof: } ba^2 + b^2a &= baa + bba = baa + bab && \text{(by 3 of def 1.1)} \\
 &= ba(a + b) && \text{(by 3 of def 1.1)} \\
 &= ba(b + a) && \text{(by 1 of def 1.1)} \\
 &= ba(b + a + \underline{ba + ba}) && \text{(by 4 of def 1.1)} \\
 &= bab + baa + baba + baba && \text{(by 3 of def 1.1)} \\
 &= ba(b + a + ba) + (ba)^2 && \text{(by 1 of def 1.1 and lemma 1.5)} \\
 &= ba + (ba)^2
 \end{aligned}$$

**Lemma 2.3:** Let  $R$  be a weak commutative Boolean like semi ring. Then

$$c(a + a^2)(b + b^2) = 0 \quad \forall a, b, c \in R.$$

$$\begin{aligned}
 \text{Proof: Consider } c(a + a^2)(b + b^2) &= c(a + a^2)b + c(a + a^2)b^2 && \text{(by 3 of def 1.1)} \\
 &= cb(a + a^2) + c b^2(a + a^2) && \text{(by def 1.3)} \\
 &= c[b(a + a^2) + b^2(a + a^2)] && \text{(by 3 of def 1.1)} \\
 &= c[ba + ba^2 + b^2a + b^2a^2] && \text{(by 3 of def 1.1)} \\
 &= c[(ba + b^2a^2) + (ba^2 + b^2a)] && \\
 &= c[(ba + (ba)^2) + (ba^2 + b^2a)] && \text{(by lemma 1.5)} \\
 &= c[ba + (ba)^2 + (ba + (ba)^2)] && \text{(by lemma 2.2)} \\
 &= c0 && \text{(by 4 of def 1.1)} \\
 &= 0 && \text{(by lemma 1.2)}
 \end{aligned}$$

**Remark 2.4:** In [8], it is observed that  $a + a^2$  is a nilpotent element in a Boolean like semi ring  $R$ .

**Theorem 2.5:** Let  $R$  be a weak commutative Boolean like semi ring and  $S$  be a multiplicatively closed subset of  $R$ . Define a relation  $\sim$  on  $R \times S$  by  $(r_1, s_1) \sim (r_2, s_2)$  if and only if  $\exists s \in S$  such that  $s(s_1 r_2 + s_2 r_1) = 0$ . Then  $\sim$  is an equivalence relation.

**Proof:** Let  $(r, s) \in R \times S$ . Then for any  $t$  in  $S$ , from 4 of definition 1.1 and lemma 1.2 we have that  $t(sr + sr) = t0 = 0$ .

Hence  $(r, s) \sim (r, s)$ . Thus  $\sim$  is reflexive.

Now suppose  $(r_1, s_1) \sim (r_2, s_2)$ . Then  $t(s_1 r_2 + s_2 r_1) = 0$  for some  $t$  in  $S$ .

$$\Rightarrow t(s_2 r_1 + s_1 r_2) = 0$$

$$\Rightarrow (r_2, s_2) \sim (r_1, s_1). \text{ Thus } \sim \text{ is symmetric.}$$

Finally let  $(r_1, s_1), (r_2, s_2)$  and  $(r_3, s_3) \in R \times S$  such that  $(r_1, s_1) \sim (r_2, s_2)$  and  $(r_2, s_2) \sim (r_3, s_3)$ . Then  $t[s_1r_2+s_2r_1] = 0 = t'[s_2r_3+s_3r_2]$  for some  $t$  and  $t'$  in  $S$

$$\begin{aligned} \Rightarrow t[s_1r_2+s_2r_1]s_3 &= 0 = t' [s_2r_3+s_3r_2]s_1 && \text{(by lemma 1.4)} \\ \Rightarrow ts_3[s_1r_2+s_2r_1] &= 0 = t' s_1[s_2r_3+s_3r_2] && \text{(by def 1.3)} \\ \Rightarrow t' ts_3[s_1r_2+s_2r_1] &= 0 = t' s_1[s_2r_3+s_3r_2]t && \text{(by def 2.3 \& lemma 1.4)} \\ \Rightarrow t' t s_3(s_1r_2) + t' t s_3(s_2r_1) &+ t' ts_1(s_2r_3) + t' ts_1(s_3r_2) = 0 \\ \Rightarrow [t' ts_1(s_3r_2) + t' t s_1(s_3r_2)] &+ [t' ts_2(s_1r_3) + t' ts_2(s_3r_1)] = 0 \\ \Rightarrow t' ts_2 (s_3r_1) + t' ts_2(s_1r_3) &= 0 \\ \Rightarrow t' ts_2[s_3r_1 + s_1r_3] &= 0 && \text{(by def 1.3)} \\ \Rightarrow t' ts_2 [s_1r_3+s_3r_1] &= 0 && \text{(by 1 of def 1.1 where } tt's_2 \text{ is in } S.) \end{aligned}$$

Hence we have some  $t' ts_2 \in S$  such that  $(r_1, s_1) \sim (r_3, s_3)$ . Thus  $\sim$  is an equivalence relation.

**Remark 2.6:** From the preceding theorem we denote the equivalence class containing  $(r, s)$  in  $R \times S$  by  $\frac{r}{s}$  and the set of all equivalence classes by  $S^{-1}R$ .

**Lemma 2.7:** Let  $R$  be a weak commutative Boolean like semi ring and  $S$  be a multiplicatively closed subset of  $R$ . Then

- (i) If  $0 \notin S$  and  $R$  has no zero divisors, then  $(r_1, s_1) \sim (r_2, s_2)$  if and only if  $s_1r_2 = s_2r_1$
- (ii)  $\frac{r}{s} = \frac{rt}{st} = \frac{tr}{st} = \frac{tr}{ts}$  for all  $r$  in  $R$  and for all  $s, t$  in  $S$
- (iii)  $\frac{rs}{s} = \frac{rs'}{s'}$  for all  $r$  in  $R$  and for all  $s, s'$  in  $S$ .
- (iv)  $\frac{s}{s} = \frac{s'}{s'}$  for all  $s, s'$  in  $S$ .
- (v) If  $0 \in S$ , then  $S^{-1}R$  contains exactly one element.

**Proof:** Routine.

**Theorem 2.8:** Let  $S$  be a multiplicatively closed subset in a weak commutative Boolean like semi ring  $R$ . Define binary operations  $+$  and  $\cdot$  on  $S^{-1}R$  as follows:

$$\frac{r_1}{s_1} + \frac{r_2}{s_2} = \frac{s_2r_1 + s_1r_2}{s_1s_2} \quad \text{and} \quad \frac{r_1}{s_1} \cdot \frac{r_2}{s_2} = \frac{r_1r_2}{s_1s_2}$$

Then  $(S^{-1}R, +, \cdot)$  is a Boolean like semi ring.

**Proof:** To prove  $+$  and  $\cdot$  are well defined let  $\frac{r_1}{s_1} = \frac{r'_1}{s'_1}, \frac{r_2}{s_2} = \frac{r'_2}{s'_2}$ . This implies  $t[s'_1r_1+s_1r'_1] = 0 = t'[s'_2r_2+s_2r'_2]$  for some  $t, t'$ , in  $S$ . [\*]

First we prove that  $\frac{s_2r_1+s_1r_2}{s_1s_2} = \frac{s'_2r'_1+s'_1r'_2}{s'_1s'_2}$  [i.e.  $\frac{r_1}{s_1} + \frac{r_2}{s_2} = \frac{r'_1}{s'_1} + \frac{r'_2}{s'_2}$ ]

$$\begin{aligned} \text{Consider } t' t [s'_1s'_2(s_2r_1 + s_1r_2) + s_1s_2(s'_2r'_1 + s'_1r'_2)] \\ = t' t(s'_1s'_2)(s_2r_1) + t' t(s'_1s'_2)(s_1r_2) + t' t(s_1s_2)(s'_2r'_1) + t' t(s_1s_2)(s'_1r'_2) &\text{ (by 3 of def 1.1)} \\ = t' t(s'_2s_2)(s'_1r_1) + t' t(s'_1s_1)(s'_2r_2) + t' t(s'_2s_2)(s_1r'_1) + t' t(s'_1s_1)(s_2r'_2) &\text{ (by def 1.3)} \\ = (t' t) (s'_2s_2)[(s'_1r_1) + (s_1r'_1)] + (t' t) (s'_1s_1)[(s'_2r_2) + (s_2r'_2)] \\ = t' (s'_2s_2)t[(s'_1r_1) + (s_1r'_1)] + t' [(s'_2r_2) + (s_2r'_2)] t(s'_1s_1) &\text{ (by def 1.3)} \end{aligned}$$

$$\begin{aligned}
 &= t' (s'_2 s_2)(0) + (0) t (s'_1 s_1) && \text{(by [*] above)} \\
 &= 0 + 0 && \text{(by lemmas 1.2 \& 1.4)} \\
 &= 0
 \end{aligned}$$

Hence + is well defined. Next we prove,  $s[(s'_1 s'_2)r_1 r_2 + s_1 s_2 (r'_1 r'_2)] = 0$  for some s in S.

Now,  $\frac{r_1}{s_1} = \frac{r'_1}{s'_1}$ ,  $\frac{r_2}{s_2} = \frac{r'_2}{s'_2}$  implies,

$$\begin{aligned}
 t[s'_1 r_1 + s_1 r'_1] &= 0 = t'[s'_2 r_2 + s_2 r'_2] \text{ for some } t \text{ and } t' \text{ in } S. \\
 \Rightarrow t[s'_1 r_1 + s_1 r'_1] s'_2 r_2 &= 0 = t'[s'_2 r_2 + s_2 r'_2] s_1 r'_1 && \text{(by lemma 1.4)} \\
 \Rightarrow t t' [s'_1 r_1 + s_1 r'_1] s'_2 r_2 &= 0 = t t' [s'_2 r_2 + s_2 r'_2] s_1 r'_1 && \text{(by lemma 1.2 \& 1.4)} \\
 \Rightarrow t t' (s'_2 r_2) [s'_1 r_1 + s_1 r'_1] &+ t t' (s_1 r'_1) [s'_2 r_2 + s_2 r'_2] = 0 && \text{(by def 1.3)} \\
 \Rightarrow t t' (s'_2 r_2) [s'_1 r_1] &+ t t' (s'_2 r_2) [s_1 r'_1] + t t' (s_1 r'_1) [s'_2 r_2] + t t' (s_1 r'_1) [s_2 r'_2] = 0 && \text{(by 3 of def 1.1)} \\
 \Rightarrow t t' (s'_1 s'_2) [r_1 r_2] &+ t t' [s_1 r'_1] (s'_2 r_2) + t t' (s_1 r'_1) [s'_2 r_2] + t t' [s_1 s_2] (r'_1 r'_2) = 0 && \text{(by def 1.3)} \\
 \Rightarrow t t' (s'_1 s'_2) [r_1 r_2] &+ t t' [s_1 s_2] (r'_1 r'_2) = 0 && \text{(by 4 of def 1.1)} \\
 \Rightarrow t t' [(s'_1 s'_2) [r_1 r_2] &+ [s_1 s_2] (r'_1 r'_2)] = 0
 \end{aligned}$$

Thus choose  $s = t t'$  which is also in S. Hence multiplication is also well defined. We observe the following trivial fact, before proceeding to the next.

Before proceeding to the next,

$$\frac{r_1}{s} + \frac{r_2}{s} = \frac{r_1 + r_2}{s} \text{ for any } r_1, r_2 \text{ in } R \text{ and } s \text{ in } S \text{ (from lemma 2.7 and theorem 2.8)}$$

**Now we claim that  $(S^{-1}R, +)$  is an abelian group.**

**A.** + is associative. Let  $\frac{r_1}{s_1}, \frac{r_2}{s_2}, \frac{r_3}{s_3} \in S^{-1}R$ . Then,

$$\begin{aligned}
 \frac{r_1}{s_1} + \left[ \frac{r_2}{s_2} + \frac{r_3}{s_3} \right] &= \frac{r_1}{s_1} + \left[ \frac{s_3 r_2 + s_2 r_3}{s_2 s_3} \right] = \frac{(s_2 s_3) r_1 + s_1 (s_3 r_2 + s_2 r_3)}{s_1 (s_2 s_3)} \\
 &= \frac{(s_2 r_1) s_3 + (s_1 r_2) s_3 + (s_1 s_2) r_3}{(s_1 s_2) s_3} = \frac{(s_2 r_1) s_3}{(s_1 s_2) s_3} + \frac{(s_1 r_2) s_3}{(s_1 s_2) s_3} + \frac{(s_1 s_2) r_3}{(s_1 s_2) s_3} && \text{(by lemma 2.7)} \\
 &= \frac{(s_2 r_1 + s_1 r_2) s_3}{(s_1 s_2) s_3} + \frac{(s_1 s_2) r_3}{(s_1 s_2) s_3} = \frac{s_2 r_1 + s_1 r_2}{s_1 s_2} + \frac{r_3}{s_3} && \text{(by lemma 2.7)} \\
 &= \left[ \frac{r_1}{s_1} + \frac{r_2}{s_2} \right] + \frac{r_3}{s_3}. \text{ Hence associative.}
 \end{aligned}$$

**B.** Existence of additive identity (zero element)

For any  $\frac{r}{s}$  in  $S^{-1}R$ ,  $\frac{r}{s} + \frac{0}{s} = \frac{r+0}{s}$  (by remark 3.4)

$$= \frac{r}{s} = \frac{0+r}{s} = \frac{0}{s} + \frac{r}{s}.$$

Hence  $\frac{0}{s}$  is the additive identity for any s in S.

**C.** + is commutative.

$$\begin{aligned} \text{Let } \frac{r_1}{s_1}, \frac{r_2}{s_2} \in S^{-1}R, \text{ then } \frac{r_1}{s_1} + \frac{r_2}{s_2} &= \frac{s_2r_1 + s_1r_2}{s_1s_2} = \frac{s_1r_2 + s_2r_1}{s_1s_2} \quad (\text{by 1 of def 1.1}) \\ &= \frac{s(s_1r_2 + s_2r_1)}{s(s_1s_2)}, \quad (\text{by lemma 2.7 for any } s \text{ in } S) \\ &= \frac{s(s_1r_2 + s_2r_1)}{s(s_2s_1)} = \frac{s_1r_2 + s_2r_1}{s_2s_1} \quad (\text{by def 1.3}) \\ &= \frac{r_2}{s_2} + \frac{r_1}{s_1} \end{aligned}$$

**D.** Existence of additive inverse:

$$\begin{aligned} \text{Let } \frac{r}{s} \text{ be in } S^{-1}R. \text{ Then } \frac{r}{s} + \frac{r}{s} = \frac{r+r}{s} = \frac{0}{s}. \quad (\text{by } \#) \\ &= \frac{s(s_1r_2 + s_2r_1)}{s(s_2s_1)} = \frac{s_1r_2 + s_2r_1}{s_2s_1} \quad (\text{by def 1.3}) \\ &= \frac{r_2}{s_2} + \frac{r_1}{s_1} \end{aligned}$$

Hence  $(S^{-1}R, +)$  is an abelian group.

**E.**  $(S^{-1}R, \cdot)$  is a semi group.

$$\begin{aligned} \text{Let } \frac{r_1}{s_1}, \frac{r_2}{s_2}, \frac{r_3}{s_3} \in S^{-1}R, \text{ then } \frac{r_1}{s_1} \cdot \left[ \frac{r_2}{s_2} \cdot \frac{r_3}{s_3} \right] &= \frac{r_1}{s_1} \cdot \left[ \frac{r_2r_3}{s_2s_3} \right] = \frac{r_1(r_2r_3)}{s_1(s_2s_3)} = \frac{(r_1r_2)r_3}{(s_1s_2)s_3} \quad (\text{by 2 of def 1.1}) \\ &= \left[ \frac{r_1r_2}{s_1s_2} \right] \cdot \frac{r_3}{s_3} = \left[ \frac{r_1}{s_1} \cdot \frac{r_2}{s_2} \right] \cdot \frac{r_3}{s_3} \end{aligned}$$

**F.**  $(\cdot)$  is distributive over +

$$\begin{aligned} \text{Let } \frac{r_1}{s_1}, \frac{r_2}{s_2}, \frac{r_3}{s_3} \in S^{-1}R, \text{ then} \\ \frac{r_1}{s_1} \cdot \left[ \frac{r_2}{s_2} + \frac{r_3}{s_3} \right] &= \frac{r_1}{s_1} \cdot \left[ \frac{s_3r_2 + s_2r_3}{s_2s_3} \right] = \frac{r_1(s_3r_2 + s_2r_3)}{s_1(s_2s_3)} = \frac{r_1(s_3r_2) + r_1(s_2r_3)}{s_1(s_2s_3)} = \frac{r_1(r_2s_3) + r_1(r_3s_2)}{(s_1s_2)s_3} \\ &= \frac{(r_1r_2)s_3 + (r_1r_3)s_2}{s_1(s_2s_3)} \quad (\text{by def 1.3}) \\ &= \frac{(r_1r_2)s_3}{(s_1s_2)s_3} + \frac{(r_1r_3)s_2}{(s_1s_3)s_2} = \frac{(r_1r_2)}{(s_1s_2)} + \frac{(r_1r_3)}{(s_1s_3)} \\ &= \left[ \frac{r_1}{s_1} \cdot \frac{r_2}{s_2} \right] + \left[ \frac{r_1}{s_1} \cdot \frac{r_3}{s_3} \right] \end{aligned}$$

**G.** Characteristic of  $S^{-1}R$  is 2.

$$\text{Let } \frac{r}{s} \text{ be in } S^{-1}R, \text{ then } \frac{r}{s} + \frac{r}{s} = \frac{r+r}{s} = \frac{0}{s}$$

$$\text{H. Let } \frac{r_1}{s_1}, \frac{r_2}{s_2} \in S^{-1}R. \text{ Then } \frac{r_1}{s_1} \cdot \frac{r_2}{s_2} = \frac{r_1}{s_1} \cdot \frac{r_2}{s_2} \left[ \frac{r_1}{s_1} + \frac{r_2}{s_2} + \frac{r_1}{s_1} \cdot \frac{r_2}{s_2} \right]$$

$$\text{Claim: } \frac{r_1r_2}{s_1s_2} = \frac{r_1r_2}{s_1s_2} \left[ \frac{s_2r_1 + s_1r_2 + r_1r_2}{s_1s_2} \right]$$

Let  $t \in S$  be any element. Then consider;

$$\begin{aligned} t[(s_1s_2)(r_1r_2)(s_2r_1 + s_1r_2 + r_1r_2) + (s_1s_2)(s_1s_2)(r_1r_2)] \\ &= t[(s_1s_2)(r_1r_2)(s_2r_1 + s_1r_2 + r_1r_2) + (s_1s_2)(r_1r_2)(s_1s_2)] \quad (\text{by def 1.3}) \\ &= t(s_1s_2)(r_1r_2)[(s_2r_1 + s_1r_2 + r_1r_2) + (s_1s_2)] \end{aligned}$$

$$\begin{aligned}
 &= t(s_1s_2)(r_1r_2)[r_1s_2 + s_1r_2 + r_1r_2 + s_1s_2] && \text{(by def 1.3)} \\
 &= t(s_1s_2)(r_1r_2)[r_1(s_2 + r_2) + s_1(r_2 + s_2)] && \text{(by 3 of def 1.1)} \\
 &= t(s_1s_2)(r_1r_2)(r_1 + s_1)(r_2 + s_2) \\
 &= t(s_1r_1)(s_2r_2)(s_1 + r_1)(s_2 + r_2) && \text{(by def 1.3)} \\
 &= t(s_1r_1)(s_1 + r_1)(s_2r_2)(s_2 + r_2) && \text{(by def 1.3)} \\
 &= t(s_1r_1 + (s_1r_1)^2)(s_2r_2 + (s_2r_2)^2) \\
 &= 0 && \text{(by lemma 2.2).}
 \end{aligned}$$

Hence we have that  $\frac{r_1r_2}{s_1s_2} = \frac{r_1r_2}{s_1s_2} \left[ \frac{s_2r_1 + s_1r_2 + r_1r_2}{s_1s_2} \right]$  which proves 5 of def1.1

Thus  $(S^{-1}R, +, \cdot)$  is a Boolean like semi ring.

Now we recall the following definition of A. L. Foster [1] regarding Boolean like ring

**Definition [1]:** A Boolean like ring  $R$  is a commutative ring with unity and is of characteristic 2 in which  $ab(1+a)(1+b) = 0$  for all  $a, b$  in  $R$

**Theorem 2.9:**  $(S^{-1}R, +, \cdot)$  is a Boolean like ring.

**Proof:** Let  $\frac{r_1}{s_1}, \frac{r_2}{s_2} \in S^{-1}R$ . Then  $\frac{r_1}{s_1} \cdot \frac{r_2}{s_2} = \frac{r_1r_2}{s_1s_2} = \frac{s(r_1r_2)}{s(s_1s_2)}$  (by lemma 2.7)

$$= \frac{s(r_2r_1)}{s(s_2s_1)} = \frac{(r_2r_1)}{(s_2s_1)} \quad \text{(by lemma 2.7)}$$

$$= \frac{r_2r_1}{s_2s_1} = \frac{r_2}{s_2} \cdot \frac{r_1}{s_1}$$

Thus  $\cdot$  is commutative.

Hence left distributive property also holds from commutative property of  $\cdot$  and F of theorem 2.8

**i. Multiplicative identity,**

Let  $\frac{r}{s}$  be any element. Then,

$$\frac{r}{s} \cdot \frac{s}{s} = \frac{rs}{ss} = \frac{r}{s} \quad \text{(by lemma 2.7)}$$

$$= \frac{sr}{ss} = \frac{s}{s} \cdot \frac{r}{s} \quad \text{(by lemma 2.7). Hence } \frac{s}{s} \text{ is the multiplicative identity.}$$

**ii. Let**  $\frac{r_1}{s_1}, \frac{r_2}{s_2} \in S^{-1}R$ . Then,

$$\frac{r_1}{s_1} \cdot \frac{r_2}{s_2} \left[ \frac{s_1}{s_1} + \frac{r_1}{s_1} \right] \left[ \frac{s_2}{s_2} + \frac{r_2}{s_2} \right] = \frac{(r_1r_2)(s_1+r_1)(s_2+r_2)}{(s_1s_2)(s_1s_2)} \quad \text{(by lemma 2.7)}$$

$$= \frac{s(s_1s_2)(r_1r_2)(s_1+r_1)(s_2+r_2)}{s(s_1s_2)(s_1s_2)(s_1s_2)} \quad \text{(by lemma 2.7)}$$

$$= \frac{s(s_1r_1)(s_1+r_1)(s_2r_2)(s_2+r_2)}{s(s_1s_2)(s_1s_2)(s_1s_2)}$$

$$= \frac{0}{s'} \quad \text{where } s' = s(s_1s_2)(s_1s_2)(s_1s_2) \quad \text{(by lemma 2.3)}$$

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